Computing quantifier scope readings by sequential or simultaneous expansion

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Denotation of names

- Let $E = \{a, b, c, d, e, f, g\}$ be the domain of entities.
- The denotation of a proper name is a set of all subsets of E containing a particular element of E:

$$\begin{array}{lll}
\|Ann\| & = \{X : X \subseteq E \wedge a \in X\} & = I_a \\
\|Bill\| & = \{X : X \subseteq E \wedge b \in X\} & = I_b \\
\|Chris\| & = \{X : X \subseteq E \wedge c \in X\} & = I_c \\
\|Dan\| & = \{X : X \subseteq E \wedge d \in X\} & = I_d \\
\|Ed\| & = \{X : X \subseteq E \wedge e \in X\} & = I_e \\
\|Fred\| & = \{X : X \subseteq E \wedge f \in X\} & = I_f \\
\|Greg\| & = \{X : X \subseteq E \wedge g \in X\} & = I_g
\end{array}$$

Coordination, disjunction and negation

$$
\begin{aligned}
\|Ann \text{ } and \text{ } Bill\| &= \|Ann\| \cap \|Bill\| \\
&= \{X : X \subseteq E \wedge a \in X\} \cap \{X : X \subseteq E \wedge b \in X\} \\
&= \{X : X \subseteq E \wedge a \in X \wedge b \in X\} \\
&= I_a \cap I_b
\end{aligned}
$$

$$
\begin{aligned}
\|Ann \text{ } or \text{ } Bill\| &= \|Ann\| \cup \|Bill\| \\
&= \{X : X \subseteq E \wedge a \in X\} \cup \{X : X \subseteq E \wedge b \in X\} \\
&= \{X : X \subseteq E \wedge (a \in X \vee b \in X)\} \\
&= I_a \cup I_b
\end{aligned}
$$

$$
\begin{aligned}
\|Ann \text{ } but \text{ } not \text{ } Chris\| &= \|Ann\| \cap \neg \|Chris\| \\
&= \{X : X \subseteq E \wedge a \in X\} \cap \{X : X \subseteq E \wedge c \notin X\} \\
&= \{X : X \subseteq E \wedge a \in X \wedge c \notin X\} \\
&= I_a \cap \overline{I_c}
\end{aligned}
$$

Denotation of common nouns

$$
\begin{aligned}
\|professor(s)\| &= \|Ann\| \cup \|Bill\| \cup \|Chris\| \\
&= I_a \cup I_b \cup I_c \\
&= \{X : X \subseteq E \wedge (a \in X \vee b \in X \vee c \in X)\}
\end{aligned}
$$

$$
\begin{aligned}
\|students(s)\| &= \|Dan\| \cup \|Ed\| \cup \|Fred\| \cup \|Greg\| \\
&= I_d \cup I_e \cup I_f \cup I_g \\
&= \{X : X \subseteq E \wedge (d \in X \vee e \in X \vee f \in X \vee g \in X)\}
\end{aligned}
$$

Denotation of noun phrases

$\|exactly\ \ two\ \ professors\| =$

$$\begin{aligned} = \quad & (\|Ann\| \cap \|Bill\| \cap \neg\|Chris\|) \cup \\ & (\|Ann\| \cap \neg\|Bill\| \cap \|Chris\|) \cup \\ & (\neg\|Ann\| \cap \|Bill\| \cap \|Chris\|) \end{aligned}$$

$$\begin{aligned} = \quad & (I_a \cap I_b \cap \overline{I_c}) \cup \\ & (I_a \cap \overline{I_b} \cap I_c) \cup \\ & (\overline{I_a} \cap I_b \cap I_c) \end{aligned}$$

Denotation of noun phrases

$$\|\textit{most professors}\| =$$

$$
\begin{aligned}
= \; & (\|Ann\| \cap \|Bill\| \cap \neg\|Chris\|) \cup \\
& (\|Ann\| \cap \neg\|Bill\| \cap \|Chris\|) \cup \\
& (\neg\|Ann\| \cap \|Bill\| \cap \|Chris\|) \cup \\
& (\|Ann\| \cap \|Bill\| \cap \|Chris\|)
\end{aligned}
$$

$$
\begin{aligned}
= \; & (I_a \cap I_b \cap \overline{I_c}) \cup \\
& (I_a \cap \overline{I_b} \cap I_c) \cup \\
& (\overline{I_a} \cap I_b \cap I_c) \cup \\
& (I_a \cap I_b \cap I_c)
\end{aligned}
$$

Denotation of noun phrases

$$
\begin{aligned}
\|\textit{no professor}\| &= \\
&= (\neg\|\textit{Ann}\| \cap \neg\|\textit{Bill}\| \cap \neg\|\textit{Chris}\|) \\
&= (\overline{I_a} \cap \overline{I_b} \cap \overline{I_c}) \\
&= \{X : X \subseteq E \wedge a \notin X \wedge b \notin X \wedge c \notin X\}
\end{aligned}
$$

Denotation of determiners

- If X is a set and E the domain of elements, let $\text{SGT}_D(X)$ be the set of elements d of E for which the singleton set $\{d\}$ is in X:

$$\text{SGT}_D(X) = \{d : d \in E \wedge \{d\} \in X\}$$

- $\text{SGT}_D(\|professor\|) =$
$$\begin{aligned} &= \{d : d \in E \wedge \{d\} \in \|professor\|\} \\ &= \{a, b, c\} \end{aligned}$$

- $\text{SGT}_D(\|student\|) =$
$$\begin{aligned} &= \{d : d \in E \wedge \{d\} \in \|student\|\} \\ &= \{d, e, f, g\} \end{aligned}$$

Denotation of determiners

- $\|exactly\ n\ N\| =$
  $= \{X : X \in \|N\| \wedge |X \cap \textsc{sgt}_D(\|N\|)| = n\}$
- $\|most\ N\| =$
  $= \{X : X \in \|N\| \wedge |X \cap \textsc{sgt}_D(\|N\|)| > \frac{1}{2}|\textsc{sgt}_D(\|N\|)|\}$
- $\|no\ N\| =$
  $= \{X : X \in \|N\| \wedge |X \cap \textsc{sgt}_D(\|N\|)| = 0\}$

Denotation of determiners

- $\|\mathit{exactly\ two\ professors}\| =$

$$
\begin{aligned}
&= \{X : X \in \|\mathit{professors}\| \wedge |X \cap \textsc{sgt}_D(\|\mathit{professors}\|)| = 2\} \\
&= \{X : X \in \|\mathit{professors}\| \wedge |X \cap \{a, b, c\}| = 2\} \\
&= (I_a \cap I_b \cap \overline{I_c}) \cup \\
&\quad (I_a \cap \overline{I_b} \cap I_c) \cup \\
&\quad (\overline{I_a} \cap I_b \cap I_c)
\end{aligned}
$$

Basic denotation of predicates

- $\|examined\| =$
  $= \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_a, I_f\rangle, \langle I_b, I_d\rangle, \langle I_b, I_e\rangle, \langle I_b, I_g\rangle\}$
- $\|praised\| =$
  $= \{\langle I_a, I_d\rangle, \langle I_b, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_a, I_f\rangle, \langle I_c, I_f\rangle\}$
- $\|criticised\| =$
  $= \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\}$

Sequential expansion – first projection

$\|examined\| = \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_a, I_f\rangle, \langle I_b, I_d\rangle, \langle I_b, I_e\rangle, \langle I_b, I_g\rangle\}$

- Base:
  $\langle I_a, I_d\rangle \in \|examined\|$
  therefore $\langle I_a, I_d\rangle \in \text{EXP}_1(\|examined\|)$
- Conjunction introduction:
  $\langle I_a, I_d\rangle \in \text{EXP}_1(\|examined\|)$ and $\langle I_b, I_d\rangle \in \text{EXP}_1(\|examined\|)$
  therefore by expansion of the first projection
  $\langle I_a \cap I_b, I_d\rangle \in \text{EXP}_1(\|examined\|)$
- Disjunction introduction:
  $\langle I_a, I_d\rangle \in \text{EXP}_1(\|examined\|)$
  therefore by expansion of the first projection
  $\langle I_a \cup I_b, I_d\rangle \in \text{EXP}_1(\|examined\|)$
- Negation introduction:
  $\langle I_c, I_d\rangle \notin \text{EXP}_1(\|examined\|)$
  therefore $\langle \overline{I_c}, I_d\rangle \in \text{EXP}_1(\|examined\|)$

Sequential expansion – second projection

$$\|examined\| = \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_a, I_f\rangle, \langle I_b, I_d\rangle, \langle I_b, I_e\rangle, \langle I_b, I_g\rangle\}$$

- Base:
  $\langle I_a, I_d\rangle \in \|examined\|$
  therefore $\langle I_a, I_d\rangle \in \text{EXP}_2(\|examined\|)$
- Conjunction introduction:
  $\langle I_a, I_d\rangle \in \text{EXP}_2(\|examined\|)$ and $\langle I_a, I_e\rangle \in \text{EXP}_2(\|examined\|)$
  therefore by expansion of the first projection
  $\langle I_a, I_d \cap I_e\rangle \in \text{EXP}_2(\|examined\|)$
- Disjunction introduction:
  $\langle I_a, I_d\rangle \in \text{EXP}_2(\|examined\|)$
  therefore by expansion of the first projection
  $\langle I_a, I_d \cup I_e\rangle \in \text{EXP}_2(\|examined\|)$
- Negation introduction:
  $\langle I_a, I_g\rangle \notin \text{EXP}_2(\|examined\|)$
  therefore $\langle I_a, \overline{I_g}\rangle \in \text{EXP}_2(\|examined\|)$

Definition: sequential expansion

Let $\|\phi\|$ be the denotation of a formula, and $\mathcal{X}, \mathcal{Y}$ stand for sets of subsets of E, i.e. $\mathcal{X}, \mathcal{Y} \subseteq \wp(E)$. Then $\textsc{exp}_i(\|\phi\|)$ is the smallest set of sequences such that:

1. for all σ,
   if $\sigma \in \|\phi\|$, then $\sigma \in \textsc{exp}_i(\|\phi\|)$,
2. for all σ, τ
   if $\sigma \sim_i \tau, \pi_i(\sigma) = \mathcal{X}, \pi_i(\tau) = \mathcal{Y}$ and $\sigma, \tau \in \textsc{exp}_i(\|\phi\|)$,
   then $\sigma_i[\mathcal{X}/\mathcal{X} \cap \mathcal{Y}] \in \textsc{exp}_i(\|\phi\|)$,
3. for all σ,
   if $\sigma \in \textsc{exp}_i(\|\phi\|), \pi_i(\sigma) = \mathcal{X}, \mathcal{X} \subseteq \mathcal{Y}$,
   then $\sigma_i[\mathcal{X}/\mathcal{Y}] \in \textsc{exp}_i(\|\phi\|)$,
4. $\forall\sigma\forall\mathcal{X}\forall\mathcal{Y}$:
   if $\sigma \in \textsc{exp}_i(\|\phi\|) \wedge \pi_i(\sigma) = \mathcal{X} \wedge \forall\tau.[\sigma \sim_i \tau \wedge \tau \in \textsc{exp}_i(\|\phi\|) \rightarrow \wedge\pi_i(\tau) \neq \mathcal{Y}]$,
   then $\sigma_i[\mathcal{X}/\overline{\mathcal{Y}}] \in \textsc{exp}_i(\|\phi\|)$,

Sequential expansion – an example

1. $\langle I_b, I_d\rangle \in \|examined\|$ therefore $\langle I_b, I_d\rangle \in \mathrm{EXP}_2(\|examined\|)$
2. $\langle I_b, I_e\rangle \in \|examined\|$ therefore $\langle I_b, I_e\rangle \in \mathrm{EXP}_2(\|examined\|)$
3. $\langle I_b, I_g\rangle \in \|examined\|$ therefore $\langle I_b, I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$
4. $\langle I_b, I_f\rangle \notin \mathrm{EXP}_2(\|examined\|)$ therefore $\langle I_b, \overline{I_f}\rangle \in \mathrm{EXP}_2(\|examined\|)$
5. $\langle I_b, I_d\rangle \in \mathrm{EXP}_2(\|examined\|)$ and $\langle I_b, I_e\rangle \in \mathrm{EXP}_2(\|examined\|)$ therefore $\langle I_b, I_d \cap I_e\rangle \in \mathrm{EXP}_2(\|examined\|)$
6. $\langle I_b, I_d \cap I_e\rangle \in \mathrm{EXP}_2(\|examined\|)$ and $\langle I_b, I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$
   therefore $\langle I_b, I_d \cap I_e \cap I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$
7. $\langle I_b, I_d \cap I_e \cap I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$ and $\langle I_b, \overline{I_f}\rangle \in \mathrm{EXP}_2(\|examined\|)$, therefore $\langle I_b, I_d \cap I_e \cap \overline{I_f} \cap I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$
8. $\langle I_b, I_d \cap I_e \cap \overline{I_f} \cap I_g\rangle \in \mathrm{EXP}_2(\|examined\|)$ therefore $\langle I_b, (I_d \cap I_e \cap I_f \cap \overline{I_g}) \cup (I_d \cap I_e \cap \overline{I_f} \cap I_g) \cup (I_d \cap \overline{I_e} \cap I_f \cap I_g) \cup (\overline{I_d} \cap I_e \cap I_f \cap I_g)\rangle \in \mathrm{EXP}_2(\|examined\|)$ so $\langle I_b, \|exactly\ three\ students\|\rangle \in \mathrm{EXP}_2(\|examined\|)$

Sequential expansion – an example

9. $\langle I_a, I_d\rangle \in \|examined\|$ therefore $\langle I_a, I_d\rangle \in \text{EXP}_2(\|examined\|)$
10. $\langle I_a, I_e\rangle \in \|examined\|$ therefore $\langle I_a, I_e\rangle \in \text{EXP}_2(\|examined\|)$
11. $\langle I_a, I_f\rangle \in \|examined\|$ therefore $\langle I_a, I_f\rangle \in \text{EXP}_2(\|examined\|)$
12. $\langle I_a, I_g\rangle \notin \text{EXP}_2(\|examined\|)$ therefore $\langle I_a, \overline{I_g}\rangle \in \text{EXP}_2(\|examined\|)$
13. $\langle I_a, I_d\rangle \in \text{EXP}_2(\|examined\|)$ and $\langle I_a, I_e\rangle \in \text{EXP}_2(\|examined\|)$ therefore $\langle I_a, I_d \cap I_e\rangle \in \text{EXP}_2(\|examined\|)$
14. $\langle I_a, I_d \cap I_e\rangle \in \text{EXP}_2(\|examined\|)$ and $\langle I_a, I_f\rangle \in \text{EXP}_2(\|examined\|)$
    therefore $\langle I_a, I_d \cap I_e \cap I_f\rangle \in \text{EXP}_2(\|examined\|)$
15. $\langle I_a, I_d \cap I_e \cap I_f\rangle \in \text{EXP}_2(\|examined\|)$ and $\langle I_a, \overline{I_g}\rangle \in \text{EXP}_2(\|examined\|)$, therefore $\langle I_a, I_d \cap I_e \cap I_f \cap \overline{I_g}\rangle \in \text{EXP}_2(\|examined\|)$
16. $\langle I_a, I_d \cap I_e \cap I_f \cap \overline{I_g}\rangle \in \text{EXP}_2(\|examined\|)$ therefore $\langle I_a, (I_d \cap I_e \cap I_f \cap \overline{I_g}) \cup (I_d \cap I_e \cap \overline{I_f} \cap I_g) \cup (I_d \cap \overline{I_e} \cap I_f \cap I_g) \cup (\overline{I_d} \cap I_e \cap I_f \cap I_g)\rangle \in \text{EXP}_2(\|examined\|)$ so $\langle I_a, \|exactly\ three\ students\|\rangle \in \text{EXP}_2(\|examined\|)$

Sequential expansion – an example

17. $\langle I_a, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_2(\|\textit{examined}\|)$, so
    $\langle I_a, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$
18. $\langle I_b, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_2(\|\textit{examined}\|)$, so
    $\langle I_b, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$
19. $\langle I_a, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$,
    $\langle I_b, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$, so
    $\langle I_a \cap I_b, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$
20. $\langle I_c, \|\textit{exactly three students}\|\rangle \notin \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$, so
    $\langle \overline{I_c}, \|\textit{exactly three students}\|\rangle \in \textsc{exp}_1(\textsc{exp}_2(\|\textit{examined}\|))$

Sequential expansion – an example

21. $\langle I_a \cap I_b, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$, $\langle \overline{I_c}, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$, so $\langle I_a \cap I_b \cap \overline{I_c}, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$
22. $\langle I_a \cap I_b \cap \overline{I_c}, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$, so $\langle (I_a \cap I_b \cap \overline{I_c}) \cup (I_a \cap \overline{I_b} \cap I_c) \cup (\overline{I_a} \cap I_b \cap I_c), \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$ and so $\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|examined\|))$

Sequential expansion

- $\|examined\| =$
  $= \quad \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_a, I_f\rangle, \langle I_b, I_d\rangle, \langle I_b, I_e\rangle, \langle I_b, I_g\rangle\}$

- **subject wide scope reading**

  $$\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \in$$

  $$\in \mathrm{EXP}_1(\mathrm{EXP}_2(\|examined\|))$$

  there are exactly two professors for which it holds that each examined exactly three students

- **object wide scope reading**

  $$\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \notin$$

  $$\notin \mathrm{EXP}_2(\mathrm{EXP}_1(\|praised\|))$$

  it is not the case that there are three students for which it holds that they were examined by exactly two professors each

Sequential expansion

- $\|\textit{praised}\| =$
  $= \quad \{\langle I_a, I_d\rangle, \langle I_b, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_a, I_f\rangle, \langle I_c, I_f\rangle\}$
- **subject wide scope reading**

  $$\langle \|\textit{exactly two professors}\|, \|\textit{exactly three students}\|\rangle \notin$$

  $$\notin \text{EXP}_1(\text{EXP}_2(\|\textit{praised}\|))$$

  it is not the case that there are exactly two professors for which it holds that each praised exactly three students
- **object wide scope reading**

  $$\langle \|\textit{exactly two professors}\|, \|\textit{exactly three students}\|\rangle \in$$

  $$\in \text{EXP}_2(\text{EXP}_1(\|\textit{praised}\|))$$

  there are three students for which it holds that they were each praised by exactly two professors

Simultaneous expansion

- projections are not expanded sequentially but simultaneously
- $\|criticised\| =$
  $= \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\}$
- Base:
  $\langle I_a, I_d\rangle \in \|criticised\|$
  therefore $\langle I_a, I_d\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$
- Cumulation:
  $\langle I_a \cap I_b, I_d \cap I_e \cap I_f\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$
- Completion:
  - Negation:
    $\langle I_c, I_d \cap I_e \cap I_f\rangle \notin \textrm{EXP}_{1,2}(\|criticised\|)$ therefore
    $\langle \overline{I_c}, I_d \cap I_e \cap I_f\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$
  - Conjunction:
    $\langle I_a \cap I_b \cap \overline{I_c}, I_d \cap I_e \cap I_f\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$
  - Disjunction:
    $\langle \|exactly\ two\ professors\|, I_d \cap I_e \cap I_f\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$
    $\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \in \textrm{EXP}_{1,2}(\|criticised\|)$

Definition: simultaneous expansion

Let $\Pi_i(\|\phi\|) = \bigcap\{I_x : x \in D \wedge \exists\sigma \in \|\phi\|.\pi_i(\sigma) = I_x\}$

Let ϕ be a formula of arity $n \leq a$, and $\mathcal{X}, \mathcal{Y} \subseteq \wp(D)$. Then $\mathbb{E}^{sim}(\|\phi\|)$ is the smallest set such that:

1. for all σ,
   if $\sigma \in \|\phi\|$ then $\sigma \in \mathbb{E}^{sim}(\|\phi\|)$
2. $\langle \Pi_1(\|\phi\|), \ldots, \Pi_{a+1}(\|\phi\| \rangle \in \mathbb{E}^{sim}(\|\phi\|)$
3. for all $i < n$ and all σ, τ:
   - if $\sigma \notin \mathbb{E}^{sim}(\|\phi\|)$ then $\sigma_i[\mathcal{X}/\overline{\mathcal{X}}] \in \mathbb{E}^{sim}(\|\phi\|)$
   - if $\sigma \in \mathbb{E}^{sim}(\|\phi\|), \mathcal{X} \subseteq \mathcal{Y}$,
     then $\sigma_i[\mathcal{X}/\mathcal{Y}] \in \mathbb{E}^{sim}(\|\phi\|)$
   - if $\sigma \sim_i \tau, \pi_i(\sigma) = \mathcal{X}, \pi_i(\tau) = \mathcal{Y}$ and $\sigma, \tau \in \mathbb{E}^{sim}(\|\phi\|)$,
     then $\sigma_i[\mathcal{X}/\mathcal{X} \cap \mathcal{Y}] \in \mathbb{E}^{sim}(\|\phi\|)$,

Combining verb and NP denotations – the challenge

- Assumption: the assignment of semantic roles is independent of the determination of scope relations. In particular, semantic roles can be assigned to quantified NPs without the scope relations being determined.
- Consequence: the combination of verb and NP denotations should result in an underspecified semantic entity which is then specified by means of semantic operations.
- Questions:
  - how do the verb and NP denotations combine?
  - what is the resulting underspecified semantic entity?
- If we can find such an underspecified semantic entity, then we can derive the direct scope, inverse scope and cumulative reading from the **same underspecified semantic entity** by means of sequential or simultaneous expansion
- What would then be underspecified is not the representation of the semantic entity, but the semantic entity itself.

Combining verb and NP denotations – the idea

- Let $\|v\|$ be a set of n-ary tuples, $n > 0$ (of sets of subsets of E), $\sigma_n^\perp$ be the n-ary tuple $\langle \perp_1, \ldots, \perp_n \rangle$, and $\sigma_n^\perp[i/x]$ be the result of replacing $\pi_i(\sigma_n^\perp)$ with x. Then

$$O_i(\|NP\|, \|V\|) = \mathfrak{c}(\|V\|) \cup \{\sigma_n^\perp[i/\|NP\|]\}$$

- If $\|V\|$ is a set of sets of subsets, we have:

$$O_1(\|NP\|, \|V\|) = \{\langle \|NP\| \rangle\} \cup \mathfrak{c}(\|V\|)$$

- If $\|V\|$ is a set of pairs, we have:

$$O_1(\|NP\|, \|V\|) = \{\langle \|NP\|, \perp \rangle\} \cup \mathfrak{c}(\|V\|)$$

$$O_2(\|NP\|, \|V\|) = \{\langle \perp, \|NP\| \rangle\} \cup \mathfrak{c}(\|V\|)$$

- If $\|V\|$ is a set of triples, we have:

$$O_1(\|NP\|, \|V\|) = \{\langle \|NP\|, \perp, \perp \rangle\} \cup \mathfrak{c}(\|V\|)$$

$$O_2(\|NP\|, \|V\|) = \{\langle \perp, \|NP\|, \perp \rangle\} \cup \mathfrak{c}(\|V\|)$$

$$O_3(\|NP\|, \|V\|) = \{\langle \perp, \perp, \|NP\| \rangle\} \cup \mathfrak{c}(\|V\|)$$

Truth definition

- a denotation $\|\phi\|$ of a formula ϕ is called **consistent** if and only if

$$\neg\exists\sigma\exists\sigma'\exists i\exists t.(\sigma \sim_i \sigma' \wedge \pi_i(\sigma) = t \wedge \pi_i(\sigma') = \overline{t})$$

- a denotation $\|\phi\|$ of a formula ϕ is called **proper** if and only if there is a sequence $\sigma \in \|\phi\|$ not containing $\perp$ such that for all sequences $\sigma' \in \|\phi\|$ containing at least one $\perp$ it holds that for all i:
  - if $\pi_i(\sigma') = \perp$ then $\pi_i(\sigma) \neq \perp$,
  - if $\pi_i(\sigma') \neq \perp$ then $\pi_i(\sigma') = \pi_i(\sigma)$
- a sentence ϕ is **true** iff $\|\phi\|$ has a consistent and proper expansion (i.e. iff there is an x such that (i) x is a result of expanding $\|\phi\|$, and (ii) x is consistent and proper).

Completion of verb denotations

- $\|criticised\| =$
  $= \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\}$
- $\mathfrak{c}(\|criticised\|) =$
  $= \{\langle\overline{I_a}, \overline{I_a}\rangle, \langle\overline{I_a}, \overline{I_b}\rangle, \langle\overline{I_a}, \overline{I_c}\rangle, \langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle\overline{I_a}, \overline{I_f}\rangle, \langle\overline{I_a}, \overline{I_g}\rangle,$
  $\langle\overline{I_b}, \overline{I_a}\rangle, \langle\overline{I_b}, \overline{I_b}\rangle, \langle\overline{I_b}, \overline{I_c}\rangle, \langle\overline{I_b}, \overline{I_d}\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f e\rangle, \langle\overline{I_b}, \overline{I_g}\rangle,$
  $\ldots\}$
- The **completion** of a verb denotation (i.e. of a set of *n*-ary tuples of I_x, with $x \in E$) is defined as follows:

  $$\mathfrak{c}(X) = X \cup \{\langle\overline{I_{x_1}}, \ldots, \overline{I_{x_n}}\rangle : x_1, \ldots, x_n \in E \wedge \langle I_{x_1}, \ldots, I_{x_n}\rangle \notin X\}$$

Why completion?

- Assume that $\|laughed\| = \{\langle I_b\rangle, \langle I_c\rangle\}$, and that $\|Ann\| = I_a$, so that the sentence *Ann laughed.* is false. Without completion we get:

$$O_1(I_a, \{\langle I_b\rangle, \langle I_c\rangle\}) = \{\langle I_a\rangle, \langle I_b\rangle, \langle I_c\rangle\}$$

  This denotation is both consistent and proper, so the sentence is wrongly predicted to be true.

- $O_1(I_a, \mathfrak{c}(\{\langle I_b\rangle, \langle I_c\rangle\}))$
  $= O_1(I_a, \{\langle\overline{I_a}\rangle, \langle I_b\rangle, \langle I_c\rangle, \langle\overline{I_d}\rangle, \langle\overline{I_e}\rangle, \langle\overline{I_f}\rangle, \langle\overline{I_g}\rangle\}$
  $= \{\langle I_a\rangle, \langle\overline{I_a}\rangle, \langle I_b\rangle, \langle I_c\rangle, \langle\overline{I_d}\rangle, \langle\overline{I_e}\rangle, \langle\overline{I_f}\rangle, \langle\overline{I_g}\rangle\}$

  This denotation is inconsistent, so the sentence *Ann laughed.* is predicted to be false.

Combining verb and NP denotations – an example

- $\|criticised\| =$
  $= \quad \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\}$

- $O_2(\|exactly\ three\ students\|, \|criticised\|) =$
  $\{\langle \bot, \|exactly\ three\ students\|\rangle\} \cup \mathfrak{c}(\|criticised\|)$

  $O_1(\|exactly\ two\ professors\|, \|criticised\ exactly\ three\ students\|) =$
  $\{\langle \|exactly\ two\ professors\|, \bot\rangle\} \cup$
  $\{\langle \bot, \|exactly\ three\ students\|\rangle\} \cup \mathfrak{c}(\|criticised\|)$

- $\text{EXP}_{1,2}(\{\langle \|exactly\ two\ professors\|, \bot\rangle\} \cup$
  $\{\langle \bot, \|exactly\ three\ students\|\rangle\} \cup \mathfrak{c}(\|criticised\|))$ is true, because
  $\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \in$
  $\text{EXP}_{1,2}(\{\langle \|exactly\ two\ professors\|, \bot\rangle\} \cup$
  $\{\langle \bot, \|exactly\ three\ students\|\rangle\} \cup \mathfrak{c}(\|criticised\|))$

Comparision with other theories

- denotation of NPs
- denotation of verbs
- syntactic ambiguity
- underspecified denotational (as opposed to representational) semantics
- various readings derived from the same underspecified semantic entity

Thank you!