CALCULATIONS ON ADDITION CHAINS

(For notation and motivation, see D. Knuth, Art of Computer Programming, Vol. 2, Section 4.6.3.)

The following new values have been found, based on an evaluation of $\ell(n)$ for $n \leq 18269$:

$$c(16) = \min\{n \mid \ell(n) = 16\} = 3583$$
$$c(17) = 6271$$
$$c(18) = 11231 = 11 \cdot 1021$$

$$d(13) = \operatorname{card}\{n \mid \ell(n) = 13\} = 772$$
$$d(14) = 1382$$
$$d(15) = 2481$$

$\ell(n) = \ell(2n)$ for $2n$ = 382, 1402, 1486, 2222, 2778, 2958, 4206, 4430, 4750, 5362, 5902, 8562, 8846, 8982, 9486, 10674, 11034, 11790, 12638, 12734, 12982, 13406, 13502, 14494, 14638, 14926, 14962, 15142, 15502, 15818, 15986, 16086, 16166, 16538, 16850, 17074, 17678, 17706, 17942, ...

$\ell(n) < \ell(p) + \ell(n/p) - 1$ (where p is least prime factor of n) and $< \ell(n - 1)$ for n = 5527, 7583, 7643, 8147, 10519, 10667, 13751, 14011, 14207, 14827, 15767, 16211, 17107, 18133, ...

No cases were observed where $\ell(n) = \ell(mn)$ for $m > 2$.

We have $\ell(2^n - 1) = \ell(n) + n - 1$ for $1 \leq n \leq 14$.

$\ell^*(n) = \ell(n) + 1$ for n = 12509, 13207, 13705, 15473, 16537, ... It is remarkable that there are only 8 chains of length 17 for $n = 16537$, namely 1, 2, 4, 8, 9, 16, 32, 64, 128, 256, 512, (521 or 1024), 1033, 2066, 4132, 8264, (8273 or 16528), 16537; and 1, 2, 4, 8, 16, 17, 32, 64, 128, 256, 512, 1024, (1041 or 2048), 2065, 4130, 8260, (8277 or 16520), 16537. These are essentially given by Hansen's construction, where $n = 2^{14} + (2^3+1)(2^4+1)$.

By contrast, there are 82 addition chains of length 16 for $n = 2^{13} + (2^3+1)(2^4+1) = 8345$, including star chains such as the factor-method chain 1, 2, 4, 5, 10, 15, 25, 50, 65, 130, 260, 520, 1040, 2080, 4160, 8320, 8345.

The above calculations were done over a long period of time during which the computer broke down occasionally, so it is possible (though unlikely) that some of the answers are erroneous. The algorithm by which the calculations were done will be described in Knuth's Vol. 4.

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