An Introduction to
Mildly Context-Sensitive Grammar Formalisms

— *Linear Context-Free Rewriting Systems* —

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- Linear context-free rewriting systems (LCFRSs) can be considered as a special case of Generalized Context-Free Grammars (GCFGs) (Pollard 1984).
- As string rewriting systems, LCFRSs provide a generalization of HGs, and are weakly equivalent to set-local MCTAGs.

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$L(G)$, the **language derivable by** G, is the language $L_G(S)$

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  and f is total and regular

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  $z_i \in \text{Strings}(X \cup V_T)$ such that each x_{ij} appears at most once

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$$\begin{aligned} conc &: \big\langle \langle x_1, x_2 \rangle \big\rangle \mapsto \langle x_1 x_2 \rangle \\ f &: \big\langle \langle x_1, x_2 \rangle \big\rangle \mapsto \langle a x_1 b, c x_2 d \rangle \\ g &: \quad \big\langle \big\rangle \quad \mapsto \langle \varepsilon, \varepsilon \rangle \end{aligned}$$

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$$L(G_1) = \Big\{ a^n b^n c^n d^n \Big| n \geq 0 \Big\}$$

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$$L(G_2) = \Big\{ a^n b^n a^m b^m c^n d^n c^m d^m \Big| m, n \geq 0 \Big\}$$