Natural Logic and Semantics

Larry Moss

Indiana University

Dedicated to Ed Keenan

What it's all about

By *natural logic* I mainly mean the study of inference, formulated in languages as close to natural language as possible, and <u>not</u> via translation to standard logical systems.

Fitch 1972

1		John is a man	Hyp
2		Any woman is a mystery to any man	Hyp
3	Jane	Jane is a woman	Hyp
4		Any woman is a mystery to any man	R, 2
5		Jane is a mystery to any man	Any Elim, 4
6		John is a man	R, 1
7		Jane is a mystery to John	Any Elim, 6
8		Any woman is a mystery to John	Any intro, 3, 7

Current interest

Fitch's work lacked a syntax or semantics.
In particular, one could not ask traditional questions about it.

There are also a series of fairly recent papers from areas like
Artificial Intelligence
Natural Language Processing
Knowledge Representation
which do propose fragments and get soundness and decidability results.
As far as I know,
the interesting mathematical question of completeness is always left open.

Logic and language: a tough example of syllogistic reasoning

Example

All xenophobics hate all Albanians
All yodelers hate all zookeepers
All non-yodelers hate all non-Albanians
All wardens are xenophobics

All wardens hate all zookeepers

Does the conclusion follow?

"Syntax"

We start with *variables* X, Y, ..., representing plural common nouns of English.
$\mathcal{V}$ is the set of variables in what follows.
We also start with verbs V.
We assume that there is a *complementation operation* $' : \mathcal{V} \to \mathcal{V}$ on the variables such that $X'' = X$ for all X.
We also also *names* J (John), M (Mary), ...
Sentences S of the following very restricted forms:

All X are Y	*J is an X*
Some X are Y	*J is M*
All X V all Y	*All X V some Y*

Model-Theoretic Semantics

A *model* $\mathcal{M} = (M, [\![\ \ ]\!])$ consists of

a set M

a subset $[\![X]\!] \subseteq M$ for each variable X

and an element $[\![J]\!] \in M$ for each name J.

We then set $[\![X']\!] = M \setminus [\![X]\!]$.

And we say

$$
\begin{array}{lll}
\mathcal{M} \models \textit{All } X \textit{ are } Y & \text{iff} & [\![X]\!] \subseteq [\![Y]\!] \\
\mathcal{M} \models \textit{Some } X \textit{ are } Y & \text{iff} & [\![X]\!] \cap [\![Y]\!] \neq \emptyset \\
\mathcal{M} \models J \textit{ is an } X & \text{iff} & [\![J]\!] \in [\![X]\!] \\
\mathcal{M} \models J \textit{ is } M & \text{iff} & [\![J]\!] = [\![M]\!]
\end{array}
$$

The Usual Semantic Definitions

If Γ is a finite or infinite set of sentences,

$\mathcal{M} \models \Gamma$

$\mathcal{M} \models S$ for all $S \in \Gamma$.

$\Gamma \models S$

Every model $\mathcal{M}$ which makes all sentences in Γ true also makes S true.
We say Γ *semantically entails* S.

Proof Trees

We are going build proof trees using various sets of rules.
The rules will be presented in a completely syntactic way.

$\Gamma \vdash S$

There is a proof tree for whose leaves are labeled with members of Γ and whose root is labeled S.
We say Γ *proves*, or *derives*, S.

All the systems in this talk will be *sound*:

$$\textit{If } \Gamma \vdash S, \textit{ then } \Gamma \models S.$$

We shall be interested in *completeness* results of the form:

$$\textit{If } \Gamma \models S, \textit{ then } \Gamma \vdash S.$$

This would say that the system is *strong enough* to represent everything about *entailment* we could possibly want.

Proof Rules: *All*

$$\frac{}{\textit{All } X \textit{ are } X} \qquad \frac{\textit{All } X \textit{ are } Z \quad \textit{All } Z \textit{ are } Y}{\textit{All } X \textit{ are } Y}$$

Proof Rules: *All*

$$\frac{}{\textit{All } X \textit{ are } X} \qquad\qquad \frac{\textit{All } X \textit{ are } Z \quad \textit{All } Z \textit{ are } Y}{\textit{All } X \textit{ are } Y}$$

Theorem

The logic above is complete.

This seems to be the simplest completeness theorem in logic.

Proof of completeness

Suppose that $\Gamma \models$ *All X are Y*.
Let M be the set of variables.
Define $A \leq B$ to mean that $\Gamma \vdash$ *All A are B*.
Check that this is reflexive and transitive, using the logic.
The semantics is via downsets:

$$[\![A]\!] \quad = \quad \downarrow A \quad = \quad \{B : B \leq A\}$$

By transitivity, $\mathcal{M} \models \Gamma$.

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By transitivity, $\mathcal{M} \models \Gamma$.
In more detail, suppose Γ contains *All C are D*.
Then if $\mathrm{W} \leq C$, then also $\mathrm{W} \leq D$.

Proof of completeness

Suppose that $\Gamma \models$ *All X are Y*.
Let M be the set of variables.
Define $A \leq B$ to mean that $\Gamma \vdash$ *All A are B*.
Check that this is reflexive and transitive, using the logic.
The semantics is via downsets:

$$[\![A]\!] \quad = \quad \downarrow A \quad = \quad \{B : B \leq A\}$$

By transitivity, $\mathcal{M} \models \Gamma$.
So $[\![X]\!] \subseteq [\![Y]\!]$.
But by reflexivity X $\in [\![X]\!]$.
And so $X \in [\![X]\!]$; this means that $X \leq Y$.

That's "all", folks!

$$\frac{}{\textit{All X are X}} \qquad \frac{\textit{All X are Z} \quad \textit{All Z are Y}}{\textit{All X are Y}}$$

Intersecting Adjectives

> The sense in which an intersecting adjective determines a property can be described as follows. If Dana is a female student and if Dana is also an athlete, then Dana is a female athlete.
>
> Keenan and Faltz, 1985, p. 123

We'll add a single intersecting adjective, *red*.
We'll add it productively, so that *red red car* counts as a N.

Intersecting Adjectives

For each intersecting adjective, say *red*, we select a set, say red, and then

$$[\![red\ X]\!] \quad = \quad red \cap [\![X]\!].$$

Proof Rules: Intersecting Adjectives

Proof Rules: Intersecting Adjectives

$$\frac{}{\textit{All red X are X}} \qquad \frac{\textit{All X are red Z} \quad \textit{All X are Y}}{\textit{All X are red Y}}$$

Proof Rules: Intersecting Adjectives

$$\frac{}{\textit{All red X are X}} \qquad \frac{\textit{All X are red Z} \quad \textit{All X are Y}}{\textit{All X are red Y}}$$

Note that we understand the letters above to be Ns, so to avoid confusion it would be better say

$$\frac{}{\textit{All red N are N}} \qquad \frac{\textit{All N are red P} \quad \textit{All N are M}}{\textit{All N are red M}}$$

A Deduction

$$\frac{\textit{All N are red P} \quad \textit{All N are M}}{\textit{All N are red M}}$$

Let's make a substitution:

$$N \mapsto \text{red } X$$
$$M \mapsto \text{red } X$$
$$P \mapsto X$$

and so we get

$$\frac{\textit{All red X are red X} \quad \textit{All red X are red X}}{\textit{All red X are red red X}}$$

A Deduction

$$\frac{\textit{All N are red P} \quad \textit{All N are M}}{\textit{All N are red M}}$$

Let's make a substitution:

$$\begin{aligned} N &\mapsto \text{red } X \\ M &\mapsto \text{red } X \\ P &\mapsto X \end{aligned}$$

and so we get

$$\frac{\textit{All red X are red X} \quad \textit{All red X are red X}}{\textit{All red X are red red X}}$$

The premises of *this* being axioms, $\vdash$*All red X are red red X*

Completeness

Fix a set Γ of sentences, and suppose that $\Gamma \models S$.
We must show that $\Gamma \vdash S$.
Construct a model $\mathcal{M}$ by

$$
\begin{array}{lcl}
M & = & \text{the variables} \\
[\![X]\!] & = & \downarrow X \\
 & (= & \{Y : \Gamma \vdash \textit{All } Y \textit{ are } X\}) \\
\text{red} & = &
\end{array}
$$

Completeness

Fix a set Γ of sentences, and suppose that $\Gamma \models S$.
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Construct a model $\mathcal{M}$ by

$$
\begin{array}{lcl}
M & = & \text{the variables} \\
[\![X]\!] & = & \downarrow X \\
 & (= & \{Y : \Gamma \vdash \mathit{All}\ Y\ \mathit{are}\ X\}) \\
\textcolor{red}{\text{red}} & = & \bigcup_X \downarrow \mathit{red}\ X
\end{array}
$$

Completeness, continued

Lemma

$[\![N]\!] = \downarrow N$ *for all nouns N.*

Proof.

It is sufficient to do this for nouns of the form *red* X.
If $\Gamma \vdash$ *All Y are red X*, then easily $Y \in [\![\textit{red } X]\!]$.
If $Y \in [\![\textit{red } X]\!] = \textit{red} \cap [\![X]\!]$, then

1. There is some Z such that $\Gamma \vdash$ *All Y are red Z.*
2. $\Gamma \vdash$ *All Y are X*

So by our logic, $\Gamma \vdash$ *All Y are red X.*
This completes the proof. $\square$

Completeness, concluded

Lemma

$[\![N]\!] = \downarrow N$ *for all nouns* N.

So at this point we have a model $\mathcal{M}$, and this lemma.
The lemma easily implies that $\mathcal{M} \models \Gamma$.
Completeness means:

$$\text{If } \Gamma \models \text{All } N \text{ are } M\text{, then } \Gamma \vdash S.$$

This follows just as in the basic logic of *All*.

Proof Rules *All* and *Some*

$$\frac{}{\textit{All } X \textit{ are } X} \qquad \frac{\textit{All } X \textit{ are } Z \quad \textit{All } Z \textit{ are } Y}{\textit{All } X \textit{ are } Y}$$

$$\frac{\textit{Some } X \textit{ are } Y}{\textit{Some } Y \textit{ are } X} \qquad \frac{\textit{Some } X \textit{ are } Y}{\textit{Some } X \textit{ are } X}$$

$$\frac{\textit{All } Y \textit{ are } Z \quad \textit{Some } X \textit{ are } Y}{\textit{Some } X \textit{ are } Z}$$

Proper Names

$$\frac{}{\textsf{\textit{J is J}}} \qquad \frac{\textsf{\textit{J is M}} \quad \textsf{\textit{M is F}}}{\textsf{\textit{J is F}}} \qquad \frac{\textsf{\textit{M is J}}}{\textsf{\textit{J is M}}}$$

$$\frac{\textsf{\textit{M is an X}} \quad \textsf{\textit{J is M}}}{\textsf{\textit{J is an X}}}$$

$$\frac{\textsf{\textit{All X are Y}} \quad \textsf{\textit{J is an X}}}{\textsf{\textit{J is a Y}}}$$

$$\frac{\textsf{\textit{J is an X}} \quad \textsf{\textit{J is a Y}}}{\textsf{\textit{Some X are Y}}}$$

A tougher example of syllogistic reasoning

Example

All xenophobics hate all Albanians
All yodelers hate all zookeepers
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All wardens are xenophobics

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A tougher example of syllogistic reasoning

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All non-yodelers hate all non-Albanians
All wardens are xenophobics

All wardens hate all zookeepers

Why does the conclusion follow?

Take a warden. He or she will be a xenophobic, and hence hate all Albanians.

If also a yodeler, he or she will certainly hate all zookeepers; if not, he or she will hate all non-Albanians and hence hate everyone whatsoever, a fortiori all zookeepers.

Syllogistic logic with complement

$$\frac{}{\textsf{All } X \textsf{ are } X} \qquad \frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } X} \qquad \frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } Y \textsf{ are } X}$$

$$\frac{\textsf{All } X \textsf{ are } Z \quad \textsf{All } Z \textsf{ are } Y}{\textsf{All } X \textsf{ are } Y}\ \textsf{Barbara}$$

$$\frac{\textsf{All } Y \textsf{ are } Z \quad \textsf{Some } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } Z}\ \textsf{Darii}$$

$$\frac{\textsf{All } Y \textsf{ are } Y'}{\textsf{All } Y \textsf{ are } X}\ \textsf{Zero} \qquad \frac{\textsf{All } Y' \textsf{ are } Y}{\textsf{All } X \textsf{ are } Y}\ \textsf{One}$$

$$\frac{\textsf{All } X \textsf{ are } Y'}{\textsf{All } Y \textsf{ are } X'}\ \textsf{Antitone} \qquad \frac{\textsf{Some } X \textsf{ are } X'}{S}\ \textsf{Ex falso quodlibet}$$

Why study this?

- Completeness results have an intrinsic value.
- This line of work may have a practical value also (cf. Sukkarieh 2005).
- There are complexity reasons.
- For this talk, the interesting point is the investigation of non-Boolean structure.

Orthoposets

Definition

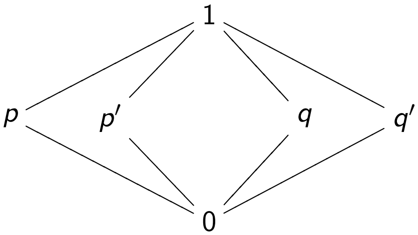
An *orthoposet* is a tuple $(P, \leq, 0, ')$ such that

Poset	$\leq$ is a reflexive, transitive, and antisymmetric relation on the set P.
Zero	$0 \leq p$ for all $p \in P$.
Antitone	If $x \leq y$, then $y' \leq x'$.
Involutive	$x'' = x$.
Inconsistency	If $x \leq y$ and $x \leq y'$, then $x = 0$.

A Key Point

Orthoposets need not have a meet or join operation.

The Chinese lantern M_2



Here and elsewhere, we understand $(x')' = x$, $0' = 1$, $1' = 0$.

Orthoposets

Example

For all sets X we have an orthoposet $(\mathcal{P}(X), \subseteq, \emptyset, ')$, where $a' = X \setminus a$ for all subsets a of X.

Orthoposets from the logic

Let Γ be any set of sentences in the fragment.
Let $\mathcal{V}$ be the set of variables.
We already know the preorder $\leq$:

$$\mathsf{X} \leq \mathsf{Y} \quad \text{iff} \quad \Gamma \vdash \textit{All X are Y}.$$

We have an induced equivalence relation $\equiv$, and we take $\mathcal{V}_\Gamma$ to be the quotient $\mathcal{V}/\!\equiv$.
If there is some X such that $\mathsf{X} \leq \mathsf{X}'$, then set 0 to be [X].
We finally define $[\mathsf{X}]' = [\mathsf{X}']$.
If there is no X such that $\mathsf{X} \leq \mathsf{X}'$, we add fresh elements 0 and 1 to $\mathcal{V}/\!\equiv$.
It is not hard to check that we have an orthoposet $\mathcal{V}_\Gamma$.

Points of orthoposets

A *point* of a orthoposet $P = (P, \leq, 0, ')$ is a subset $\mathcal{S} \subseteq P$ with the following properties:

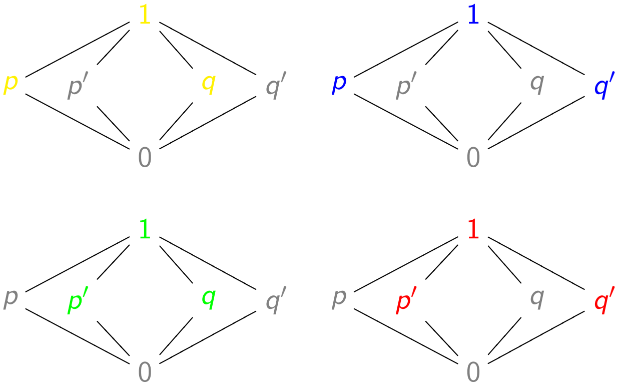
UP-CLOSED If $p \in S$ and $p \leq q$, then $q \in \mathcal{S}$.

COMPLETE For all p, either $p \in S$ or $p' \in \mathcal{S}$.

PAIRWISE COMPATIBLE For all $p, q \in \mathcal{S}$, $p \not\leq q'$.

Points are sets

There are four points here ●, ●, ●, and ●:



Points need not be filters

Let $X = \{1, 2, 3\}$, and let $\mathcal{P}(X)$ be the power set orthoposet. Then $\mathcal{S}$ is a point, where

$$\mathcal{S} \quad = \quad \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

It is easy to check that the points on this $\mathcal{P}(X)$ are exactly $\mathcal{S}$ as above and the three principal ultrafilters.
$\mathcal{S}$ shows that a point of a boolean algebra need not be a filter.

The Extension Lemma for pairwise consistent sets

Lemma

Let $\mathcal{S} \subseteq P$ be pairwise consistent: $(\forall p, q \in \mathcal{S}) p \not\leq q'$.
Then for all $x \in P$, either $\mathcal{S} \cup \{x\}$ or $\mathcal{S} \cup \{x'\}$ is again pairwise consistent.

Proof.

Suppose not. Then x and x' figure in to problems both times.
There is some $p \in \mathcal{S}$ such that $p \leq x'$.
There is some $q \in \mathcal{S}$ such that $q \leq x'' = x$.
And now: $q \leq x \leq p'$. Ooops! □

The Extension Lemma for pairwise consistent sets

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Then for all $x \in P$, either $\mathcal{S} \cup \{x\}$ or $\mathcal{S} \cup \{x'\}$ is again pairwise consistent.

Lemma

If $x \not\leq y$, then $\{x, y'\}$ is pairwise consistent.
Thus there is a point $\mathcal{S}$ containing x but not y.

Representation Theorem

Let $P = (P, \leq, ')$ be an orthoposet.
Let $points(P)$ be the set of points of P.
We already know how to endow this with an orthoposet structure.
Let $m : P \to \mathcal{P}(points(P))$ be given by

$$m(p) \quad = \quad \{\mathcal{S} : p \in \mathcal{S}\}.$$

Theorem

m is a strict morphism of orthoposets:
it preserves 0, $'$, and $\leq$ in a strict way: $p \leq q$ iff $m(p) \subseteq m(q)$.

Representation Theorem

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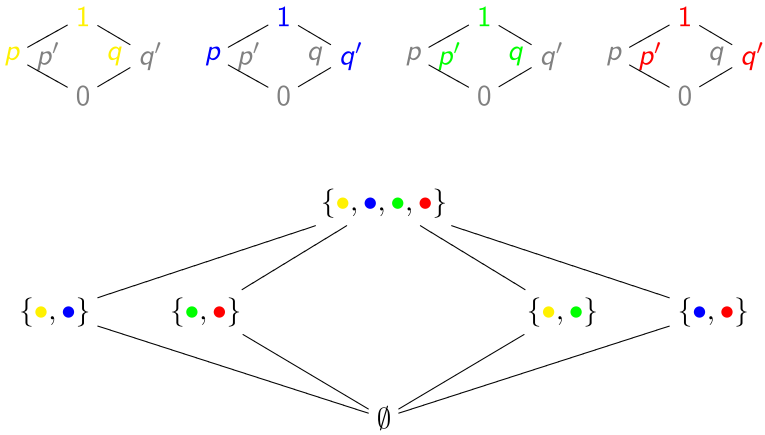
Corollary

Every orthoposet is isomorphic to a sub-orthoposet of a power set orthoposet.

How the representation works



Sources the Representation Theorem

N. Zierler and M. Schlessinger

Boolean embeddings of orthomodular sets and quantum logic.
Duke Mathematical Journal 32 (1965), 251–262.

F. Katrnoška

On the representation of orthocomplemented posets.
Comment. Math. Univ. Carolinae 23 (1982), 489–498.

C. S. Calude, P. H. Hertling, K. Svozil

Embedding quantum universes into classical ones.
Foundations of Physics, 29, 3 (1999), 349-379.

A system which we have seen

$$\frac{}{\textit{All } X \textit{ are } X} \qquad \frac{\textit{Some } X \textit{ are } Y}{\textit{Some } X \textit{ are } X} \qquad \frac{\textit{Some } X \textit{ are } Y}{\textit{Some } Y \textit{ are } X}$$

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The canonical model

Lemma

Let Γ be consistent in $\vdash$. There is a canonical model $\mathcal{M} = (M, [\![\ ]\!])$ such that

1. $\mathcal{M} \models \Gamma$.
2. *If $\mathcal{M} \models$ All X are Y, then $\Gamma \vdash$ All X are Y.*

Proof.

Let $\mathcal{V}_\Gamma$ be the syntactic orthoposet for Γ. Let $M = points(\mathcal{V}_\Gamma)$.
The interpretation $[\![\ ]\!] : \mathcal{V} \to \mathcal{P}(M)$ is given by

$$\mathcal{V} \xrightarrow{n} \mathcal{V}_\Gamma \xrightarrow{m} \mathcal{P}(points(\mathcal{V}_\Gamma)) = \mathcal{P}(M)$$

Key point If Γ contains *Some U are V*, need a point including $\{[\mathrm{U}], [\mathrm{V}]\}$.
If none exists, then wlog $\mathrm{U} \leq \mathrm{V}'$. But then Γ is inconsistent. $\square$

If Γ is consistent and $\Gamma \models$ *Some X are Y*, then $\Gamma \vdash$ *Some X are Y*

If Γ is consistent and $\Gamma \models$ *Some X are Y*, then $\Gamma \vdash$ *Some X are Y*

Lemma (Ian Pratt-Hartmann 2007)

There is some existential sentence in Γ, say Some A are B, such that

$$\Gamma_{all} \cup \{\textit{Some } A \textit{ are } B\} \models \textit{Some } X \textit{ are } Y.$$

Proof.

If not, then for every $T \in \Gamma_{some}$, there is a model $\mathcal{M}_T \models \Gamma_{all} \cup \{T\}$ and $\mathcal{M}_T \models$ *All X are Y'*. Take the disjoint union of the models $\mathcal{M}_T$ to get a model of $\Gamma_{all} \cup \Gamma_{some} = \Gamma$ where S fails. $\square$

If Γ is consistent and $\Gamma \models$ *Some X are Y*, then $\Gamma \vdash$ *Some X are Y*

Fix A and B as in the now-proven claim.
Consider the model $\mathcal{M} = \mathcal{M}(\mathcal{V}_{\Gamma_{all}})$ of points on $\mathcal{V}_{\Gamma_{all}}$. $\mathcal{M} \models \Gamma_{all}$.
Consider $\{[A], [B], [X']\}$. If this set were a subset of a point x, then consider $\{x\}$ as a one-point submodel of $\mathcal{M}$. In the submodel, $\Gamma_{all} \cup \{\textit{Some A are B}\}$ would hold, and yet *Some X are Y* would fail since $[\![X]\!] = \emptyset$.
We use a lemma to divide into cases:

1. $A \leq A'$.
2. $A \leq B'$.
3. $A \leq X$.
4. $B \leq B'$.
5. $B \leq X$.
6. $X' \leq X$.

If Γ is consistent and $\Gamma \models$ *Some X are Y*, then $\Gamma \vdash$ *Some X are Y*

Next, consider $\{A, B, Y'\}$. The same analysis gives two other cases: $A \leq Y$ and $B \leq Y$.
Putting these together with the other two gives four pairs.
The case when $A \leq X$ and $B \leq Y$ is representative:

$$\dfrac{\textit{All } B \textit{ are } Y \qquad \dfrac{\dfrac{\dfrac{\textit{All } A \textit{ are } X \qquad \textit{Some } B \textit{ are } A}{\textit{Some } B \textit{ are } X}}{\textit{Some } X \textit{ are } B}}{}}{\textit{Some } X \textit{ are } Y}$$

The other cases are similar. This completes the proof.

Boolean connectives are (more than) ok

1. All substitution instances of propositional tautologies.
2. *All X are X*
3. (*All X are Z*) $\wedge$ (*All Z are Y*) $\rightarrow$ *All X are Y*
4. (*All Y are Z*) $\wedge$ (*Some X are Y*) $\rightarrow$ *Some Z are X*
5. *Some X are Y* $\rightarrow$ *Some X are X*
6. $\neg$(*Some X are X*) $\rightarrow$ *All X are Y*
7. *Some X are Y'* $\leftrightarrow$ $\neg$(*All X are Y*)

See Łukasiewicz (1951) and Westerståhl (1989) for related completeness results.

Verbs again, using *see* for the verb

$$\frac{\textit{All } X \textit{ see all } Y \qquad \textit{All } X' \textit{ see all } Y}{\textit{All } Z \textit{ see all } Y}\ \textit{LEM}$$

$$\frac{\textit{All } X \textit{ see all } Y \qquad \textit{All } X \textit{ see all } Y'}{\textit{All } X \textit{ see all } Z}\ \textit{LEM}'$$

$$\frac{\textit{All } X \textit{ see all } A \qquad \textit{All } Y \textit{ see all } Z \qquad \textit{All } Y' \textit{ see all } A'}{\textit{All } X \textit{ see all } Z}\ \textit{3pr}$$

A Fitch-style system is more elegant.
But this system seems to be easier to analyze.

1		*All xenophobics hate all Albanians*	Hyp
2		*All yodelers hate all zookeepers*	Hyp
3		*All non-yodelers hate all non-Albanians*	Hyp
4		*All wardens are xenophobics*	Hyp
5	*Jane*	*Jane is a warden*	Hyp
6		*All wardens are xenophobics*	R, 4
7		*Jane is a xenophobic*	All Eliim, 6
8		*All xenophobics hate all Albanians*	R, 2
9		*Jane hates all Albanians*	All Elim, 8
10		*Jane is a yodeler*	Hyp
11		*Jane hates all zookeepers*	Easy from 2
12		*Jane is not a yodeler*	Hyp
13		*Jane hates all zookeepers*	See below
14		*Jane hates all zookeepers*	Cases 10-11, 12-13
15		*All wardens hate all zookeepers*	All Intro

1		*Jane is not a yodeler*	Hyp
2		*Jane hates all Albanians*	R, above
3		*All non-yodelers hate all non-Albanians*	R, above
4		*Jane hates all non-Albanians*	All Elim, 1, 3
5	*Bob*	*Bob is a zookeeper*	Hyp
6		*Bob is Albanian*	Hyp
7		*Jane hates Bob*	All Elim, 2
8		*Bob is not Albanian*	Hyp
9		*Jane hates Bob*	All Elim, 4
10		*Jane hates Bob*	Cases
11		*Jane hates all zookeepers*	All Intro

The Canonical Model, again

Let M be the set of all points.
Let $[\![X]\!] = \{\mathcal{S} \in M : X \in \mathcal{S}\}$, and let

$$[\![\mathit{see}]\!] \quad = \quad \{(\mathcal{S}, \mathcal{T}) : (\exists A \in \mathcal{S})(\exists B \in \mathcal{T})\ \Gamma \vdash \mathit{All\ A\ see\ all\ B}\}.$$

The Canonical Model, again

Let M be the set of all points.
Let $[\![X]\!] = \{\mathcal{S} \in M : X \in \mathcal{S}\}$, and let

$$[\![\mathit{see}]\!] \quad = \quad \{(\mathcal{S}, \mathcal{T}) : (\exists A \in \mathcal{S})(\exists B \in \mathcal{T})\ \Gamma \vdash \mathit{All}\ A\ \mathit{see}\ \mathit{all}\ B\}.$$

Lemma

Fix Γ, and also fix X and Y such that

$$\Gamma \nvdash \mathit{All}\ X\ \mathit{see}\ \mathit{all}\ Y.$$

Then there are points $\mathcal{S}^*$ and $\mathcal{T}^*$ such that $X \in \mathcal{S}^*$, $Y \in \mathcal{T}^*$, and for all $A \in \mathcal{S}^*$ and $B \in \mathcal{T}^*$, $\Gamma \nvdash \mathit{All}\ A\ \mathit{see}\ \mathit{all}\ B$.

This completes the proof!

Summary

The completeness results in this talk

All

All + an intersective adjective

All + *Some* + noun-level complements

All + verbs + noun-level complements

Natural logic and semantics: some programs

– there's still more to do on the extensional fragments
– intensionality would be a very natural next step
– there are several ways in which one would want to align this work with psychological studies of inference
– we have a model-theoretic syntax, why not a proof-theoretic semantics?
– it's clear that inference/entailment is not really the right notion, so we should propose other ones, perhaps using defaults, probability, revision, etc.