CONTRARIETY VS. POST-COMPLEMENT
Logico-linguistic issues around classical and modern squares of opposition

(for Ed Keenan)

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MoL10 Workshop, UCLA
July 28–30, 2007

Outline

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  and thus, for $Q = \textit{between p/q and (q-p)/q of the}$, $Q\neg = Q$.
- It illustrates one thing I will argue today: post-complement is, logically as well as linguistically, more interesting than other forms of 'inner' negation proposed, notably contrariety.

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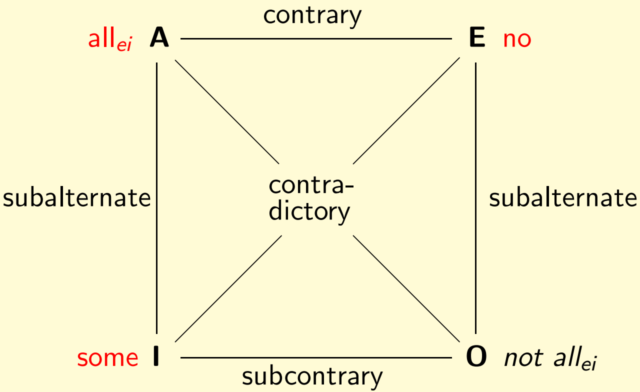
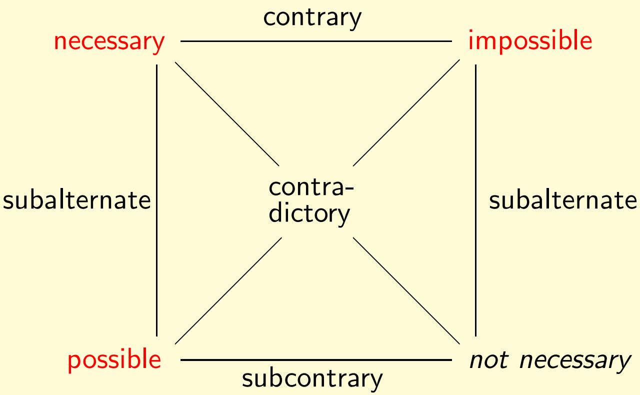
- Some of the remarks here originate in S. Peters and D. Westerståhl, *Quantifiers in Language and Logic*, OUP, 2006, chs. 1 and 4.

Aristotle's (quantified) Square



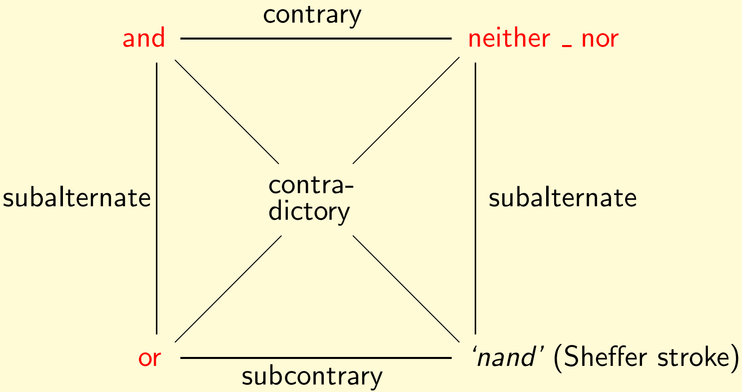
(Red color indicates lexicalization.)

A modal square (also Aristotle)



And there is a *deontic square* (obligatory, forbidden, permitted, not obligatory), and a *temporal adverb square* (always, never, sometimes, not always), . . .

And a propositional square



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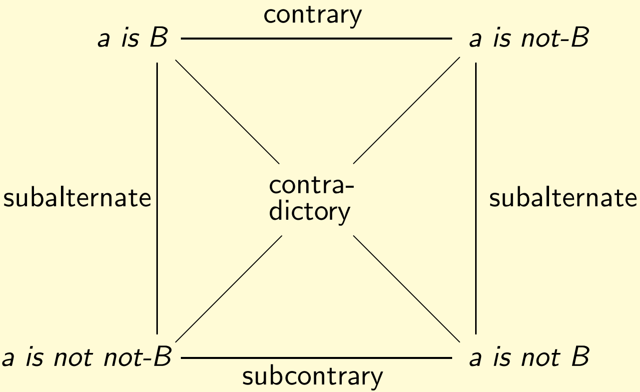
- So we focus on the quantified square.
- However, Aristotle considered yet another square:

The singular square



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- So, back to Aristotle's (general, quantified) square:

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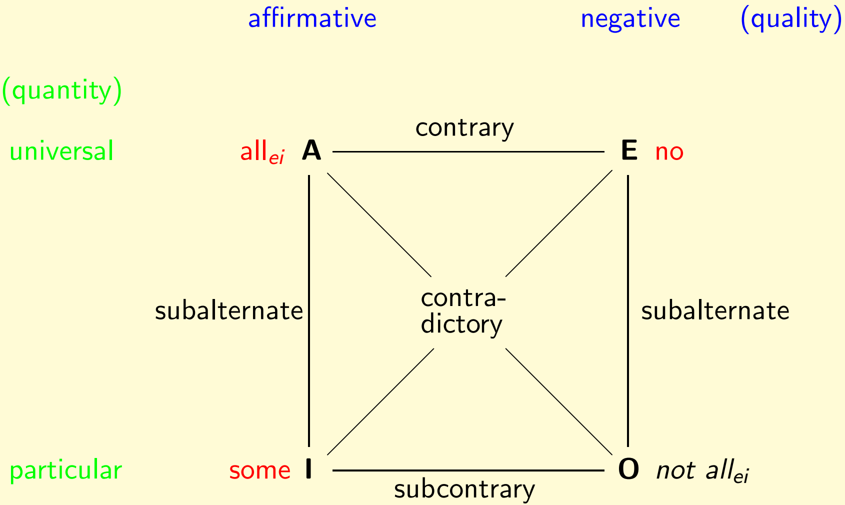
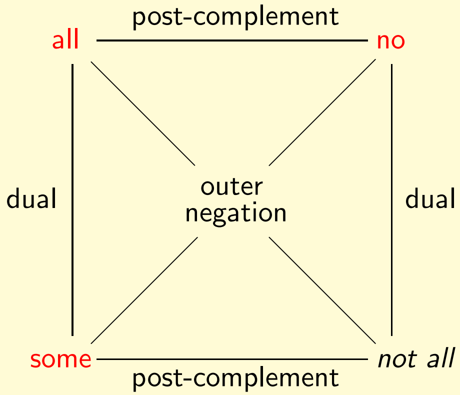


The Modern Square

The modern square (spanned by *all*) differs from Aristotle's *only* in the fact that *all* replaces all_{ei}. But this apparently small difference is in fact huge.



Logical relations in the squares

- Aristotle's square:
  1. contradictoriness (diagonal) = outer negation: $Q' = \neg Q$
  2. contrariety (upper horizontal): $Q(A, B) \Rightarrow \neg Q'(A, B)$
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- (No quarrel about the meaning of negation here. The quarrel is about which of these relations are fundamental.)
- Existential import $[Q(A, B) \Rightarrow A \neq \emptyset]$ at *affirmative* positions in Aristotle's square; at *particular* positions in modern square.

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Another main issue:

- I will, however, say something about existential import (EI). Three traditional positions (Horn 1989, 1997):
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  2. EI stems from (particular) *quantity* (Frege, modern logic).
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- My own position (to be argued) is that (a) for the squares discussed so far, EI is a pragmatic matter (Gricean implicature; this is roughly Horn's position), although the notion of EI is too narrow, but (b) semantic issues of the EI type exist for certain *other squares*.

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- So while Aristotle's (quantificational) square illustrates some of the behavior of a few selected quantifiers, the modern square reveals more general *patterns of quantification*.
- To see this, we need one more preliminary.

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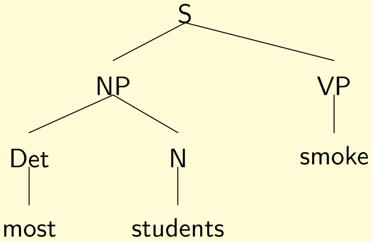

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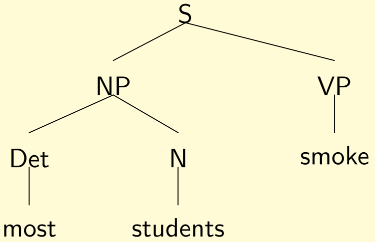

- Such denotations are $\textsc{Conserv}$ $[Q_M(A, B) \Leftrightarrow Q_M(A, A \cap B)]$ and $\textsc{Ext}$ [if $A, B \subseteq M \subseteq M'$ then $Q_M(A, B) \Leftrightarrow Q_{M'}(A, B)$], and often $\textsc{Isom}$. $\textsc{Conserv} + \textsc{Ext}$ (as well as $\textsc{Isom}$) is *preserved* under the three kinds of negation (but *not* under e.g. contrariness.)

Examples of quantifiers denoted by determiners:

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$$
\begin{array}{l}
\mathit{all}(A,B) \iff A \subseteq B \\
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\mathit{at\ least\ two}(A,B) \iff |A \cap B| \geq 2 \\
\mathit{exactly\ five}(A,B) \iff |A \cap B| = 5 \\
\mathit{all\ but\ three}(A,B) \iff |A - B| = 3 \\
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  $an\ even\ number\ of(A,B) \iff |A \cap B| \text{ is even}$
- Let $\mathbf{1}$ ($\mathbf{0}$) be the trivially true (false) quantifier (*at least zero*, *fewer than zero*).

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  (c) If $Q' \in \mathit{square}(Q)$, then $\mathit{square}(Q) = \mathit{square}(Q')$. So any two squares are either identical or disjoint.
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  (d) $square(Q)$ has either two or four members.
- *Ad* (d): Normally four, but consider Keenan's *between one-third and two-thirds of the*!

This fails completely for classical squares

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- Each position determines the diagonally opposed quantifier , i.e, its outer negation, but *not* the quantifiers at the other two positions. For example:
- **FACT 2**: The square

  [**A**: *at least five*; **E**: *no*; **I**: *some*; **O**: *at most four*]

  is classical. More generally, for $n \geq k$,

  [**A**: *at least n*; **E**: *fewer than k*; **I**: *at least k*; **O**: *fewer than n*]

  is classical.

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- What is the role of existential import (and other kinds of import)?
- Such a project cannot even be formulated with classical squares (since these are not determined).
- Which is not to say that the classical logical oppositions always fail for squares other than $\{all_{\mathrm{ei}}, no, not\ all_{\mathrm{ei}}, some\}$. When do they hold and when don't they?

Qualitative and quantitative aspects of modern squares?

- Since, for $Q' \in \mathit{square}(Q)$, $\mathit{square}(Q') = \mathit{square}(Q)$, have we lost all qualitative and quantitative aspects in modern squares? I.e. can *any* quantifier occupy the **A** corner?

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- Some familiar monotonicity properties:

  $\textsc{Mon}\uparrow$: $Q(A, B)$ & $B \subseteq B' \Rightarrow Q(A, B')$
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  etc.
- **FACT 3:** The monotonicity behavior of Q determines the monotonicity behavior of all elements of $square(Q)$:
  1. Q is $\mathrm{MON}\uparrow$ iff $Q\neg$ is $\mathrm{MON}\downarrow$ iff $\neg Q$ is $\mathrm{MON}\downarrow$ iff Q^{d} is $\mathrm{MON}\uparrow$
  2. Q is $\uparrow\mathrm{MON}$ iff $Q\neg$ is $\uparrow\mathrm{MON}$ iff $\neg Q$ is $\downarrow\mathrm{MON}$ iff Q^{d} is $\downarrow\mathrm{MON}$
  3. So if Q is *doubly* monotone, all four combinations are exemplified in its square.

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- However, many quantifiers are only *right monotone*, e.g. the proportional quantifiers. So we would know, for example, that *at least two-thirds of the* is affirmative: either **A** or **I**.
- And this seems right, since the dual of *at least two-thirds of the* is *more than one-third of the*, and it seems arbitrary which of these two should go into the **A** position.

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  Q is symmetric if $Q(A,B) \Rightarrow Q(B,A)$.
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- **FACT 4:** The symmetry behavior of Q determines the symmetry behavior of all elements of *square*(Q):
  1. Q is symmetric iff $Q\neg$ is co-symmetric iff $\neg Q$ symmetric iff Q^{d} is co-symmetric
  2. Also, under $\textsc{Conserv}$, symmetry is the same as *intersectivity* (Keenan's term): if $A \cap B = A' \cap B'$ then $Q(A,B) \Leftrightarrow Q(A',B')$. So co-symmetry = co-intersectivity: if $A - B = A' - B'$ then $Q(A,B) \Leftrightarrow Q(A',B')$.

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- For example, *at most ten* is at the **E** position.
- But we already knew that, since *at most ten* is $\downarrow\textsc{Mon}\downarrow$. Indeed, if Q is right monotone and symmetric, it is clearly also left monotone.
- The cases where symmetry would give extra information are rather limited. This follows from the next result.

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- So we get extra information for cases like *an even number of*, which is logical and symmetric but not right monotone, and *no _ except John*, which is non-logical ($\textsc{Conserv}$ and $\textsc{Ext}$, but not $\textsc{Isom}$) and symmetric but not right monotone.

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- I.e. there are two possible configurations of *square*(*an even number of*) and *square*(*no _ except John*).

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- **FACT 9:** If Q is logical and $\uparrow\mathrm{MON}$, it is a finite disjunction of quantifiers of the form $|A \cap B| \geq n$ & $|A - B| \geq k$.

Left and double monotonicity cont.

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- And *(all of) John's* is $\mathrm{M}\textsc{on}\uparrow$ and *weakly* $\downarrow\mathrm{M}\textsc{on}$ (you can decrease A as long as something belonging to John remains), so it goes in the **A** corner.
- On the other hand, *most of John's* is $\mathrm{M}\textsc{on}\uparrow$ but not left monotone, so it is affirmative, but there seems to be no logical indication of whether it is **A** or **I**.

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- [But note that only *an odd number of* has existential import, so perhaps it has to be in the **I** corner?]
- With this, it's time to look at a few examples.

Example I: numerical quantifiers 1

$Q =$ *at least six*, ($\uparrow$Mon$\uparrow$)

(Now red color means 'realizable' as a (possible complex) determiner.)

all but at most five A are B $\lvert A - B \rvert \leq 5$	at most five A are B $\lvert A \cap B \rvert \leq 5$
at least six A are B $\lvert A \cap B \rvert \geq 6$	"all but at least six A are B" $\lvert A - B \rvert \geq 6$

Example I: numerical quantifiers 2

$Q =$ (*exactly*) *five*. Symmetric, not monotone, so two configurations possible. Choose the one below, since for $n = 0$ it becomes the (modern) Aristotelian square.

all but five A are B $\lvert A - B\rvert = 5$	(exactly) five A are B $\lvert A \cap B\rvert = 5$
"not five A are B" $\lvert A \cap B\rvert \neq 5$	all but five A are not B $\lvert A - B\rvert \neq 5$

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- The modern Aristotelian square (*square*(*all*)) becomes classical if $A \neq \emptyset$ is presupposed.
- But here we would need very odd 'existential imports':
- **FACT 10:**
  (a) *square*(*at least n+1*) is classical iff $|A| > 2n$ is presupposed.
  (b) *square*(*exactly n*) is classical iff $|A| \neq 2n$ is presupposed.

Digression: standard EI too weak even for Aristotelian square

- Compare:
  (4) Every natural satellite of the Earth has been visited by humans.
  (5) Some natural satellite of the Earth has been visited by humans.
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- Conclusion: relegate facts like these to pragmatics.

Example II: proportional quantifiers

Q = *at least 2/3 of the*. $\textsc{Mon}\uparrow$, but not left monotone or symmetric, so two configurations possible. No obvious choice between them, i.e. whether Q or Q^{d} is **A**.

at least 2/3 of the A are B $\lvert A\cap B\rvert \geq 2/3\cdot\lvert A\rvert$	at most 1/3 of the A are B $\lvert A\cap B\rvert \leq 1/3\cdot\lvert A\rvert$
more than 1/3 of the A are B $\lvert A\cap B\rvert > 1/3\cdot\lvert A\rvert$	fewer than 2/3 of the A are B $\lvert A\cap B\rvert < 2/3\cdot\lvert A\rvert$

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- If we do, this square is both classical and modern.

Example III: an 'exceptive' square

$Q =$ *no _ except John*. Symmetric, not monotone; two configurations possible. Is Q **I** or **E**? The latter choice makes the **O** corner 'unrealized' (as usual), and reduces to the (modern) Aristotelian square when the exception set is empty:

every A except John is B
$A - B = \{j\}$

no A except John is B
$A \cap B = \{j\}$

'if John then some other' A is B
$A \cap B \neq \{j\}$

$A - B \neq \{j\}$

Example IV: possessive quantifiers

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1 Universal, existential and other readings:

(10) John's books are missing.
(11) Six students' books are missing. [All books missing: universal]
(12) No students' books are missing. [No books missing: existential]
(13) Most students' books are missing. [Various readings possible.]

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2 'Freedom' of the possessor relation: "John's books" could be the books he owns, wrote, borrowed, recently read, is standing on to reach the top shelf: implicit relation parameter.

Possessive quantifiers cont.

- (17) Most people's grandchildren hate them.

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- Here $R_a = \{b : R(a, b)\}$, and $dom_A(R) = \{a : A \cap R_a \neq \emptyset\}$.

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- But this does not seem to hold for the class of quantifiers denoted by possessive determiners.
- For example, "Mary's cars are nice" (universal reading) is

  $$\emptyset \neq car \cap own_m \subseteq nice$$

  (so *Mary's* has existential import), but the outer negation, "Either Mary has no cars or some car of hers is not nice", seems hard to get with a possessive Det.

Possessive squares 1

$Q =$ (all of) *Mary's* $= Poss(all_{\mathrm{ei}}, \{m\}, all, R)$. Q is $\mathrm{Mon}\!\uparrow$ and weakly $\downarrow\!\mathrm{Mon}$, so it belongs in the **A** corner:

Mary's A are B $\emptyset \neq A \cap R_m \subseteq B$	none of Mary's A are B $\emptyset \neq A \cap R_m \subseteq \overline{B}$
Mary has no A or some of her A are B $A \cap R_m = \emptyset \ \vee \ A \cap R_m \not\subseteq \overline{B}$	Mary has no A or not all of her A are B $A \cap R_m = \emptyset \ \vee \ A \cap R_m \not\subseteq B$

This square happens to be classical as well.

Mary's: comments

- However: (a) the **I** and **O** corners are unnatural, and (b) all quantifiers of the form Q_2 *of Mary's* have 'possessive existential import', in the sense that their truth conditions entail $A \cap R_m \neq \emptyset$.

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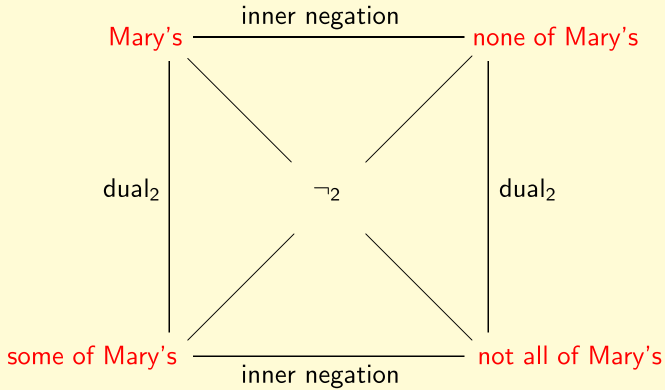
- Now, we obtain:

Mary's once again



This can be seen as either *square*(*Mary's*) under the presupposition $A \cap R_m \neq \emptyset$, or as *square*$^{\text{poss}}$(*Mary's*), where, for possessive Q,

$$square^{\text{poss}}(Q) = \{Q, Q\neg, \neg_2 Q, Q^{d_2}\}.$$

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- **FACT 12:** If Q' belongs to $square^{\text{poss}}(Q)$, then $square^{\text{poss}}(Q') = square^{\text{poss}}(Q)$.

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- (Of course it cannot mean that some of her friends are nice either (Q^{d_2}), but that holds for all squares: dual is not a negation, but a form of *double* negation.)

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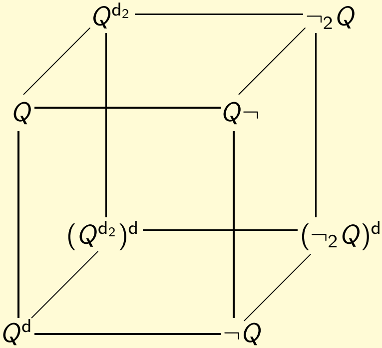
- **FACT 15:** Of the 16 combinations of $Q = Q_1, Q_2$ in $Poss(Q_1,C,Q_2,R)$ (Q, $Q\neg$, $\neg Q$, Q^{d}), only 8 yield distinct results.

A cube of opposition

This allows arranging all the ‘possessive oppositions’ in a cube of opposition, with inner negation along 4 sides, dual along 4 sides, and dual$_2$ along the remaining 4:



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- Let us look at the possible squares for these.

square(*no car's*) (existential)

$Q(A,B) \Leftrightarrow Poss(no, C, some, R)(A,B) \Leftrightarrow C \subseteq \{a\colon A \cap R_a \subseteq \overline{B}\}$. By a theorem in P & W 2006 (or directly), Q is $\downarrow$Mon$\downarrow$; hence in the **E** corner. In contrast with *Mary's*, Q has no 'possessive existential import'.

each car's A were (all) B
$C \subseteq \{a\colon A \cap R_a \subseteq B\}$
(cf. Fact 13)

no car's A were B
$C \subseteq \{a\colon A \cap R_a \subseteq \overline{B}\}$

at least one of some car's A was B
$C \not\subseteq \{a\colon A \cap R_a \subseteq \overline{B}\}$

not all of some car's A were B
$C \not\subseteq \{a\colon A \cap R_a \subseteq B\}$

$\textit{square}^{\text{poss}}(\textit{no car's})$ (existential)

The possessive square for *no car's* (existential) differs at the **I** and **O** corner. Interestingly, at the **O** corner we get the universal reading of *no car's*; the one I claimed was sometimes reasonable for *No student's books were returned*. Analogously for the **A** and **I** corners:

$$
\begin{array}{ccc}
\text{each car's } A \text{ were } B \text{ (univ.)} & \xrightarrow{\text{inner neg.}} & \text{no car's } A \text{ were } B \text{ (exist.)} \\
C \subseteq \{a\colon A \cap R_a \subseteq B\} & & C \subseteq \{a\colon A \cap R_a \subseteq \overline{B}\} \\
\text{dual}_2 & \neg_2 & \text{dual}_2 \\
\text{each car's } A \text{ were } B \text{ (exist.)} & \xrightarrow[\text{inner neg.}]{} & \text{no car's } A \text{ were } B \text{ (univ.)} \\
C \cap dom_A(R) \subseteq \{a\colon A \cap R_a \cap B \neq \emptyset\} & & C \cap dom_A(R) \subseteq \{a\colon A \cap R_a \cap \overline{B} \neq \emptyset\}
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- But we saw that, in spite of this, all eight quantifiers occupy a unique corner in *cube*(*no students'*) (since sufficiently many of them are doubly monotone).