Proof-Theoretic Semantics for a Natural Language Fragment*

Nissim Francez† and Roy Dyckhoff‡

July 10, 2007

1 Introduction

We propose a *Proof-Theoretic Semantics (PTS)* for a fragment E_0^+ (delineated below, and extended in the sequel) of *Natural Language (NL)*. This semantics is intended to be *incorporated into type-logical grammars(TLG)* [4], constituting an alternative to the traditional *model-theoretic semantics (MTS)*, originating in Montague's seminal work [3], used in TLG. The essence of our proposal is:
1. *For sentences, replace* **truth conditions** *(in arbitrary models) by* **canonical derivability conditions** *(from suitable assumptions).* In particular, this involves a "dedicated" proof-system (in natural deduction form), based on which the derivability conditions obtain. The system should be *harmonious*, in that its rules satisfy certain balance between introduction and elimination, in order to qualify as meaning conferring. Two notions of harmony are shown to be satisfied by the proposed rules. The approach put forward here is different from a related one by Ranta (e.g., [9]), who relates NL constructs to constructive type-theory in Martin-Löf's theory.
2. For *sub-sentential phrases*, down to lexical units (words), replace their denotations (in arbitrary models) as conferring meaning, by their **contributions** to the meanings (i.e. derivability conditions) of sentences in which they occur. This adheres to Frege's *context principle*, the latter made more specific by the incorporation into a TLG. For lack of space, extraction of sub-sentential phrase meanings is not shown here.
To the best of our knowledge, there has been no attempt[1] to develop PTS as part of a grammar for NL. The following quotation from [13] (p. 2) emphasizes this lack of applicability to NL, the original reason for considering PTS to start with:

> Although the *"meaning as use"* approach has been quite prominent for half a century now and provided one of the cornerstones of philosophy of language, in particular of ordinary language philosophy, it has never become prevailing in the *formal* semantics of artificial and natural languages. In formal semantics, the *denotational* approach which starts with interpretations of singular terms and predicates, then fixes the meaning of sentences in terms of truth conditions, and finally defines logical consequence as truth preservation under all interpretations, has always dominated.

In attempting to incorporate PTS into the grammar of NL, we are not *committing* ourselves to the accompanying philosophical position w.r.t. semantics, as put forward prominently by, for example, Dummett and Brandom [discussion in the full paper].

*Work carried out while the first author was on leave at St Andrews university
†Computer Science dept., the Technion-IIT, Haifa, Israel (francez@cs.technion.ac.il)
‡School of Computer Science, University of St Andrews, Scotland, UK (rd@st-andrews.ac.uk)
[1] [1] provides natural deduction for English, some similar to the rules here, at least in spirit; however they are not claimed to confer meaning.

There are some differences in the way our PTS is conceived, due to the difference between E_0 and traditional formal calculi for which ND-systems were proposed in logic as a basis for PTS.
– Logical calculi are *recursive*, in that each operator (connective, quantifier) is applied to (one or two) formulas of the calculus. Thus, there is a natural notion of the *dominant* (or *main*) operator which is being introduced/eliminated from a formula. In E_0^+, on the other hand, there is no such notion of a dominant operator. Furthermore, the operators are introduced *according to their grammatical function*; for example, 'every' may be introduced either into the subject or into the object of a transitive verb, or into both.
– Formal calculi are usually taken to be *semantically unambiguous*, while E_0^+ (and NL in general) is semantically ambiguous. In a PTS, the semantic ambiguity manifests itself via different derivation (from same assumptions). [The full paper will show how traditional *scope ambiguity* manifests itself in PTS].
– Since E_0^+ has no non-trivial (formal) theorems (derived from no assumptions) [shown in the full paper], a different notion of *PTS-validity* ([7],[12]) is needed [presented in the full paper].

2 The natural deduction proof system

2.1 The NL core fragment E_0^+

The fragment E_0^+ has sentences headed by intransitive and transitive verbs, and noun phrases with a noun and a determiner (proper names are included in the full paper). In addition, there is the copula (later, adjectives and relative clauses are added). This is a typical fragment of many NLs, syntactically focusing on *subcategorization*, and semantically focusing on *predication* and *quantification*. Some typical sentences: every/some girl smiles, some/every girl is a student, every/some girl loves every/some boy. Anticipating the TLG for E_0^+, we refer to expressions such as every X, some Y as *np*s (noun-phrases). In E_0^+, *np*s can be endowed with *grammatical functions (gf)* within sentences in a fairly obvious way, fitting the pre-theoretic use of this term: *subj* (subject) and *obj* (object).

The proof system N_0^+ is defined over a language L_0^+, extending (and schematizing over) E_0^+. We use $X, Y, ...$ to schematize over nouns, P, Q to schematize over intransitive verbs, and R to schematize over transitive verbs. In addition, L_0^+ has a countable set P of *individual parameters*, ranged over by meta-variables (in boldface font)n**j, r**. We refer to L_0^+ sentences as *pseudo-sentences*. Pseudo-sentences inherit the grammatical functions in the obvious way. For an L_0^+ pseudo-sentence S, we use the notation $S_{[np/gf]}$ to indicate that S has *np* in its *gf*-position. For example, $S_{[every\ X/subj]}$ indicates a pseudo-sentence with subject *every* X. These pseudo-sentences include *every* X P, *every* X R *every/some* Y and *every* X R **j**. Similarly, $S_{[\mathbf{j}/obj]}$ indicates a pseudo-sentence with object **j**, like *some* X R **j**, or **r** R **j**. Also, $S_{[-/gf]}$ denotes a sentence "missing" an *np* in its *gf*-position (e.g., $S_{[-/subj]}$ is a verb-phrase). A pseudo-sentence is *ground* if it has parameters in *all* its *gf*-positions, otherwise *non-ground*.

2.2 The natural deduction proof-system N_0^+

We start with a BHK-like justification of the forthcoming rules.

- every [**evidence transforming**]: A proof of $S_{[every\ X/gf]}$ is a function mapping each proof of **j** *is an* X (for some *fresh* **j**) into a proof of $S_{[\mathbf{j}/gf]}$. For example, a proof of *every* X P is a function mapping each proof of **j** *is an* X to a proof of **j** P. Similarly, a proof of *every* X R **r** is a *function* mapping each proof of **j** *is an* X to a proof of **j** R **r**.

$$\Gamma, S \vdash S \quad (Ax)$$

$$\frac{\Gamma, [\mathbf{j}\ is\ an\ X]_i \vdash S_{[\mathbf{j}/gf]}}{\Gamma \vdash S_{[every\ X/gf]}}\ (eI^i_{gf}) \quad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ X \quad \Gamma \vdash S_{[\mathbf{j}/gf]}}{\Gamma \vdash S_{[some\ X/gf]}}\ (sI_{gf})$$

$$\frac{\Gamma \vdash S_{[every\ X/gf]} \quad \Gamma \vdash \mathbf{j}\ is\ an\ X}{\Gamma \vdash S_{[\mathbf{j}/gf]}}\ (eE_{gf}) \qquad \frac{\Gamma \vdash S_{[some\ X/gf]} \quad \Gamma, [\mathbf{j}\ is\ an\ X]_j, [S_{[\mathbf{j}/gf]}]_i \vdash S'}{\Gamma \vdash S'}\ (sE^{i,j}_{gf})$$

where $\mathcal{F}(\Gamma\ S_{[every\ X/gf]},\ \mathbf{j})$ in (eI_{gf}), and $\mathcal{F}(\Gamma\ S_{[some\ X/gf]}S',\ \mathbf{j})$ in (sE_{gf}).

Figure 1: The meta-rules for N_0^+

- some [**evidence combining**]: A proof of $S_{[some\ X/gf]}$ is a pair of proofs, one of $\mathbf{j}$ *is an* X and the other of $S_{[\mathbf{j}/gf]}$. For example, a proof of *some* X P is a pair of proofs, one of $\mathbf{j}$ *is an* X, and the other of $\mathbf{j}$ P. A proof of *some* X R $\mathbf{r}$ is a pair of proofs, one of $\mathbf{j}$ *is an* X and the other of $\mathbf{j}$ R $\mathbf{r}$.

We next present the ND-rules for N_0^+. As is traditional, we enclose *discharged assumptions* in (indexed) square brackets, using the index to mark the rule-application responsible for the discharge. The presentation is in Gentzen-style *ND*, using (single consequent) sequents (with shared contexts), formed over L_0^+ pseudo-sentences. There is an *introduction-rule* and *elimination-rule* for each determiner forming an *np*, in each gf. For a (finite) set Γ of L_0^+ sentences and a parameter $\mathbf{j}$, let $\mathcal{F}(\Gamma, \mathbf{j})$ mean that $\mathbf{j}$ is *fresh* for Γ, i.e. not mentioned in Γ.
We present in Figure 1 the N_0^+-rules via *meta-rules*, parameterized by grammatical functions (gf). In the rule names, we abbreviate '*every*' and '*some*' to '*e*' and '*s*', respectively. For example, the meta-rule (eI^i_{gf}) generates rules (eI^i_{subj}), (eI^i_{obj}). We assume the usual definition of a (tree-shaped) ND-derivation. A derivation is *canonical* if it ends with an application of an introduction-rule. Let $\Gamma \vdash S$ denote derivability of S from Γ, with $\mathcal{D}^{\Gamma \vdash S}$ denoting such a derivation; $\Gamma \vdash^c S$ and $\mathcal{D}^{\Gamma \vdash^c S}$ stand for the canonical counterpart. Also, $[\![S]\!]^*_\Gamma$ denotes the collection of all derivations of S from Γ (possibly empty).
Below we present a sample N_0^+ non-canonical) derivation, establishing

$$some\ U\ is\ an\ X,\ every\ X\ R\ some\ Y,\ every\ Y\ is\ a\ Z \vdash some\ U\ R\ some\ Z$$

$$\frac{\dfrac{some\ U\ is\ an\ X \quad \dfrac{[\mathbf{r}\ is\ a\ U]_1 \quad \dfrac{[\mathbf{r}\ is\ an\ X]_2 \quad every\ X\ R\ some\ Y}{\mathbf{r}\ R\ some\ Y}\ (eE_{subj})}{some\ U\ R\ some\ Y}\ (sI_{subj})}{some\ U\ R\ some\ Y}\ (sE^{1,2}_{subj}) \quad \dfrac{[some\ U\ R\ \mathbf{j}]_3 \quad \dfrac{[\mathbf{j}\ is\ a\ Y]_4 \quad every\ Y\ is\ a\ Z}{\mathbf{j}\ is\ a\ Z}\ (eE_{subj})}{(some\ U\ R\ some\ Z}\ (sI_{obj})}{some\ U\ R\ some\ Z}\ (sE^{3,4}_{obj})$$

Lemma: If $\Gamma \vdash S$, than $\Gamma, \Gamma' \vdash S$.

3 The sentential proof-theoretic meaning

In the discussions of PTS in logic, it is usually stated that '*the ND-rules* **determine** *the meanings (of the connectives/quantifiers)*'. However, no *explicit* denotational meaning ("semantic value") is defined (*proof-theoretic*, not *model-theoretic*, denotation). In other words, there is no explicit definition of the result of this determination. If one wants to apply Frege's context principle to those PTS-meanings, and derive meanings for sub-sentential phrases (including lexical words) as *contributions to sentential meanings*, such an explication is needed. We take here the PTS-meaning of an E_0^+ sentence S, as well as of an L_0^+ non-ground pseudo-sentence S, to be the function from contexts Γ returning the collection of all

the *canonical derivations* in N_0^+ of S from Γ. In accordance with many views in philosophy of language, such a collection of derivations can be viewed as providing $G[\![S]\!]$, *grounds of asserting* S.
Definition (PTS-meaning, grounds): For a sentence S, or a non-ground pseudo-sentence S, in L_0^+:

$$[\![S]\!]^{PTS}_{L_0^+} =^{df.} \lambda\Gamma.[\![S]\!]_\Gamma \quad [= \lambda\Gamma.\{\mathcal{D}^{\Gamma\vdash^c S}\}] \qquad G[\![S]\!] =^{df.} \{\Gamma \mid \Gamma\vdash^c S\}$$

where:

1. For S a sentence in E_0^+, Γ consists of E_0^+-sentences only. Parameters are not "observable" in grounds for assertion.
2. For S a pseudo-sentence in L_0^+, Γ may also contain pseudo-sentences with parameters.

The set of canonical derivations of *ground pseudo-sentences* **is** *given***, and meaning is defined relative to them!**
[The full paper shows how to reconstruct in PTS the scope ambiguity inherent in E_0^+].

4 Properties of N_0^+

The origin of PTS for logic is already in the work of Gentzen [2], who invented the *natural deduction* proof-system for 1st-order logic. He hinted there, that introduction-rules could be seen as the *definition* of the logical constant serving as the main connective, while the elimination-rules are nothing more than *consequences of this definition*. This was later refined into the *Inversion Principle* by Prawitz, which shows how the introduction-rules determine the elimination-rules. The introduction-rules were taken as a definition of *the meaning* of logical constants, instead of model-theoretic interpretation, that appeals to *truth in a model*.
However, in view of Prior's [8] attack on this approach, by presenting a connective '*tonk*', whose introduction-rule was that of a disjunction, while its elimination-rule was that of conjunction, trivializing the whole deductive theory by rendering every two sentences inter-derivable, it became apparent that not every combination of ND-rules can serve as a basis for PTS.
The property of *harmony* of the ND-rules, taken in a broad sense to express a certain balance between elimination and introduction rules (absent in the *tonk* rules) became a serious contender for an appropriateness condition for ND-rules to serve as a basis for a PTS. One approach to this property, known also as *intrinsic harmony*, was already relied upon by Prawitz's observations about "reducing proofs", whereby every proof with a *maximal formula*, i.e. one resulting from an application of an introduction-rule, immediately followed by an application of its elimination-rule, can be simplified to a more "direct" form. Prawitz showed that for FOL proofs *normalize*, avoiding any such "detour". See [10, 11] for a critical discussion of *tonk*'s disharmony, and a different conception of harmony as a condition for admitting deduction-rules as defining meaning, based on what is called in [6] *generalized elimination-rules*. According to this approach, an elimination-rule allows drawing an *arbitrary conclusion*, provided it is derivable from the premisses of the introduction-rule. We show N_0^+ satisfies both criteria.
Intrinsic Harmony One way to explicate harmony is by the following properties of rules [5], complementing Prawitz's approach:
Local Soundness: Every introduction followed *directly* by an elimination can be reduced. This shows that the elimination-rules are not *too strong* w.r.t. the introduction-rules. This is basically Prawitz's normalizability.
Local Completeness: There is a way to eliminate and to reintroduce. This shows that the elimination-rules are not *too weak* w.r.t. the introduction-rules. Here, introduction/elimination refer to the *same* gf. These properties lead to *reduction* and *expansion* transformations on derivations.

$$\frac{}{\Gamma, S \vdash S}\ (ID)$$

$$\frac{\Gamma, \mathbf{j}\ is\ an\ X, S_{[every\ X/gf]}, S_{[\mathbf{j}/gf]} \vdash S'}{\Gamma, \mathbf{j}\ is\ an\ X, S_{[every\ X/gf]} \vdash S'}\ (eL_{gf}) \qquad \frac{\Gamma, \mathbf{j}\ is\ an\ X \vdash S_{[\mathbf{j}/gf]}}{\Gamma \vdash S_{[every\ X/gf]}}\ (eR_{gf}) \quad (fresh\ \mathbf{j})$$

$$\frac{\Gamma, \mathbf{j}\ is\ an\ X, S_{[\mathbf{j}/gf]} \vdash S'}{\Gamma, S_{[some\ X/gf]} \vdash S'}\ (sL_{gf}) \quad (fresh\ \mathbf{j}) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ X \quad \Gamma \vdash S_{[\mathbf{j}/gf]}}{\Gamma \vdash S_{[some\ X/gf]}}\ sR_{gf}$$

Figure 2: A sequent-calculus for N_0^+

The reduction and expansion for every are shown below. [The full paper also shows them for some.]

$$\frac{\dfrac{\dfrac{\dfrac{[\mathbf{j}\ is\ an\ X]_i}{\mathcal{D}_1}}{S_{[\mathbf{j}/gf]}}}{S_{[every\ X/gf]}}\ (eI^i_{gf}) \quad \dfrac{\mathcal{D}_2}{\mathbf{k}\ is\ an\ X}}{S_{[\mathbf{k}/gf]}}\ (Ee_{gf}) \quad \leadsto_r \quad \dfrac{\dfrac{\mathcal{D}_2}{\mathbf{k}\ is\ an\ X}}{\dfrac{\mathcal{D}_1[\mathbf{k}/\mathbf{j}]}{S_{[\mathbf{k}/gf]}}} \qquad \dfrac{\mathcal{D}}{S_{[every\ X/gf]}} \quad \leadsto_e \quad \dfrac{\dfrac{\dfrac{\mathcal{D}}{S_{[every\ X/gf]}} \quad [\mathbf{j}\ is\ an\ X]_i}{S_{[\mathbf{j}/gf]}}\ eE_{gf}}{S_{[every\ X/gf]}}\ eI^i_{gf}$$

Generalized-Elimination based Harmony According to this view ([10, 11]), harmony is captured by the elimination-rules having a specific form (often, simplifiable to the stated form). Suppose an operator δ has $n \geq 1$ introduction-rules, schematically presented as $\frac{\Pi_i}{A}\ (\delta I)_i$, $\quad 1 \leq i \leq n$ The corresponding elimination-rule takes the form $\frac{A \quad \overset{[\Pi_1]_1}{C} \quad \cdots \quad \overset{[\Pi_n]_n}{C}}{C}\ (\delta E_{1,\cdots,n})$ where the hypothetical premiss derivations are discharged. Thus, C is deducible from A (formed with δ as its main operator) precisely when it is deducible from the grounds of introducing A, namely $\Pi_1, \ \ldots, \ \Pi_n$. Note that *all* grounds are used. We now establish generalized-elimination harmony for N_0^+.

1. For every, the generalized elimination-rule takes the following form (see [10] for the explanation how hypothetical derivations are expressed by such rules): $\frac{\Gamma \vdash S_{[every\ X/gf]} \quad \Gamma \vdash \mathbf{j}\ is\ an\ X \quad \Gamma, S_{[\mathbf{j}/gf]} \vdash S'}{\Gamma \vdash S'}\ (eGE_{fg})$

By choosing S' to be $S_{[\mathbf{j}/fg]}$, (eGE_{gf}) reduces to our original rule (eE_{gf}). The reduced rule suffices, since the generalized one is derivable from it [shown in the full paper].

2. For some, the presented rule *has* already the form of the generalized elimination-rule that corresponds to the sI_{gf} introduction-rule.

4.1 Decidability of N_0^+ derivability

We now attend to the issue of *decidability* of derivability in N_0^+. The positive result provided here makes PTS-based meaning effective for L_0^+. Figure 2 displays a sequent calculus presentation of L_0^+, easily shown equivalent to the natural deduction presentation considered thus far. The rules are arranged in the usual way of L-rules (introduction of an antecedent) and R-rules (introduction of a succedent). In view of the ambiguity of the $S_{[\ldots/gf]}$-notation, it is here understood that the gf indicated is that which is to be analyzed first during proof-search. Admissibility of *weakening* (W), *Contraction* (C) and (Cut) is routinely established for this calculus. The existence of a terminating proof-search procedure follows, since the number of parameter occurrences generated by the rules is bounded [more details in the full paper].

5 Extending the fragment

We now consider two simple extensions of E_0^+, related to extending the notion of nouns. In E_0^+, we had only primitive nouns. We add two forms of *compound noun*: one formed by *adjectives* and the other by *relative clauses*. In both cases, in the corresponding extensions of N_0^+, we let X, Y schematize over compound nouns also in the original rules.

Note that, while the N_0^+ rules may still seem to be introducing/eliminating some kind of logical constants (albeit not as main operator), the following rules (for adjectives and relative clauses) clearly transcend logical rules, and are more strictly meaning-defining for natural-language constructions.

Adding intersective adjectives: *Intersective adjectives* are schematized by A. Typical sentences are: /every girl/some/girl is beautiful, every beautiful girl/some beautiful girl smiles, every beautiful girl/some beautiful girl loves every clever boy/some clever boy. A noun preceded by an adjective is again a (compound) noun (the syntax will be treated more precisely once the grammar is presented). Denote this extension by $E_{0,adj}^+$. Recall that in the N_0^+ rules, the noun schematization should be taken over compound nouns too. Note that $E_{0,adj}^+$ is no longer finite, as an unbounded number of adjectives may precede a noun. We augment N_0^+ with the following ND-rules for adjectives.

$$\frac{\Gamma_1 \vdash \mathbf{j}\ is\ an\ X \quad \Gamma_2 \vdash \mathbf{j}\ is\ A}{\Gamma_1\Gamma_2 \vdash \mathbf{j}\ is\ an\ A\ X}\ (adjI) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ A\ X}{\Gamma \vdash \mathbf{j}\ is\ an\ X}\ (adjE_1) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ A\ X}{\Gamma \vdash \mathbf{j}\ is\ A}\ (adjE_2)$$

Let the resulting system be $N_{0,adj}^+$.

As an example of derivations using the rules for adjectives, consider the following derivation for

$$\mathbf{j}\ \mathsf{loves\ every\ girl} \vdash \mathbf{j}\ \mathsf{loves\ every\ beautiful\ girl}$$

In model-theoretic semantics terminology, the corresponding entailment is a witness to the *downward monotonicity* of the meaning of every in its second argument. We use an obvious schematization.

$$\frac{\dfrac{\dfrac{\dfrac{[\mathbf{r}\ is\ an\ A\ Y]_1}{\mathbf{r}\ is\ a\ Y}\ (adjE_1) \quad \mathbf{j}\ R\ every\ Y}{\mathbf{j}\ R\ \mathbf{r}}\ (eE_{subj})}{}}{\mathbf{j}\ R\ every\ A\ Y}\ (eI_{obj}^1)$$

[The full paper shows harmony for the adjective rules.]

Adding Relative Clauses Typical sentences include the following. every boy/some boy loves every/some girl who(m) smiles/loves a boy who smiles, every girl/some girl is a girl who loves some/every boy, some boy loves every girl who loves every boy who smiles (nested relative clause). So, girl who smiles and girl who loves every boy are compound nouns. We relate somewhat loosely to the case of the relative pronoun, in the form of who(m), abbreviating either *who* or *whom*, as the case requires. Denote the resulting fragment $E_{0,r}^+$ (or $E_{0,a,r}^+$ if both adjectives and relative clauses are considered). The corresponding ND-system $N_{0,r}^+$ extends N_0^+ by adding the following introduction and elimination-rules.

$$\frac{\Gamma_1 \vdash \mathbf{j}\ is\ an\ X \quad \Gamma_2 \vdash S_{[\mathsf{j}/gf]}}{\Gamma_1\Gamma_2 \vdash \mathbf{j}\ is\ an\ X\ who\ S_{-gf}}\ (relI) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ X\ who\ S_{-gf}}{\Gamma \vdash \mathbf{j}\ is\ an\ X}\ (relE_1) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ X\ who\ S_{-gf}}{\Gamma \vdash S_{[\mathsf{j}/gf]}}\ (relE_2)$$

As an example of a derivation in this fragment, consider some girl who smiles sings $\vdash_{N_{0,r}^+}$ some girl sings exhibiting the model-theoretical *upward monotonicity* of some in its first argument.

$$\frac{some\ X\ who\ P_1\ P_2 \quad \dfrac{\dfrac{[\mathbf{r}\ is\ an\ X\ who\ P_1]_1}{\mathbf{r}\ is\ an\ X}\ (relE_1) \quad [\mathbf{r}\ P_2]_2}{some\ X\ P_2}\ (sI_{subj})}{some\ X\ P_2}\ (sE_{subj}^{1,2})$$

[The full paper will show harmony for these rules.]

Once again, decidability of derivability is shown by means of the following additional sequent-calculus rules:

$$\frac{\Gamma, \mathbf{j}\ is\ an\ X, S_{[\mathsf{j}/gf]} \vdash S'}{\Gamma, \mathbf{j}\ is\ an\ X\ who\ S_{[-/gf]} \vdash S'}\ (Lrel) \qquad \frac{\Gamma \vdash \mathbf{j}\ is\ an\ X \quad \Gamma \vdash S_{[\mathsf{j}/gf]}}{\Gamma \vdash \mathbf{j}\ is\ an\ X\ who\ S_{[-/gf]}}\ (Rrel)$$

References

[1] Frederic B. Fitch. Natural deduction rules for English. *Philosophical Studies*, 24:89–104, 1973.

[2] Gerhard Gentzen. Investigations into logical deduction. In M.E. Szabo, editor, *The collected papers of Gerhard Gentzen*, pages 68–131. North-Holland, Amsterdam, 1935. English translation of the 1935 paper in German.

[3] Richard Montague. The proper treatment of quantification in ordinary english. In Jaco Hintikka, J. Moravcsik, and Patrick Suppes, editors, *Approaches to natural language*. Reidl, Dordrecht, 1973. Proceedings of the 1970 Stanford workshop on grammar and semantics.

[4] Michael Moortgat. Categorial type logics. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, pages 93–178. North Holland, 1997.

[5] Frank Pfenning and Rowan Davies. A judgmental reconstruction of modal logic. *Mathematical Structures in Computer Science*, 11:511–540, 2001.

[6] Jan Von Plato. Natural deduction with general elimination rules. *Archive Mathematical Logic*, 40:541–567, 2001.

[7] Dag Prawitz. Ideas and results in proof theory. In J. Fenstad, editor, *Proc. 2nd Scandinavian Symposium*. North-Holland, 1971.

[8] Arthur N. Prior. The roundabout inference-ticket. *Analysis*, 21:38–39, 1960.

[9] Aarne Ranta. *Type-Theoretical Grammar*. Oxford University Press, 1994.

[10] Stephen Read. Harmony and autonomy in classical logic. *Journal of Philosophical Logic*, 29:123–154, 2000.

[11] Stephen Read. Harmony and modality. In *UNILOG 2005: World Congress on Universal Logic*, 2005. To Appear.

[12] Peter Schroeder-Heister. On the notion of *assumption* in logical systems. In R. Bluhm and C. Nimtz, editors, *Philosophy-Science-Scientific Philosophy*. Mentis, Paderborn, 2004. Selected papers of the 5th int. congress of the society for Analytic Philosophy, Bielfield, September 2003.

[13] Peter Schroeder-Heister. Validity concepts in proof-theoretic semantics. In R. Kale and Peter Schroeder-Heister, editors, *Proof-Theoretic Semantics*, volume 148, pages 525–571. February 2006. Special issue of *Synthese*.