Computing quantifier scope by sequential or simultaneous expansion

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Abstract

The paper develops, based on Akiba (2009), an analysis of quantification in which quantifier scope readings as well as the cumulative readings are derived from the same semantic entity (the underspecified linguistic meaning of the clause) by applying to it different semantic expansion operations. Applying the sequential expansion operations to the underspecified linguistic meaning in different orders yields the different scope readings, while applying the simultaneous expansion operation yields the cumulative reading. Depending on how the denotation of a formula is expanded, this formula may receive different interpretations, so meaning assignment is not functional but relational. Three important aspects will be emphasized: (i) NP denotations are assigned their semantic role without simultaneously fixing the scope relations, (ii) there is no need to postulate different constituent structures for different quantifier scope readings, and (iii) no storage is needed.

1 Introduction

The starting point of this paper is the theory of quantification and in particular the idea of denotation expansion presented in Akiba (2009). This novel theory of quantification (and distributivity, which shall be ignored here) is based on a novel kind of entities called shadows. Shadows may be conjunctive, disjunctive or negative, are *individual-like* in the sense that they are not abstract entities, they exist in space-time and have other properties. Despite the fact that shadows have unusual properties compared to individuals in that they may be incomplete (the shadow denoted by *Ann and Bill* is neither male nor female) or inconsistent (the shadow of *Ann or Bill* is both male and female), the first main motivation for shadow theory is the construction a semantic theory that sustains the compositionality principle while keeping the denotations of proper names and similar singular terms at the individual level. The second main motivation for adopting shadows is that they allow for a uniform and unified analysis of distributivity, since a shadow may have properties that none of its constituents has.

In order to simplify things for the sake of presentation, terms are not interpreted by shadows, but by sets of subsets of elements in the domain D. The main innovations of the present paper concern three properties of Akiba's theory. First, in Akiba's theory term denotations can only be combined after the (sequential) expansion of the verb denotation. We shall propose a mechanism which allows the NP denotation to be combined with the verb denotation before expanding the verb denotation. Second, we shall propose an operation of simultaneous expansion, which allows the derivation of cumulative readings from the same underspecified meaning from which the direct and inverse scope readings are computed. And finally, the present paper will propose a relational meaning assignment as opposed to the functional meaning assignment proposed by Akiba. The present paper will not presuppose familiarity with this theory.

Section 2 will present the proper name, common noun, determiner and NP denotations. Section 3 will present the denotation of verbs as well as the operations combining NP and V denotations, section 4 will then introduce and illustrate sequential and simultaneous expansion of denotations, section 5 presents the truth definition, section 6 presents the relational meaning assignment, and section 7 concludes.

2 NP denotations

Let $D = \{a, b, c, d, e, f, g, \bot\}$[1] be the domain of entities and *Ann, Bill, Chris, Dan, Ed, Fred, Greg* be proper names, then one possibility would be to interpret names as the set of all subsets in which a particular entity in D occurs, as done e.g. in Montague (1974):

$$\begin{array}{lll} \|Ann\| & = \{X : X \subseteq D \wedge a \in X\} & = I_a \\ \|Bill\| & = \{X : X \subseteq D \wedge b \in X\} & = I_b \\ \|Chris\| & = \{X : X \subseteq D \wedge c \in X\} & = I_c \\ \|Dan\| & = \{X : X \subseteq D \wedge d \in X\} & = I_d \\ \|Ed\| & = \{X : X \subseteq D \wedge e \in X\} & = I_e \\ \|Fred\| & = \{X : X \subseteq D \wedge f \in X\} & = I_f \\ \|Greg\| & = \{X : X \subseteq D \wedge g \in X\} & = I_g \end{array}$$

Common nouns are interpreted as follows:

$$\begin{array}{ll} \|professor(s)\| & = \{X : X \subseteq D \wedge (a \in X \vee b \in X \vee c \in X)\} \\ & = I_a \cup I_b \cup I_c \\ & = \|Ann\| \cup \|Bill\| \cup \|Chris\| \end{array}$$

$$\begin{array}{ll} \|students(s)\| & = \{X : X \subseteq D \wedge (d \in X \vee e \in X \vee f \in X \vee g \in X)\} \\ & = I_d \cup I_e \cup I_f \cup I_g \\ & = \|Dan\| \cup \|Ed\| \cup \|Fred\| \cup \|Greg\| \end{array}$$

the underlying idea being that *professor(s)* denotes the set of all subsets of D which contain a professor. The interpretation of noun phrases is illustrated below:

$$\begin{array}{lll} \|exactly\,two\,professors\| & = & (\|Ann\| \cap \|Bill\| \cap \neg\|Chris\|) \cup \\ & & (\|Ann\| \cap \neg\|Bill\| \cap \|Chris\|) \cup \\ & & (\neg\|Ann\| \cap \|Bill\| \cap \|Chris\|) \\ & = & (I_a \cap I_b \cap \overline{I_c}) \cup \\ & & (I_a \cap \overline{I_b} \cap I_c) \cup \\ & & (\overline{I_a} \cap I_b \cap I_c) \\ \|most\,professors\| & = & (\|Ann\| \cap \|Bill\| \cap \neg\|Chris\|) \cup \\ & & (\|Ann\| \cap \neg\|Bill\| \cap \|Chris\|) \cup \\ & & (\neg\|Ann\| \cap \|Bill\| \cap \|Chris\|) \cup \\ & & (\|Ann\| \cap \|Bill\| \cap \|Chris\|) \\ & = & (I_a \cap I_b \cap \overline{I_c}) \cup \\ & & (I_a \cap \overline{I_b} \cap I_c) \cup \\ & & (\overline{I_a} \cap I_b \cap I_c) \cup \\ & & (I_a \cap I_b \cap I_c) \\ \|no\,professor\| & = & \neg\|Ann\| \cap \neg\|Bill\| \cap \neg\|Chris\| \\ & = & \overline{I_a} \cap \overline{I_b} \cap \overline{I_c} \end{array}$$

Terms are defined as follows:

1. proper names are terms
2. if t is a term, then $f_\neg(t) = not\ t$ is a term

[1]The use of $\bot$ will be explained below.

3. if t_1 and t_2 are terms, then $f_\wedge(t_1, t_2) = t_1\ and\ t_2$ is a term

4. if t_1 and t_2 are terms, then $f_\vee(t_1, t_2) = t_1\ or\ t_2$ is a term

5. if cn is a common noun and $n > 0$, then $f_{=n}(cn) = exactly\ n\ cn$ is a term

6. if cn is a common noun, then $f_{most}(cn) = most\ cn$ is a term

7. if cn is a common noun, then $f_{=0}(cn) = no\ cn$ is a term

8. ...

To define the denotation of determiners we use the following auxiliary predicate. If X is a subset of the domain D of elements, let $\text{SGT}_D(X)$ be the set of elements d of D for which the singleton set $\{d\}$ is in X:

$$\text{SGT}_D(X) = \{d : d \in D \wedge \{d\} \in X\}$$

For example,

$$\begin{aligned}\text{SGT}_D(\|professor\|) &= \{d : d \in D \wedge \{d\} \in \|professor\|\} \\ &= \{a, b, c\} \\ \text{SGT}_D(\|student\|) &= \{d : d \in D \wedge \{d\} \in \|student\|\} \\ &= \{d, e, f, g\}\end{aligned}$$

Then the denotation of determiners can be formulated as follows, where $n > 0$:

$$\begin{aligned}\|f_{=n}(N)\| &= \|exactly\ n\ N\| &&= \{X : X \in \|N\| \wedge |X \cap \text{SGT}_D(\|N\|)| = n\} \\ \|f_{most}(N)\| &= \|most\ N\| &&= \{X : X \in \|N\| \wedge |X \cap \text{SGT}_D(\|N\|)| > \tfrac{1}{2}|\text{SGT}_D(\|N\|)|\} \\ \|f_{=0}(N)\| &= \|no\ N\| &&= \{X : X \in \|N\| \wedge |X \cap \text{SGT}_D(\|N\|)| = 0\}\end{aligned}$$

So the denotation of *exactly two professors* ends up being the set of subsets of D which contain exactly two professors:

$$\begin{aligned}\|exactly\ two\ professors\| &= \{X : X \in \|professors\| \wedge |X \cap \text{SGT}_D(\|professors\|)| = 2\} \\ &= \{X : X \in \|professors\| \wedge |X \cap \{a, b, c\}| = 2\} \\ &= (I_a \cap I_b \cap \overline{I_c}) \cup \\ &\quad\ (I_a \cap \overline{I_b} \cap I_c) \cup \\ &\quad\ (\overline{I_a} \cap I_b \cap I_c)\end{aligned}$$

The denotation of terms can now be defined as follows:

1. if p is a proper name, then $\|p\| = I(p)$

2. if t is a term, then $\|f_\neg(t)\| = \wp(D) - \|t\|$

3. if t_1 and t_2 are terms, then $\|f_\wedge(t_1, t_2)\| = \|t_1\| \cap \|t_2\|$

4. if t_1 and t_2 are terms, then $\|f_\vee(t_1, t_2)\| = \|t_1\| \cup \|t_2\|$

5. if cn is a common noun and $n > 0$, then $\|f_{=n}(cn)\| = \{X : X \in \|cn\| \wedge |X \cap \text{SGT}_D(\|cn\|)| = n\}$

6. if cn is a common noun, then $\|f_{most}(cn)\| = \{X : X \in \|cn\| \wedge |X \cap \text{SGT}_D(\|cn\|)| > \frac{1}{2}|\text{SGT}_D(\|cn\|)|\}$

7. if cn is a common noun, then $\|f_{=0}(cn)\| = \{X : X \in \|N\| \wedge |X \cap \text{SGT}_D(\|N\|)| = 0\}$

8. ...

Instead of assuming that the denotation of proper names is a set of subsets containing a particular element in D, one could also assume that the denotation of proper names is simply an element of D, e.g. $\|Ann\| = a$. The trade-off is that we now need a slightly more complicated definition of the semantics of conjunction and disjunction (clause 3 and 4), e.g.:

$$\|f_\wedge(t_1, t_2)\| = A \cap B$$

where

$$A = \begin{cases} \{X : X \subseteq D \wedge \|t_1\| \in X\} & \text{if } t_1 \text{is a proper name} \\ \|t_1\| & \text{otherwise} \end{cases}$$

and

$$B = \begin{cases} \{X : X \subseteq D \wedge \|t_2\| \in X\} & \text{if } t_2 \text{is a proper name} \\ \|t_2\| & \text{otherwise} \end{cases}$$

Under this alternative analysis one could then also simplify the denotation of common nouns and of determiners, so that e.g. $\|\mathit{professor(s)}\| = \{a, b, c\}$ and $\|f_{=0}(cn)\| = \{X : X \subseteq D \wedge |X \cap \|cn\|\,| = 0\}$.

3 VP denotations

We assume that verbs denote sets of tuples of sets of subsets of elements in D, e.g.:

$$\begin{array}{lcl} \|laughed\| & = & \{\langle I_b\rangle, \langle I_c\rangle\} \\ \|examined\| & = & \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_a, I_f\rangle, \langle I_b, I_d\rangle, \langle I_b, I_e\rangle, \langle I_b, I_g\rangle\} \\ \|praised\| & = & \{\langle I_a, I_d\rangle, \langle I_b, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_a, I_f\rangle, \langle I_c, I_f\rangle\} \\ \|criticized\| & = & \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\} \end{array}$$

We assume that the assignment of semantic roles to NP denotations is independent of the determination of scope relations. In particular, semantic roles can be assigned to quantified NPs without the scope relations being determined. Consequently, the combination of verb and NP denotations should result in an underspecified semantic entity which is then specified by means of semantic operations. We now address the question how verb and NP denotations combine and what the resulting underspecified denotation of a clause is.

Let $\|V\|$ be the denotation of a verb (i.e. a set of n tuples of sets of subsets of elements in D, with $n > 0$), and let $\perp$ be an element in D which does not occur in any noun phrase or verb denotation. One could think of this element as the 'unspecified entity', since it will be used to implement underspecification. Let $\sigma_n^\perp$ be the n-ary sequence of $\perp$, i.e. $\langle \perp_1, \ldots, \perp_n\rangle$, and $\sigma_n^\perp[i/x]$ be the result of replacing the i-th projection $\pi_i(\sigma_n^\perp)$ of $\sigma_n^\perp$ with x. Then the combination of NP denotations with V denotations could be defined preliminarily as:

$$O_i(\|NP\|, \|V\|) = \|V\| \cup \{\sigma_n^\perp[i/\|NP\|]\}$$

That is, if an NP denotation is to saturate the i-th position of a verb denotation, we add to the verb denotation the $\perp$-sequence where the entity $\perp$ in the i-th position has been replaced by the NP denotation. For example $\|V\|$ is a set of pairs, we have:

$$O_1(\|NP\|, \|V\|) = \{\langle \|NP\|, \perp\rangle\} \cup \|V\|$$

$$O_2(\|V\|, \|NP\|) = \{\langle \perp, \|NP\|\rangle\} \cup \|V\|$$

To give an example consider the denotation of (1):

(1) *Exactly two professors examined exactly three students.*

Combining $\|$*Exactly two professors*$\|$ with $\|$*examined*$\|$ by means of O_1 results in:

$$O_1(\|\textit{exactly two professors}\|, \|\textit{examined}\|) = \|\textit{examined}\| \cup \{\langle \|\textit{exactly two professors}\|, \perp\rangle\}$$

and combining this with $\|$*exactly three students*$\|$ results in the following meaning for (1):

$$\|(1)\| = O_2(\|exactly\ two\ professors\ examined\|, \|\textit{exactly three students}\|) =$$

$$\|\textit{examined}\| \cup \{\langle \|\textit{exactly two professors}\|, \bot\rangle\} \cup \{\langle \bot, \|\textit{exactly three students}\|\rangle\}$$

This, then, is the underspecified meaning of the sentence (1).

4 Expansion

The basic idea is that a sentence ϕ is true iff $\|\phi\| \neq \varnothing$ and all sequences $\sigma \in \phi$ containing $\bot$ are eliminable. First we define expansion, then eliminabilitiy and finally we give the truth definition. Formula denotations can be expanded in different ways. We shall introduce two, namely sequential and simultaneous expansion. Finally we define the eliminability of sequences.

4.1 Sequential expansion

Sequential expansion consists in the (repeated) application of the sequential expansion operation $\textsc{exp}_i$. The idea is that this operation expands the sentence denotation with certain sequences $\sigma' \sim_i \sigma$ (i.e. with new sequences which are like old sequences except for the element in position i).

The basic idea behind sequential expansion is that expanding the i-th projection of $\|(1)\|$ results in a denotation which satisfies the following conditions:

- Base condition: if a tuple belongs to the initial denotation, then it also belongs to its i-expansion.

  $\langle I_a, I_d\rangle \in \|(1)\|$
  therefore $\langle I_a, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$

- Conjunction introduction: if the expansion contains two tuples σ_1, σ_2 which are the same except that $\pi_i(\sigma_1) = X$ and $\pi_2(\sigma_2) = Y$, then the i-expansion also contains the tuple σ_3 which is like σ_1 except that $\pi_i(\sigma_3) = X \cap Y$:

  $\langle I_a, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$ and $\langle I_b, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$
  therefore by expansion of the first projection $\langle I_a \cap I_b, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$

- Disjunction introduction: if the expansion contains a tuple σ with $\pi_i(\sigma) = X$ and $Y \subseteq \wp(D)$, then the expansion also contains σ' which is like σ except that $\pi_i(\sigma') = X \cup Y$:

  $\langle I_a, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$
  therefore by expansion of the first projection $\langle I_a \cup I_b, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$

- Negation introduction: if the expansion contains a tuple σ with $\pi_i(\sigma) = X$, and there is a set Y with $Y \neq X$ such that there is no σ' like σ except that $\pi_i(\sigma') = Y$, then the expansion contains the tuple σ'' like σ except that $\pi_i(\sigma'') = \overline{Y}$

  $\langle I_a, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$ and $\langle I_c, I_d\rangle \notin \textsc{exp}_1(\|(1)\|)$
  therefore $\langle \overline{I_c}, I_d\rangle \in \textsc{exp}_1(\|(1)\|)$

More formally, sequential expansion is defined as follows: Let $\|\phi\|$ be the denotation of a formula, and $\mathcal{X}, \mathcal{Y}$ stand for sets of subsets of E, i.e. $\mathcal{X}, \mathcal{Y} \subseteq \wp(E)$. Then $\textsc{exp}_i(\|\phi\|)$ is the smallest set of sequences such that:

1. for all σ,
   if $\sigma \in \|\phi\|$, then $\sigma \in \textsc{exp}_i(\|\phi\|)$,

2. for all σ, τ
   if $\sigma \sim_i \tau, \pi_i(\sigma) = \mathcal{X}, \pi_i(\tau) = \mathcal{Y}$ and $\sigma, \tau \in \text{EXP}_i(\|\phi\|)$,
   then $\sigma_i[\mathcal{X}/\mathcal{X} \cap \mathcal{Y}] \in \text{EXP}_i(\|\phi\|)$,

3. for all σ,
   if $\sigma \in \text{EXP}_i(\|\phi\|), \pi_i(\sigma) = \mathcal{X}, \mathcal{X} \subseteq \mathcal{Y}$,
   then $\sigma_i[\mathcal{X}/\mathcal{Y}] \in \text{EXP}_i(\|\phi\|)$,

4. $\forall\sigma\forall\mathcal{X}$:
   if $\sigma \in \text{EXP}_i(\|\phi\|) \wedge \pi_i(\sigma) = \mathcal{X} \wedge \exists\mathcal{Y}[\mathcal{X} \neq \mathcal{Y} \wedge \neg\exists\tau[\sigma \sim_i \tau \wedge \pi_i(\tau) = \mathcal{Y}]]$,
   then $\sigma_i[\mathcal{X}/\overline{\mathcal{Y}}] \in \text{EXP}_i(\|\phi\|)$,

To illustrate expansion, we show first that

$$\langle \|exactly\ two\ professors\|, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|(1)\|))$$

1. $\langle I_b, I_d \rangle \in \|(1)\|$ therefore $\langle I_b, I_d \rangle \in \text{EXP}_2(\|(1)\|)$

2. $\langle I_b, I_e \rangle \in \|(1)\|$ therefore $\langle I_b, I_e \rangle \in \text{EXP}_2(\|(1)\|)$

3. $\langle I_b, I_g \rangle \in \|(1)\|$ therefore $\langle I_b, I_g \rangle \in \text{EXP}_2(\|(1)\|)$

4. $\langle I_b, I_f \rangle \notin \text{EXP}_2(\|(1)\|)$ therefore $\langle I_b, \overline{I_f} \rangle \in \text{EXP}_2(\|(1)\|)$

5. $\langle I_b, I_d \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_b, I_e \rangle \in \text{EXP}_2(\|(1)\|)$
   therefore $\langle I_b, I_d \cap I_e \rangle \in \text{EXP}_2(\|(1)\|)$

6. $\langle I_b, I_d \cap I_e \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_b, I_g \rangle \in \text{EXP}_2(\|(1)\|)$
   therefore $\langle I_b, I_d \cap I_e \cap I_g \rangle \in \text{EXP}_2(\|(1)\|)$

7. $\langle I_b, I_d \cap I_e \cap I_g \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_b, \overline{I_f} \rangle \in \text{EXP}_2(\|(1)\|)$, therefore $\langle I_b, I_d \cap I_e \cap \overline{I_f} \cap I_g \rangle \in \text{EXP}_2(\|(1)\|)$

8. $\langle I_b, I_d \cap I_e \cap \overline{I_f} \cap I_g \rangle \in \text{EXP}_2(\|(1)\|)$ therefore $\langle I_b, (I_d \cap I_e \cap I_f \cap \overline{I_g}) \cup (I_d \cap I_e \cap \overline{I_f} \cap I_g) \cup (I_d \cap \overline{I_e} \cap I_f \cap I_g) \cup (\overline{I_d} \cap I_e \cap I_f \cap I_g) \rangle \in \text{EXP}_2(\|(1)\|)$ so $\langle I_b, \|exactly\ three\ students\| \rangle \in \text{EXP}_2(\|(1)\|)$

9. $\langle I_a, I_d \rangle \in \|(1)\|$ therefore $\langle I_a, I_d \rangle \in \text{EXP}_2(\|(1)\|)$

10. $\langle I_a, I_e \rangle \in \|(1)\|$ therefore $\langle I_a, I_e \rangle \in \text{EXP}_2(\|(1)\|)$

11. $\langle I_a, I_f \rangle \in \|(1)\|$ therefore $\langle I_a, I_f \rangle \in \text{EXP}_2(\|(1)\|)$

12. $\langle I_a, I_g \rangle \notin \text{EXP}_2(\|(1)\|)$ therefore $\langle I_a, \overline{I_g} \rangle \in \text{EXP}_2(\|(1)\|)$

13. $\langle I_a, I_d \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_a, I_e \rangle \in \text{EXP}_2(\|(1)\|)$
    therefore $\langle I_a, I_d \cap I_e \rangle \in \text{EXP}_2(\|(1)\|)$

14. $\langle I_a, I_d \cap I_e \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_a, I_f \rangle \in \text{EXP}_2(\|(1)\|)$
    therefore $\langle I_a, I_d \cap I_e \cap I_f \rangle \in \text{EXP}_2(\|(1)\|)$

15. $\langle I_a, I_d \cap I_e \cap I_f \rangle \in \text{EXP}_2(\|(1)\|)$ and $\langle I_a, \overline{I_g} \rangle \in \text{EXP}_2(\|(1)\|)$, therefore $\langle I_a, I_d \cap I_e \cap I_f \cap \overline{I_g} \rangle \in \text{EXP}_2(\|(1)\|)$

16. $\langle I_a, I_d \cap I_e \cap I_f \cap \overline{I_g} \rangle \in \text{EXP}_2(\|(1)\|)$ therefore $\langle I_a, (I_d \cap I_e \cap I_f \cap \overline{I_g}) \cup (I_d \cap I_e \cap \overline{I_f} \cap I_g) \cup (I_d \cap \overline{I_e} \cap I_f \cap I_g) \cup (\overline{I_d} \cap I_e \cap I_f \cap I_g) \rangle \in \text{EXP}_2(\|(1)\|)$ so $\langle I_a, \|exactly\ three\ students\| \rangle \in \text{EXP}_2(\|(1)\|)$

17. $\langle I_a, \|exactly\ three\ students\| \rangle \in \text{EXP}_2(\|(1)\|)$, so $\langle I_a, \|exactly\ three\ students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|(1)\|))$

18. $\langle I_b, \|exactly\, three\, students\| \rangle \in$ EXP$_2(\|(1)\|)$, so $\langle I_b, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$

19. $\langle I_a, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$, $\langle I_b, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$, so $\langle I_a \cap I_b, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$

20. $\langle I_c, \|exactly\, three\, students\| \rangle \notin$ EXP$_1($EXP$_2(\|(1)\|))$, so $\langle \overline{I_c}, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$

21. $\langle I_a \cap I_b, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$, $\langle \overline{I_c}, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$, so $\langle I_a \cap I_b \cap \overline{I_c}, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$

22. $\langle I_a \cap I_b \cap \overline{I_c}, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$, so $\langle (I_a \cap I_b \cap \overline{I_c}) \cup (I_a \cap \overline{I_b} \cap I_c) \cup (\overline{I_a} \cap I_b \cap I_c), \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$ and so
$\langle \|exactly\, two\, professors\|, \|exactly\, three\, students\| \rangle \in$ EXP$_1($EXP$_2(\|(1)\|))$

The notion of truth will be defined such that (1) is true if the second projection of the clause denotation is expanded before the first projection. What this means is that the sentence is true under the wide scope reading of the subject NP. However, the sentence is false under the narrow scope reading of the subject NP. The reason we cannot derive

$$\langle \|exactly\, two\, professors\|, \|exactly\, three\, students\| \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

is that we cannot find three students which were examined by two (possibly different) professors. Both I_d and I_e were examined by two professors, namely by I_a, I_b, but I_f and I_g were examined only by one professor each, namely I_a and I_b respectively. That is, if we expand the first projection first, then we can only derive that

$$\langle \|exactly\, two\, professors\|, I_e \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

and

$$\langle \|exactly\, two\, professors\|, I_e \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

but we cannot derive that

$$\langle \|exactly\, two\, professors\|, I_f \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

or

$$\langle \|exactly\, two\, professors\|, I_g \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

and therefore we cannot derive

$$\langle \|exactly\, two\, professors\|, \|exactly\, three\, students\| \rangle \in \text{EXP}_2(\text{EXP}_1(\|examined\|))$$

In the same way it can be shown that for the sentence

(2) *Exactly two professors praised exactly three students.*

we can derive the reading with wide scope for the object *exactly three students* (by expanding the first projection before the second projection),

$$\langle \|exactly\, two\, professors\|, \|exactly\, three\, students\| \rangle \in \text{EXP}_2(\text{EXP}_1(\|(2)\|))$$

but not the reading with narrow scope for the object.

$$\langle \|exactly\, two\, professors\|, \|exactly\, three\, students\| \rangle \in \text{EXP}_1(\text{EXP}_2(\|(2)\|))$$

4.2 Simultaneous expansion

Consider next the sentence

(3) *Exactly two professors criticized exactly three students.*

It can easily be seen that if $\|criticized\| = \{\langle I_a, I_d\rangle, \langle I_a, I_e\rangle, \langle I_b, I_e\rangle, \langle I_b, I_f\rangle\}$ then neither the subject narrow scope nor the subject wide scope reading can be derived. On the other hand, it is equally obvious that there are two professors examining, and there are exactly three students being examined. Thus (3) is true under the so-called cumulative reading.

To derive this reading we propose an expansion operation which expands the projections simultaneously. Let $\Pi_i(\|\phi\|) = \bigcap\{I_x : x \in D \wedge \exists\sigma \in \|\phi\|.\pi_i(\sigma) = I_x\}$, and ϕ be a formula of arity $n \leq a$, and $\mathcal{X}, \mathcal{Y} \subseteq \wp(D)$. Then $\mathrm{E}^{sim}(\|\phi\|)$ is the smallest set such that:

1. for all σ,
   if $\sigma \in \|\phi\|$ then $\sigma \in \textsc{exp}^{sim}(\|\phi\|)$
2. $\langle\Pi_1(\|\phi\|), \ldots, \Pi_{a+1}(\|\phi\|\rangle \in \textsc{exp}^{sim}(\|\phi\|)$
3. for all $i < n$ and all σ, τ:
   (a) if $\sigma \sim_i \tau, \pi_i(\sigma) = \mathcal{X}, \pi_i(\tau) = \mathcal{Y}$ and $\sigma, \tau \in \textsc{exp}^{sim}(\|\phi\|)$,
   then $\sigma_i[\mathcal{X}/\mathcal{X} \cap \mathcal{Y}] \in \textsc{exp}^{sim}(\|\phi\|)$,
   (b) if $\sigma \in \textsc{exp}^{sim}(\|\phi\|), \mathcal{X} \subseteq \mathcal{Y}$,
   then $\sigma_i[\mathcal{X}/\mathcal{Y}] \in \textsc{exp}^{sim}(\|\phi\|)$
   (c) if $\sigma \notin \textsc{exp}^{sim}(\|\phi\|)$ then $\sigma_i[\mathcal{X}/\overline{\mathcal{X}}] \in \textsc{exp}^{sim}(\|\phi\|)$

We can now show that

$$\langle\|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \in \textsc{exp}^{sim}\|(3)\|))$$

1. by (1) $\langle I_a \cap I_b, I_d \cap I_e \cap I_f\rangle \in \textsc{exp}^{sim}(\|(3)\|)$
2. by (3c) $\langle I_c, I_d \cap I_e \cap I_f\rangle \notin \textsc{exp}^{sim}(\|(3)\|)$ therefore
   $\langle\overline{I_c}, I_d \cap I_e \cap I_f\rangle \in \textsc{exp}^{sim}(\|(3)\|)$
3. by (3a) $\langle I_a \cap I_b \cap \overline{I_c}, I_d \cap I_e \cap I_f\rangle \in \textsc{exp}_{1,2}(\|(3)\|)$
4. by (3b) $\langle\|exactly\ two\ professors\|, I_d \cap I_e \cap I_f\rangle \in \textsc{exp}^{sim}(\|(3)\|)$
   $\langle\|exactly\ two\ professors\|, \|exactly\ three\ students\|\rangle \in \textsc{exp}^{sim}(\|(3)\|)$

5 Truth

A denotation $\|\phi\|$ of a formula ϕ is called **proper** if and only if there is a sequence $\sigma \in \|\phi\|$ not containing $\perp$ such that for all sequences $\sigma' \in \|\phi\|$ containing at least one $\perp$ it holds that for all i:

- if $\pi_i(\sigma') = \perp$ then $\pi_i(\sigma) \neq \perp$,
- if $\pi_i(\sigma') \neq \perp$ then $\pi_i(\sigma') = \pi_i(\sigma)$

The idea is that a a sentence ϕ should be **true** iff $\|\phi\|$ has a proper expansion (i.e. iff there is an x such that (i) x is a result of expanding $\|\phi\|$, and (ii) x is proper).

Before this definition can be finalized we need to address the following problem. Assume that $\|laughed\| = \{\langle I_b\rangle, \langle I_c\rangle\}$, and that $\|Ann\| = I_a$, so that the sentence *Ann laughed.* should be predicted to be false. The result of combining the denotation of *Ann* with the denotation of *laughed* is $\|laughed\| \cup \{\langle\|Ann\|\rangle\}$, and therefore $\langle\|Ann\|\rangle \in EXP_1(\|laughed\| \cup \{\langle\|Ann\|\rangle\})$ predicting *Ann laughed.* to be true. We block this by introducing the notion of a (negative) completion of a verb denotation, according to which if $\|laughed\| = \{\langle I_b\rangle, \langle I_c\rangle\}$ then $\mathbf{c}(\|laughed\|) = \{\langle\overline{I_a}\rangle, \langle I_b\rangle, \langle I_c\rangle, \langle\overline{I_d}\rangle, \langle\overline{I_e}\rangle, \langle\overline{I_f}\rangle, \langle\overline{I_g}\rangle\}$. Now that we have $\overline{I_a}$ in the denotation, we can predict the sentence *Ann laughed.* to be false by requiring that true sentences be precisely those whose denotations are consistent and proper. A denotation $\|\phi\|$ of a formula ϕ is called **consistent** if and only if $\neg\exists\sigma\exists\sigma'\exists i\exists t.(\sigma \sim_i \sigma' \wedge \pi_i(\sigma) = t \wedge \pi_i(\sigma') = \bar{t})$. The **completion** of a verb denotation (i.e. of a set of n-ary tuples of I_x, with $x \in D$) is defined as follows:

$$\mathfrak{c}(X) = X \cup \{\langle\overline{I_{x_1}}, \ldots, \overline{I_{x_n}}\rangle : x_1, \ldots, x_n \in D \wedge \langle I_{x_1}, \ldots, I_{x_n}\rangle \notin X\}$$

Alternatively, one could proceed as follows. First, assume that the denotations of verbs are sets of tuples of elements in D, secondly assume that the expansion operations contain an additional clause to the effect that if $x \in \|\phi\|$ then $\{X : X \subseteq D \wedge x \in X\} \in EXP_i(\|\phi\|)$, and thirdly the combination operations has to be adjusted, such that the combination of an element x of D with a verb denotation $\|V\|$ results in all the tuples in $\|V\|$ which have x in the i-th projection, and the combination of an element of a different type results in the addition of the tuple $\{\sigma_n^{\perp}[i/x]\}$ to the verb denotation:

$$O_i(\|NP\|, \|V\|) = \begin{cases} \{\sigma : \sigma \in \|V\| \wedge \pi_i(\sigma) = \|NP\|\}, & \text{if } \|NP\| \in D \\ \|V\| \cup \{\sigma_n^{\perp}[i/\|NP\|]\}, & \text{otherwise} \end{cases}$$

6 Relational meaning assignment

The set of signs can be defined by stating what the basic signs are (first two clauses) and how complex signs can be derived from (basic or complex) signs (the rest of the clauses).

The set of signs is the smallest set such that:

1. if t is a term whose denotation is $\|t\|$, then $\langle t, \|t\|\rangle$ is a (term) sign
2. if P is an n-ary predicate symbol with $\|P\| = I(P)$, then $\langle P(x_0, \ldots x_{n-1}), \|P\|\rangle$ is a sign.
3. if $\langle\phi, \|\phi\|\rangle$ is a sign, then $\langle\neg\phi, (\wp(\wp(D)))^{a+1} - \|\phi\|\rangle$ is a sign
4. if $\langle\phi, \|\phi\|\rangle$ and $\langle\psi, \|\psi\|\rangle$ are signs, then $\langle(\phi \wedge \psi), \|\phi\| \cap \|\psi\|\rangle$ is a sign
5. if $\langle\phi, \|\phi\|\rangle$ and $\langle\psi, \|\psi\|\rangle$ are signs, then $\langle(\phi \vee \psi), \|\phi\| \cup \|\psi\|\rangle$ is a sign
6. if $\langle\phi[x_i], \|\phi\|\rangle$ is a sign and $\langle t, \|t\|\rangle$ a term sign, then
   $\langle\phi[x_i/t], \|\phi\| \cup \{\sigma_i^{\perp}[\perp/\|t\|]\}\rangle$ is a sign
7. if $\langle\phi, \|\phi\|\rangle$ is a sign and $\alpha(\|\phi\|)$ an expansion of $\|\phi\|$, then $\langle\phi, \alpha(\|\phi\|)\rangle$ is a sign

7 Conclusion

We presented in this paper a theory of quantification, based on the idea of expansion presented in Akiba (2009). NP denotations can be combined with the verb denotation without simultaneously fixing the scope dependencies. The scope relations are determined by applying sequential or simultaneous expansion operations to the same semantic entity, the meaning of the entire clause.

We will conclude with two observations. First, the present theory does not require that sentences with different quantifier scope readings have different syntactic (constituent) structures. And secondly, the cumulative reading is derived from the same underspecified semantic entity from which the other quantifier scope readings are also derived.

References

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