Segmenting Temporal Intervals for Tense and Aspect

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Abstract

Timelines interpreting interval temporal logic formulas are segmented into strings which serve as semantic representations for tense and aspect. The strings have bounded but refinable granularity, suitable for analyzing (im)perfectivity, durativity, telicity, and various relations including branching.

1 Introduction

A sentence in the simple past, such as (1a), uttered at (speech) time S can be pictured as a timeline (1b), describing an event E (Ernest explaining) prior to S.

(1) a. Ernest explained.

b. $\boxed{E}\boxed{S}$ (depicting $E \prec S$)

We can view the event E in (1b) as an unbroken point, wholly to the left of S, $E \prec S$. By contrast, in the timeline (2a) for the progressive (2b), E splits into three boxes, the middle of which contains also a *reference time* R (Reichenbach, 1947).[1]

(2) a. $\boxed{E}\boxed{E,R}\boxed{E}$ (depicting $R \sqsubset E$)

b. Ernest explaining

The relation of R *inside* E, $R \sqsubset E$, breaks E apart, moving us away from conceptualizing E as a point. Indeed, it has become common practice in linguistic semantics since (Bennett and Partee, 1972) to evaluate temporal formulas at intervals, rather than simply points. Interval temporal logics are, however, notoriously more complex than ordinary (pointwise) temporal logics (Halpern and Shoham, 1991; Marcinkowski and Michaliszyn, 2013). That said, for linguistic applications to tense and aspect, the present paper derives strings such as (1b) and (2a) from timelines for interval temporal logic, in effect reducing these timelines to finite models of ordinary temporal logic. This reduction rests on certain assumptions that require explanation and defense.

We begin with temporal formulas, which for the sake of brevity, we hereafter call *fluents*. A fluent such as E, R or S can occur as a whole, as E and S do in (1b), or as segmented, as E does in (2a). We formulate the notions of *whole* and *segmented* model-theoretically in section 2, defining a map $\varphi \mapsto \varphi_\circ$ on fluents φ through which the picture (2a) is sharpened to (3) with $E_\circ$ segmented.

(3) $\boxed{E_\circ}\boxed{E_\circ,R}\boxed{E_\circ}$ (segmented $E_\circ$, whole R)

The map $\varphi \mapsto \varphi_\circ$ is essentially a universal grinder (the right half of an adjoint pair with a universal packager, max)

$$\frac{\text{whole}}{\text{segmented}} \approx \frac{\text{count}}{\text{mass}}$$

pointing to well-known "parallels between the mass-count distinction in nominal systems and the aspectual classification of verbal expressions" (Bach, 1986a). The aspectual classification to which the whole/segmented contrast pertains is that of perfectives and imperfectives

$$\frac{\text{whole}}{\text{segmented}} \approx \frac{\text{perfective}}{\text{imperfective}}$$

as opposed to Aktionsart. A variant of the Aristotle-Ryle-Kenny-Vendler aspectual classes (Dowty, 1979) which can be reduced to durativity and telicity (Comrie, 1976; Moens and Steedman, 1988; Pulman, 1997) is analyzed in section 3 through strings that arise naturally in the investigation of grinding in section 2.

Some restraint on grinding is called for, as the simplest strings are the most coarse-grained. Section 4 enshrines this restraint as a principle, whilst

[1] Boxes are drawn instead of the usual curly braces $\{,\}$ around the elements of a set to reinforce a reading of (1b) and (2a) as comic strips, with time standing still within a box, but between boxes, progressing from left to right.

accommodating refinements as required. The idea is that strings can be refined by enlarging some contextually supplied set X of (interesting) fluents: the larger X is, the finer the grain becomes. An inverse system of string functions π_X indexed by different finite sets X of fluents is constructed, and applied for an account of relations between strings as well as branching time. The relations here go beyond the familiar order $\prec$ for tense, stretching to the progressive and the perfect, from a variety of perspectives.

2 Segmented versus whole fluents

Fix a set Φ of fluents. Fluents in Φ are interpreted relative to a Φ-*timeline*, a triple $\mathfrak{A} = \langle T, \prec, \models \rangle$ consisting of a linear order $\prec$ on a non-empty set T of (temporal) points, and a binary relation $\models$ between intervals I (over $\prec$) and fluents $\varphi \in \Phi$. An interval is understood here to be a nonempty subset I of T with no holes — i.e. $t \in I$ whenever $t_1 \prec t \prec t_2$ for some pair of points t_1, t_2 in I.[2] $I \models \varphi$ is pronounced "φ holds at I" or "I satisfies φ" (in $\mathfrak{A}$).

A fluent φ is said to be $\mathfrak{A}$-*segmented* if for all intervals I and I' such that $I \cup I'$ is an interval, φ holds at I and at I' precisely if it does at their union

$$I \models \varphi \text{ and } I' \models \varphi \iff I \cup I' \models \varphi.$$

A simple way for a fluent φ to be $\mathfrak{A}$-segmented is by holding at an interval I precisely if it holds at all points of I

$$I \models \varphi \iff (\forall t \in I)\ \{t\} \models \varphi$$

in which case we say φ is $\mathfrak{A}$-*pointed*.[3] A fluent is $\mathfrak{A}$-*singular* if at most one interval satisfies it. Generalizing $\mathfrak{A}$-singular fluents, we call a fluent φ $\mathfrak{A}$-*whole* if for all intervals I and I' such that $I \cup I'$ is an interval,

$$I \models \varphi \text{ and } I' \models \varphi \text{ implies } I = I'.$$

That is, any number of intervals may satisfy a $\mathfrak{A}$-whole fluent so long as no two form an interval. A $\mathfrak{A}$-whole fluent φ defines a *quantized* predicate (Krifka, 1998) insofar as *no* two distinct intervals can satisfy φ if one is a subset of the other. But the ban on pairs of intervals satisfying φ is wider under $\mathfrak{A}$-wholeness. For example, over $T = \{1, 2\}$, a fluent holding at exactly $\{1\}$ and $\{2\}$ is *not* whole, even though $\{\{1\}, \{2\}\}$ is quantized.

$\mathfrak{A}$-wholeness shares half of $\mathfrak{A}$-segmentedness: a fluent φ is $\mathfrak{A}$-*summable* if for all intervals I and I' in $\mathfrak{A}$ such that $I \cup I'$ is an interval,

$$I \models \varphi \text{ and } I' \models \varphi \text{ implies } I \cup I' \models \varphi.$$

Apart from the restriction that $I \cup I'$ is an interval, $\mathfrak{A}$-summability coincides with additivity in (Bach, 1981), illustrated in (4).

(4) Ed slept from 3 to 5pm, Ed slept from 4 to 6pm $\vdash$ Ed slept from 3 to 6pm

The other half of $\mathfrak{A}$-segmentedness (differentiating it from $\mathfrak{A}$-wholeness) is the subinterval property (Bennett and Partee, 1972), enjoyed by states and activities.

(5) Ed slept from 3 to 6 $\vdash$ Ed slept from 3 to 5

A fluent φ is $\mathfrak{A}$-*subinterval-persistent* ($\mathfrak{A}$-sip) if for all intervals I and I' in $\mathfrak{A}$,

$$I \subseteq I' \text{ and } I' \models \varphi \text{ implies } I \models \varphi\ .$$

It is useful to associate with any fluent φ a fluent $\varphi_\circ$ that holds precisely at subintervals of intervals satisfying φ

$$I \models \varphi_\circ \iff (\exists I' \supseteq I)\ I' \models \varphi\ .$$

We say φ is $\mathfrak{A}$-*equivalent to* ψ and write $\varphi \equiv_{\mathfrak{A}} \psi$ if for every interval I,

$$I \models \varphi \iff I \models \psi.$$

Clearly, φ is $\mathfrak{A}$-sip iff $\varphi \equiv_{\mathfrak{A}} \varphi_\circ$. Also, $\varphi_\circ$ is $\mathfrak{A}$-sip and we can say more if φ is $\mathfrak{A}$-whole.

2.1 An adjoint pair

The map $\varphi \mapsto \varphi_\circ$ is one half of a pair for breaking down and building up fluents. To describe the other half, more definitions are helpful. Given a fluent φ and a relation r between intervals, let us form the modal fluent $\langle \mathrm{r} \rangle \varphi$ that holds at an interval r-related to one satisfying φ

$$I \models \langle \mathrm{r} \rangle \varphi \iff (\exists I')\ I \mathrel{\mathrm{r}} I' \text{ and } I' \models \varphi.$$

Note $\varphi_\circ$ is just $\langle \subseteq \rangle \varphi$. Apart from $\subseteq$, other useful examples of relations r between intervals I and I' include *full precedence* $\prec$

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\ t \prec t'$$

[2] Not much would be lost were we to take an interval I, as in (Halpern and Shoham, 1991), to be a pair of points t, t' with $t \preceq t'$, or, as in (Allen, 1983), $t \prec t'$.

[3] For finite T, $\mathfrak{A}$-segmented is the same as $\mathfrak{A}$-pointed.

and a relation $\mathtt{m}$ called *meet* in (Allen, 1983) and *abutment* in (Hamblin, 1971).

$$I \,\mathtt{m}\, I' \iff I \prec I' \text{ and } I \cup I' \text{ is an interval.}$$

Now, let $\mathtt{mi}$ be the inverse of $\mathtt{m}$

$$I \,\mathtt{mi}\, I' \iff I' \,\mathtt{m}\, I$$

and max be a function on fluents that maps a fluent ψ to its conjunction with $\neg\langle\mathtt{mi}\rangle\psi$ and $\neg\langle\mathtt{m}\rangle\psi$

$$\max(\psi) \;=\; \psi \wedge \neg\langle\mathtt{mi}\rangle\psi \wedge \neg\langle\mathtt{m}\rangle\psi.$$

Proposition 1.

(a) For all $\mathfrak{A}$-whole φ, $\varphi_\circ$ is $\mathfrak{A}$-segmented and $\varphi \equiv_{\mathfrak{A}} \max(\varphi_\circ)$.

(b) For all $\mathfrak{A}$-segmented ψ, $\max(\psi)$ is $\mathfrak{A}$-whole and $\psi \equiv_{\mathfrak{A}} (\max(\psi))_\circ$.

As to the promised adjunction, let us agree to write $\varphi_{\mathfrak{A}}$ for the set of intervals satisfying φ

$$\varphi_{\mathfrak{A}} \;=\; \{I \mid I \models \varphi\}$$

(so $\varphi \equiv_{\mathfrak{A}} \psi$ iff $\varphi_{\mathfrak{A}} = \psi_{\mathfrak{A}}$) from which we define two pre-orders on fluents

$$\begin{aligned} \psi \subseteq_{\mathfrak{A}} \psi' &\iff \psi_{\mathfrak{A}} \subseteq \psi'_{\mathfrak{A}} \\ \varphi \subseteq^{\mathfrak{A}} \varphi' &\iff (\forall I \in \varphi_{\mathfrak{A}})(\exists I' \in \varphi'_{\mathfrak{A}})\; I \subseteq I' \end{aligned}$$

that apply to $\mathfrak{A}$-segmented fluents ψ and $\mathfrak{A}$-whole fluents φ respectively, for the equivalence

$$\max(\psi) \subseteq^{\mathfrak{A}} \varphi \iff \psi \subseteq_{\mathfrak{A}} \varphi_\circ$$

making max left (lower) adjoint to (of) $\cdot_\circ$.

Next, we turn to linguistic applications and the correspondences

$$\frac{\text{whole}}{\text{segmented}} \approx \frac{\text{count}}{\text{mass}} \approx \frac{\text{perfective}}{\text{imperfective}}.$$

The notion that imperfectives are mass and perfectives count is argued in (Herweg, 1991), building on (Galton, 1984; Galton, 1987) for a concept of *pofective event-type* very close to that of $\mathfrak{A}$-whole fluent above. Perfectives contrast with imperfectives according to (6).

(6) a. viewed from outside, completed, closed

b. viewed from inside, ongoing, open-ended

Towards formalizing (6), let us say an interval I is *inside* an interval I', written $I \sqsubset I'$, if I' extends to the left and also to the right of I

$$\begin{aligned} I \sqsubset I' \iff & (\exists t' \in I')\, \{t'\} \prec I \text{ and} \\ & (\exists t'' \in I')\, I \prec \{t''\} \end{aligned}$$

(called *during* in (Allen, 1983)). Next, we introduce an $\mathfrak{A}$-whole fluent V for viewpoint to picture a perfective view (6a) of E and an imperfective view (6b) as (7a) and (7b) respectively.

(7) a. $\boxed{\mathrm{V}_\circ}\boxed{\mathrm{E},\mathrm{V}_\circ}\boxed{\mathrm{V}_\circ}$ (depicting E $\sqsubset$ V)

b. $\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ}$ (depicting V $\sqsubset$ E)

The idea now is to spell out what strings such as (7a) and (7b) mean.

2.2 Segmentations and strings

A *segmentation (seg)* is a sequence $\mathbb{I} = I_1 I_2 \cdots I_n$ of intervals such that

$$I_i \,\mathtt{m}\, I_{i+1} \text{ for } 1 \leq i < n$$

or equivalently,

$$\bigcup_{i=1}^{n} I_i \text{ is an interval, and } I_i \prec I_{i+1} \text{ for } 1 \leq i < n.$$

Given a sequence $\mathbb{I} = I_1 I_2 \cdots I_n$ of intervals and an interval I, we write $\mathbb{I} \nearrow I$ to mean

$$\mathbb{I} \text{ is a seg and } I = \bigcup_{i=1}^{n} I_i,$$

in which case we say $\mathbb{I}$ is a seg(mentation) of I. We extend satisfaction $\models$ to segs $I_1 \cdots I_n$ and strings $\alpha_1 \cdots \alpha_m$ of finite subsets α_i of Φ, requiring that the lengths be the same and that I_i satisfy every fluent in α_i

$$\begin{aligned} I_1 \cdots I_n \models \alpha_1 \cdots \alpha_m \iff & n = m \text{ and} \\ & (\forall \varphi \in \alpha_i)\; I_i \models \varphi \text{ for } 1 \leq i \leq n. \end{aligned}$$

For example, if E and V are $\mathfrak{A}$-singular (or just $\mathfrak{A}$-whole)

$$\begin{aligned} (\exists \mathbb{I})\, \mathbb{I} \models \boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ} \iff & (\exists I \models \mathrm{E}) \\ & (\exists J \models \mathrm{V})\; J \sqsubset I. \end{aligned}$$

Next, $\mathbb{I} \models s$ extends from a string s to a set L of strings, with L *holding at* $\mathbb{I}$ if some string in L does

$$\mathbb{I} \models L \iff (\exists s \in L)\; \mathbb{I} \models s.$$

We then define φ to be $\mathfrak{A}$-*segmentable as* L if an interval I in $\mathfrak{A}$ satisfies φ iff every, or equivalently, some seg of I satisfies L

$$\begin{aligned} I \models \varphi &\iff (\forall \mathbb{I} \nearrow I)\ \mathbb{I} \models L \\ &\iff (\exists \mathbb{I} \nearrow I)\ \mathbb{I} \models L\,. \end{aligned}$$

Proposition 2. *If* φ *is* $\mathfrak{A}$*-summable,* $\varphi_\circ$ *is* $\mathfrak{A}$*-segmentable as the infinite language*

$$\boxed{\varphi_\circ}^+ = \boxed{\varphi_\circ} + \boxed{\varphi_\circ}\boxed{\varphi_\circ} + \boxed{\varphi_\circ}\boxed{\varphi_\circ}\boxed{\varphi_\circ} + \cdots$$

of strings $\boxed{\varphi_\circ}^n$, $n \geq 1$*. Moreover, the following five conditions are pairwise equivalent.*

(i) φ *is* $\mathfrak{A}$*-segmented*

(ii) φ *is* $\mathfrak{A}$*-segmentable as* $\boxed{\varphi_\circ}^+$

(iii) φ *is* $\mathfrak{A}$*-segmentable as* $\boxed{\varphi}^+$

(iv) φ *is* $\mathfrak{A}$*-sip and* $\mathfrak{A}$*-summable*

(v) $\varphi \equiv_{\mathfrak{A}} \max(\varphi)_\circ$.

As for $\mathfrak{A}$-whole fluents, we bound the strings in $\boxed{\varphi_\circ}^+$, adding $\neg\langle\mathtt{mi}\rangle\varphi_\circ$ to the initial boxes and $\neg\langle\mathtt{m}\rangle\varphi_\circ$ to the final boxes to form the language

$$\begin{aligned} \mathcal{L}(\varphi) \ = \ & \boxed{\varphi_\circ, \neg\langle\mathtt{mi}\rangle\varphi_\circ, \neg\langle\mathtt{m}\rangle\varphi_\circ} + \\ & \boxed{\varphi_\circ, \neg\langle\mathtt{mi}\rangle\varphi_\circ}\boxed{\varphi_\circ}^*\boxed{\varphi_\circ, \neg\langle\mathtt{m}\rangle\varphi_\circ}. \end{aligned}$$

Proposition 3. *The following conditions (i)-(iv) are pairwise equivalent.*

(i) φ *is* $\mathfrak{A}$*-whole*

(ii) $\varphi \equiv_{\mathfrak{A}} \max(\varphi_\circ)$

(iii) φ *is* $\mathfrak{A}$*-segmentable as* $\mathcal{L}(\varphi)$

(iv) $\mathbb{I} \models \boxed{\varphi}\boxed{\varphi_\circ} + \boxed{\varphi_\circ}\boxed{\varphi}$ *for no seg* $\mathbb{I}$.

3 Durative and/or telic strings

For any integer $n > 1$, an interval may have a wide variety of segmentations of length n, Propositions 2 and 3 notwithstanding. Even if

$$I \models \mathrm{V} \wedge \langle\sqsupset\rangle\mathrm{E},$$

a seg $I_1 I_2$ of I need not satisfy

$$\boxed{\mathrm{V}_\circ, \langle\supseteq\rangle\mathrm{E}}\boxed{\mathrm{V}_\circ} + \boxed{\mathrm{V}_\circ}\boxed{\mathrm{V}_\circ, \langle\supseteq\rangle\mathrm{E}}$$

(as E may straddle the line between I_1 and I_2), and if E is $\mathfrak{A}$-singular, the string

$$\boxed{\mathrm{V}_\circ}\boxed{\mathrm{E,V}_\circ}\boxed{\mathrm{V}_\circ}$$

holds in only one out of a possible multitude of segs of I with length 3. The choice of a seg can be a delicate matter. A string of sets of fluents expresses such a choice. The present section links that choice to aspect, stepping from a fluent φ to a set L of strings of finite sets of fluents, with*out* requiring that L hold at every seg of every interval satisfying φ. That is, the account of aspect given below makes essential use of the string representations over and above the fluents from which the strings are formed. Fluents/intervals describe objective matters of fact; strings/segmentations embody, in addition, particular perspectives on these matters.

A concrete linguistic illustration is provided by the notion that some events are *punctual* — i.e., lacking in internal structure. (Comrie, 1976) discusses the example of *cough*, noting that "the inherent punctuality of *cough* would restrict the range of interpretations that can be given to imperfective forms of this verb" to an iterative reading (of a series of coughs), as opposed to a single cough, which he refers to as *semelfactive*. Comrie concedes, however, that, in fact, one can imagine

> a situation where someone is commenting on a slowed down film which incorporates someone's single cough, as for instance in an anatomy lecture: here, it would be quite appropriate for the lecturer to comment on the relevant part of the film *and now the subject is coughing*, even in referring to a single cough, since the single act of coughing has now been extended, and is clearly durative, in that the relevant film sequence lasts for a certain period of time. (page 43)

The earlier contention that *coughing* can only be read iteratively suggests that the intervals spanned by single coughs are too small for our "normal" segmentations. These segmentations consist of intervals too big for "punctual" events, leading to a representation of a φ-semelfactive as $\boxed{\langle\sqsupset\rangle\varphi}$ rather than say, (8), with a middle box $\boxed{\varphi_\circ}$ of internal structure supporting the progressive.

(8) $\quad \boxed{\varphi_\circ, \neg\langle\mathtt{mi}\rangle\varphi_\circ}\boxed{\varphi_\circ}\boxed{\varphi_\circ, \neg\langle\mathtt{m}\rangle\varphi_\circ}$

The special context provided above by an anatomy lecture overturns this restriction, making (8) available after all. The punctual-durative distinction is evidently not cast in stone. But just what is durative? The simple proposal this section explores is that what is durative is a string $\alpha_1\alpha_2\cdots\alpha_n$ of sets α_i of fluents with $n \geq 3$. Between the first box α_1 and the last box α_n is a string $\alpha_2\cdots\alpha_{n-1}$ representing internal structure that, for $n \geq 3$, is non-empty.[4]

Apart from the length n of a string $\alpha_1\cdots\alpha_n$, there is also the matter of what fluents to box in a string, describing the interior as well as the immediate exterior of the situation the string represents. (The string in (8) is just an example to flesh out or otherwise revise.) Of particular relevance to temporal extent are any fluents chosen to mark the boundaries of the situation. An example in (9) is the fluent ψ which makes the string "telic" by appearing in all its boxes negated, except for the rightmost box, which it marks.

(9) $\boxed{\varphi_\circ, \neg\psi}\boxed{\varphi_\circ, \neg\psi}\boxed{\psi}$

Whether or not the intervals described by α_1 and α_n count as part of the situation represented by the string is independent of (10).

(10) a. $\alpha_1\cdots\alpha_n$ is *durative* if it has length $n \geq 3$

b. $\alpha_1\cdots\alpha_n$ is *telic* if the negation of some ψ in α_n appears in α_i for $1 \leq i < n$.

While (10a) says $\alpha_1\cdots\alpha_n$ has internal structure, (10b) says $\alpha_1\cdots\alpha_n$ culminates in some fluent $\psi \in \alpha_n$. (10b) is even more representational than (10a) in that it depends not only on segmenting an interval but on the choice of fluents we put into a string describing that segmentation. As Krifka notes, the telic-atelic distinction lies not "in the nature of the object described, but in the description applied to the object" as

> one and the same event of running can be described by running (i.e. by an atelic predicate) or by running a mile (i.e. a telic, or delimited, predicate)

(Krifka, 1998, page 207).[5] Krifka goes on to locate telicity not in objects but in sets P of objects meeting the condition in (11a), based on a proper part relation $<$ on objects induced by a sum operation $\oplus$ according to (11b).

(11) a. P is *quantized* if there are *no* $x, y \in P$ such that $x < y$

b. $x < y \iff x \neq y$ and $x \oplus y = y$

Under (11), the predicate *run a mile* is quantized, whereas the predicate *run* is not, even though one and the same run may belong to both predicates. But what about *run to the post office*? Surely, the second half of any run to the post office is also a run to the post office. A telic string may fail to be quantized because its left boundary (inception) has not been specified.

3.1 Subsumption and superposition

Some notation from (Fernando, 2004) will come in handy in what follows. Given strings s and s' of sets, we say s *subsumes* s' and write $s \unrhd s'$ if they have the same length and are related componentwise by inclusion

$$\alpha_1\cdots\alpha_n \unrhd \alpha'_1\cdots\alpha'_m \iff n = m \text{ and } \alpha_i \supseteq \alpha'_i \text{ for } 1 \leq i \leq n.$$

For instance,

$$\boxed{\varphi, \neg\psi}\boxed{\varphi, \neg\psi}\boxed{\varphi, \neg\psi}\boxed{\psi} \unrhd \boxed{\varphi}\boxed{\varphi}\boxed{\varphi}\boxed{}.$$

We extend subsumption $\unrhd$ to languages L existentially (just as we did with $\models$)

$$s \unrhd L \iff (\exists s' \in L)\ s \unrhd s'$$

so that a string s is durative iff $s \unrhd \boxed{}\boxed{}\boxed{}^+$ and telic iff $s \unrhd \boxed{\neg\psi}^+\boxed{\psi}$ for some ψ. A binary operation on strings of the same length complementing subsumption $\unrhd$ is *superposition* & obtained by componentwise union

$$\alpha_1\cdots\alpha_n \,\&\, \alpha'_1\cdots\alpha'_n = (\alpha_1 \cup \alpha'_1)\cdots(\alpha_n \cup \alpha'_n).$$

[4] Segmentations of the full linear order T into 2 or 3 intervals are central to the interpretation of *event radicals* in (Galton, 1987). A *formal occurrence* is defined there to be a pair B, A of intervals such that either $AB \nearrow T$ or $AIB \nearrow T$ where I is the complement $T - (A \cup B)$. The intuition is that B is *b*efore, and A *a*fter the situation with temporal extent $T - (A \cup B)$. The first box α_1 and last box α_n of a string $\alpha_1\cdots\alpha_n$ above (with $n \geq 3$) represent final and initial subintervals of B and A, respectively (constituting external structure). The middle bit $\alpha_2\cdots\alpha_{n-1}$ describes a segmentation of $T - (A \cup B)$. Segs generalize formal occurrences, elaborating on internal as well as external structure.

[5] Notice that the condition (10b) for telicity is not met by (8), but by the string

$$\boxed{\varphi_\circ, \neg\langle \mathtt{mi}\rangle\varphi_\circ, \langle \mathtt{m}\rangle\varphi_\circ}\boxed{\varphi_\circ, \langle \mathtt{m}\rangle\varphi_\circ}\boxed{\varphi_\circ, \neg\langle \mathtt{m}\rangle\varphi_\circ}$$

provided $\langle \mathtt{m}\rangle\varphi_\circ$ is understood to be the negation of $\neg\langle \mathtt{m}\rangle\varphi_\circ$. An alternative to leaving ψ existentially quantified in (10b) is to specify the fluent ψ and work with the notion of "culiminating in ψ."

For instance, $\boxed{\varphi}\boxed{\varphi}\boxed{\varphi}$ & $\boxed{\neg\psi}\boxed{\neg\psi}\boxed{\psi}$ $=$ $\boxed{\varphi,\neg\psi}\boxed{\varphi,\neg\psi}\boxed{\varphi,\psi}$ and for strings s and s' of the same length,

$$s \trianglerighteq s' \iff s = s \,\&\, s'$$
$$s \,\&\, s' = \text{least } \trianglerighteq\text{-upper bound of } s \text{ and } s' .$$

We extend & to sets L and L' of strings (of possibly different lengths) by collecting superpositions of strings from L and L' of the same length

$$L \,\&\, L' = \{s \,\&\, s' \mid s \in L,\ s' \in L' \text{ and length}(s)\text{=length}(s')\}$$

(a regular language provided L and L' are (Fernando, 2004)). Notice that

$$\{s\}\&\{s'\} = \{s\&s'\} \quad \text{if length}(s)\text{= length}(s')$$

and the durative strings in L can be obtained by superposing L with $\boxed{}\boxed{}\boxed{}^{+}$

$$L\&\boxed{}\boxed{}\boxed{}^{+} = \{s \in L \mid s \trianglerighteq \boxed{}\boxed{}\boxed{}^{+}\}.$$

3.2 Application to Aktionsart

Semelfactives, activities (= processes), achievements (= culminations) and accomplishments (= culminated processes) are commonly differentiated on the basis of durativity and telicity (Moens and Steedman, 1988; Pulman, 1997).

(12) a. A semelfactive is non-durative and atelic

b. An activity is durative but atelic

c. An achievement is non-durative but telic

d. An accomplishment is telic and durative

Under the present approach based on strings, (12) can be sharpened to (13).

(13) a. A φ-semelfactive $\trianglerighteq$ $\boxed{\langle\supset\rangle\varphi}$

b. A φ-activity $\trianglerighteq$ $\boxed{\varphi}\boxed{\varphi}\boxed{\varphi}^{+}$ (presupposing φ is $\mathfrak{A}$-segmented)

c. A ψ-achievement $\trianglerighteq$ $\boxed{\neg\psi}\boxed{\psi}$

d. An accomplishment built from a φ-activity culminating in ψ

$$\trianglerighteq \boxed{\varphi,\neg\psi}\boxed{\varphi,\neg\psi}\boxed{\varphi,\neg\psi}^{+}\boxed{\psi}$$

(presupposing φ is $\mathfrak{A}$-segmented)

(Bach, 1986a) argues that processes are mass and events are count, raising the question: how does the $\mathfrak{A}$-segmented/whole opposition sit with our account (13) of semelfactives, activities, achievements and accomplishments? Bach's processes are the activities in (13b), represented by the durative strings in the language $\boxed{\varphi}^{+}$ that a $\mathfrak{A}$-segmented fluent φ is $\mathfrak{A}$-segmentable as. Where $\mathfrak{A}$-whole fluents fit in (13) is, however, not immediately obvious. But as pointed out by (Comrie, 1976) for coughs and by (Krifka, 1998) for (mile-long) runs, there is an element of perspective (over and above pure, objective facts) that makes Aktionsart pliable. An achievement may, for instance, be coerced into an accomplishment to interpret the progressive in (14).

(14) The train was arriving when Anna went to order a drink.

A seg II' satisfying an achievement $\boxed{\neg\psi}\boxed{\psi}$ might, for some segmentation $I_1I_2I_3$ of I, be refined to the seg $I_1I_2I_3I'$ satisfying an accomplishment $\boxed{\varphi,\neg\psi}\boxed{\varphi,\neg\psi}\boxed{\varphi,\neg\psi}\boxed{\psi}$ with preparatory process/activity $\boxed{\varphi}\boxed{\varphi}\boxed{\varphi}$, for some $\mathfrak{A}$-segmented φ.

As representations, strings are slippery in a way that fixed pairs $\mathfrak{A}, \mathbb{I}$ are not; a shorter string might describe a larger interval than a longer string does. Strings are not so much finished objects as makeshift representations subject to refinement. So should $\mathfrak{A}$-whole fluents go into these strings? The simplest examples of $\mathfrak{A}$-whole-fluents are $\mathfrak{A}$-singular fluents (harking back to Davidson's events as particulars). Conceptualizing event time at some level of abstraction as an interval is reason enough to form a fluent picking out that interval. And with an $\mathfrak{A}$-singular fluent φ comes the $\mathfrak{A}$-segmented fluent $\varphi_\circ$, and the fluents $\neg\langle\mathtt{mi}\rangle\varphi_\circ$ and $\neg\langle\mathtt{m}\rangle\varphi_\circ$ from which to form the language $\mathcal{L}(\varphi)$ which φ is $\mathfrak{A}$-segmentable as (Proposition 3).

(Dowty, 1979) explores the hypothesis that

> the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates – stative predicates – plus three or four sentential operators and connectives

(page 71). A simplified event-based reformulation (15) of the Vendler classes in terms of Dowty's operators DO, BECOME and CAUSE is given in (Rothstein, 2004), page 35.

(15) states $\lambda e.P(e)$

activities $\lambda e.(\mathrm{DO}(P))(e)$

achievements $\lambda e.(\mathrm{BECOME}(P))(e)$

accomplishments $\lambda e.\exists e_1, e_2.[e = e_1 \oplus_S e_2 \wedge (\mathrm{DO}(P))(e_1) \wedge \mathrm{Cul}(e) = e_2]$

Dowty's CAUSE operator is reworked in (15) with a sum operation $\oplus_S$ producing singular entities, and a culmination function Cul. The resulting accomplishment e is the sum $e_1 \oplus_S e_2$ of its preparatory process (activity) e_1 and culmination e_2. To bring (13) in line with (15), we put

$$\begin{aligned} \mathrm{DO}(P) &\approx \boxed{P}\boxed{P}\boxed{P}^+ \\ \mathrm{BECOME}(P) &\approx \boxed{\neg P}\boxed{P} \end{aligned}$$

and require that P be $\mathfrak{A}$-segmented. Defining

$$\begin{aligned} \mathrm{du}(L) &= L \& \boxed{}\boxed{}^+ \\ \mathrm{cu}(L, \psi) &= (L \& \boxed{\neg\psi}^+)\boxed{\psi} \end{aligned}$$

yields

$$\begin{aligned} \boxed{P}\boxed{P}\boxed{P}^+ &= \mathrm{du}(\boxed{P}^+) \\ \boxed{\neg P}\boxed{P} &= \mathrm{cu}(\boxed{}, P) \end{aligned}$$

and for accomplishments as culiminated activities,

$$\begin{aligned} \mathrm{cu}(\mathrm{du}(\boxed{\varphi}^+), \psi) &= \boxed{\varphi, \neg\psi}\boxed{\varphi, \neg\psi}\boxed{\varphi, \neg\psi}^+\boxed{\psi} \\ &= \mathrm{du}(\boxed{\varphi, \neg\psi})\boxed{\psi}. \end{aligned}$$

Left out of (13) are the states in (15), which can be compared to $\mathfrak{A}$-segmented fluents in the present framework. As noted in (Dowty, 1986), one might also require that stative fluents be inertial, for which see (Fernando, 2008).

4 Desegmenting and branching time

Why segment an interval? The two reasons given above are (1) to get a handle on durativity and telicity, and (2) to unwind an interval fluent such as E$\wedge\langle\sqsupset\rangle$R to a string $\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{R}}\boxed{\mathrm{E}_\circ}$ interpreted against segmentations (i.e. finite timelines). Neither reason justifies grinding indefinitely. The thrust of the present section is to leave segs as coarse as possible, segmenting only if necessary, leading to a notion of time that may branch.

4.1 Desegmenting via π

Quantifying the model $\mathfrak{A}$ out of the notion of $\mathfrak{A}$-segmentability and weakening the connection between intervals and segs, let us agree that a language L *depicts* φ if for all models $\mathfrak{A}$, L is $\mathfrak{A}$-satisfiable precisely if φ is

$$(\exists \text{ seg } \mathbb{I})\ \mathbb{I} \models L \iff (\exists \text{ interval } I)\ I \models \varphi.$$

Trivially, $\boxed{\varphi}$ depicts φ, but there are more interesting examples. Unwinding the modal operator $\langle\succ\rangle$ and conjunction $\wedge$ in the fluent S $\wedge\langle\succ\rangle\varphi$,

$$\boxed{\varphi}\boxed{\mathrm{S}} + \boxed{\varphi}\boxed{}\boxed{\mathrm{S}} \quad \text{depicts} \quad \mathrm{S} \wedge \langle\succ\rangle\varphi.$$

The language $\boxed{\varphi}\boxed{\mathrm{S}} + \boxed{\varphi}\boxed{}\boxed{\mathrm{S}}$ reduces the infinite language $\boxed{}^*\boxed{\varphi}\boxed{}^*\boxed{\mathrm{S}}\boxed{}^*$ depicting S $\wedge\langle\succ\rangle\varphi$ to two strings. This reduction illustrates the possibility that under suitable assumptions on a language L depicting φ, the strings in L can be simplified in two ways:

(w1) any initial or final empty boxes can be stripped off, and

(w2) all repeating blocks α^n (for $n \geq 1$) of a box α can be compressed to α.

More precisely, we implement (w1) by a function *unpad* defined on strings s by

$$\mathit{unpad}(s) = \begin{cases} \mathit{unpad}(s') & \text{if } s = \boxed{}s' \text{ or} \\ & \quad \text{else } s = s'\boxed{} \\ s & \text{otherwise} \end{cases}$$

so that $\mathit{unpad}(s)$ neither begins nor ends with $\boxed{}$. For (w2), all blocks α^{n+1} in s are compressed in $\mathit{bc}(s)$ to α

$$\mathit{bc}(s) = \begin{cases} \mathit{bc}(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha\ \mathit{bc}(\beta s') & \text{if } s = \alpha\beta s' \text{ with} \\ & \qquad \alpha \neq \beta \\ s & \text{otherwise} \end{cases}$$

so that if $\mathit{bc}(s) = \alpha_1 \cdots \alpha_n$ then $\alpha_i \neq \alpha_{i+1}$ for i from 1 to $n-1$. We then compose bc with *unpad* for π

$$\pi(s) = \mathit{unpad}(\mathit{bc}(s)).$$

One can check that

$$\{\pi(s) \mid s \in \boxed{}^*\boxed{\varphi}\boxed{}^*\boxed{\mathrm{S}}\boxed{}^*\} = \boxed{\varphi}\boxed{\mathrm{S}} + \boxed{\varphi}\boxed{}\boxed{\mathrm{S}}.$$

Clearly, $\pi(s)$ is never longer than s, and $\pi(s) = \pi(\pi(s))$ for all strings s.

As for the "suitable assumptions" on L under which L can be reduced to $\{\pi(s) | s \in L\}$, it is helpful to consider the fluent R $\wedge \langle\sqsubset\rangle\varphi$. Can we unwind $\langle\sqsubset\rangle$ in $\boxed{\mathrm{R}, \langle\sqsubset\rangle\varphi}$? Assuming $\varphi_\circ$ is $\mathfrak{A}$-summable for all models $\mathfrak{A}$,

$$\boxed{\varphi_\circ}\boxed{\mathrm{R},\varphi_\circ}\boxed{\varphi_\circ} \text{ depicts R } \wedge \langle\sqsubset\rangle\varphi.$$

Now, let us call a string $s = \alpha_1 \cdots \alpha_n$ of sets α_i of fluents $\mathfrak{A}$-*reducible* if every fluent appearing in two consecutive string positions $\alpha_i\alpha_{i+1}$ in s (for some $1 \leq i < n$) is $\mathfrak{A}$-summable. (For example, $\boxed{\varphi_\circ}\boxed{\mathrm{R},\varphi_\circ}\boxed{\varphi_\circ}$ is $\mathfrak{A}$-reducible, provided $\varphi_\circ$ is $\mathfrak{A}$-summable.) Let us say a seg $\mathbb{I}$ *refines* a seg $I_1 \cdots I_n$ if for all i from 1 to n, I_i is the union of some subsequence of $\mathbb{I}$.

Proposition 4. *For any $\mathfrak{A}$-reducible string s, every seg $\mathbb{I}$ that satisfies s refines some seg $\mathbb{I}'$ that satisfies $\pi(s)$. Consequently, if for all $s \in L$, s is $\mathfrak{A}$-reducible and $\pi(s) \in L$, then L is $\mathfrak{A}$-satisfiable iff $\{\pi(s) | s \in L\}$ is*

$$(\exists \textit{ seg } \mathbb{I})\ \mathbb{I} \models L \iff (\exists \textit{ seg } \mathbb{I})\ \mathbb{I} \models \{\pi(s) \mid s \in L\}.$$

4.2 Relativizing π to a finite set X of fluents

Next, we fix a notion of bounded granularity through a finite set X of fluents of interest, which we can expand to refine granularity or contract to coarsen granularity. An instructive example for orientation is the representation of a calendar year of twelve months as the string

$$s_{mo} = \boxed{\mathsf{Jan}}\boxed{\mathsf{Feb}}\boxed{\mathsf{Mar}}\cdots\boxed{\mathsf{Dec}}$$

of length 12, or, were we also interested in days d1,d2...,d31, the string

$$s_{mo,dy} = \boxed{\mathsf{Jan,d1}}\boxed{\mathsf{Jan,d2}}\cdots\boxed{\mathsf{Jan,d31}} \boxed{\mathsf{Feb,d1}}\cdots\boxed{\mathsf{Dec,d31}}$$

of length 365 (for a non-leap year). Unlike the points in the real line $\mathbb{R}$, a box can split, as $\boxed{\mathsf{Jan}}$ in s_{mo} does (30 times) to $\boxed{\mathsf{Jan,d1}}\boxed{\mathsf{Jan,d2}}\cdots\boxed{\mathsf{Jan,d31}}$ in $s_{mo,dy}$, on introducing days d1, d2,..., d31 into the picture. Reversing direction and generalizing from $mo =$ {Jan,Feb,...Dec} to any set X of fluents, we define the function ρ_X on strings (of sets) to componentwise intersect with X

$$\rho_X(\alpha_1 \cdots \alpha_n) = (\alpha_1 \cap X) \cdots (\alpha_n \cap X)$$

throwing out non-X's from each box (keeping only the elements of X) so that

$$\rho_{mo}(s_{mo,dy}) = \boxed{\mathsf{Jan}}^{31}\boxed{\mathsf{Feb}}^{28}\cdots\boxed{\mathsf{Dec}}^{31}.$$

Next, we compose ρ_X and π for the function π_X mapping a string s of sets to

$$\pi_X(s) = \pi(\rho_X(s)) = \mathit{unpad}(bc(\rho_X(s)))$$

so that for example,

$$\begin{aligned}\pi_{mo}(s_{mo,dy}) &= \pi(\boxed{\mathsf{Jan}}^{31}\boxed{\mathsf{Feb}}^{28}\cdots\boxed{\mathsf{Dec}}^{31}) \\ &= s_{mo}\end{aligned}$$

and

$$\begin{aligned}\pi_{\{\mathrm{E}_\circ\}}(\boxed{\mathrm{E}_\circ}\boxed{\mathrm{R,E}_\circ}\boxed{\mathrm{E}_\circ}) &= \pi(\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ}) \\ &= \boxed{\mathrm{E}_\circ}.\end{aligned}$$

In general, a description s_X of granularity X can be refined to one $s_{X'}$ of granularity $X' \supseteq X$ provided π_X maps $s_{X'}$ to s_X. More precisely, given some large set Φ of fluents, let $\mathit{Fin}(\Phi)$ be the set of finite subsets of Φ. A function f with domain $\mathit{Fin}(\Phi)$ mapping $X \in \mathit{Fin}(\Phi)$ to a string $f(X)$ over the alphabet 2^X is said to be π-*consistent* if whenever $X \subseteq X' \in \mathit{Fin}(\Phi)$,

$$f(X) = \pi_X(f(X')).$$

Let us write $\mathfrak{IL}_\pi(\Phi)$ for the set of all π-consistent functions. "IL" here stands not for intensional logic but for inverse limit — to be precise, the *inverse limit of* the restrictions of π_X to $(2^{X'})^*$ for $X \subseteq X' \in \mathit{Fin}(\Phi)$ (all computable by finite-state transducers). That said, $\mathfrak{IL}_\pi(\Phi)$ is intensional: time branches under the relation $\prec_\Phi$ between $f, f' \in \mathfrak{IL}_\pi(\Phi)$ given by

$$f \prec_\Phi f' \iff f \neq f' \text{ and } (\forall X \in \mathit{Fin}(\Phi)) \; f(X) \text{ is a prefix of } f'(X)$$

(where s is a *prefix of* s' if $s' = s\hat{s}$ for some possibly empty string $\hat{s}$). The intuition is that a temporal moment comes with its past, and that an $f \in \mathfrak{IL}_\pi(\Phi)$ encodes the moment that is X-approximated, for each $X \in \mathit{Fin}(\Phi)$, by the last box in $f(X)$, with past given by the remainder of $f(X)$ (leading to that box). More precisely, $\prec_\Phi$ is tree-like in the sense of (Dowty, 1979).

Proposition 5. *$\prec_\Phi$ is transitive and left linear: for every $f \in \mathfrak{IL}(\Phi)$,*

$$(\forall f_1 \prec_\Phi f)(\forall f_2 \prec_\Phi f)\ f_1 \prec_\Phi f_2 \text{ or } f_2 \prec_\Phi f_1 \text{ or } f_1 = f_2.$$

Moreover, no element of $\mathfrak{IL}_\pi(\Phi)$ is $\prec_\Phi$-maximal.

Maximal chains, called *histories* in (Dowty, 1979), figure prominently in possible worlds semantics. While we can pick one out in $\mathfrak{IL}_\pi(\Phi)$ to represent an actual history, it is far from obvious what significance maximal $\prec_\Phi$-chains have in the present framework, which is closer in spirit to *situation semantics* (Bawise and Perry, 1983), updated in (Cooper, 2005; Ginzburg, 2005).

Tha handbook chapter (Thomason, 1984) opens with the declaration

> Physics should have helped us to realise that a temporal theory of a phenomenon X is, in general, more than a simple combination of two components: the statics of X and the ordered set of temporal instants. The case in which all functions from times to world-states are allowed is uninteresting; there are too many such functions, and the theory has not begun until we have begun to restrict them. And often the principles that emerge from the interaction of time with the phenomena seem new and surprising.

For a non-empty set W of worlds, and a linearly ordered set T of time instants, Thomason compares $T \times W$-frames, not unlike that in (Montague, 1973), unfavorably to tree-like frames, of which $\prec_\Phi$ above is an example, when paired with a $\subseteq$-maximal $\prec_\Phi$-chain. The crudeness of the cartesian product $\times$ aside, one may ask where T comes from, as Bach pointedly does in page 69 of (Bach, 1981), to say nothing of W. The answer from $\mathfrak{IL}_\pi(\Phi)$ involves strings formed from fluents. The projective system $(\pi_X)_{X \in Fin(\Phi)}$ gives for every finite subset X of Φ, a choice of X-approximations in $(2^X)^*$, including for $X = \{e, e'\}$ with $e \neq e'$, 13 strings s_r corresponding to the Allen interval relations r between intervals e and e' (Allen, 1983); see Table 1 (Fernando, 2012). Under the projections π_X, these strings are most naturally viewed as *indices* for evaluating an expression φ as an *extension* or *denotation*, as prescribed by Carnap-Montague *intensions* (Fernando, 2011). In (Bach, 1986b), an event type such as KISSING induces a function EXT(KISSING) that maps histories to subparts that are temporal manifestations of KISSING, treating input histories as indices and output manifestations as extensions. Under the current framework, EXT(KISSING) can for any $X \in Fin(\Phi)$, be given as a binary relation between strings in $(2^X)^*$ that X-approximate indices and extensions.

$r \in$ **Allen**	$s_r \in (2^{\{e,e'\}})^+$
$e = e'$	$\boxed{e,e'}$
e s e'	$\boxed{e,e'}\boxed{e'}$
e si e'	$\boxed{e,e'}\boxed{e}$
e f e'	$\boxed{e'}\boxed{e,e'}$
e fi e'	$\boxed{e}\boxed{e,e'}$
e d e'	$\boxed{e'}\boxed{e,e'}\boxed{e'}$
e di e'	$\boxed{e}\boxed{e,e'}\boxed{e}$
e o e'	$\boxed{e}\boxed{e,e'}\boxed{e'}$
e oi e'	$\boxed{e'}\boxed{e,e'}\boxed{e}$
e m e'	$\boxed{e}\boxed{e'}$
$e < e'$	$\boxed{e}\boxed{}\boxed{e'}$
e mi e'	$\boxed{e'}\boxed{e}$
$e > e'$	$\boxed{e'}\boxed{}\boxed{e}$

Table 1: The Allen relations in $(2^{\{e,e'\}})^+$

5 Conclusion

Segmentations arise naturally in the view from (Klein, 2009) that

> The expression of time in natural languages relates a *clause-internal temporal structure* to a *clause-external temporal structure*. The latter may shrink to a single interval, for example, the time at which the sentence is uttered; but this is just a special case. The clause-internal temporal structure may also be very simple – it may be reduced to a single interval without any further differentiation, the 'time of the situation'; but if this ever happens, it is only a borderline case. As a rule, the clause-internal struc-

ture is much more complex. (page 75)

The simplest case described by the passage is illustrated by the picture (16) of the clause-internal event (or situation) time E preceding the clause-external speech (utterance) time S.

(16) $\boxed{E}\boxed{S} + \boxed{E}\boxed{\ }\boxed{S}$ depicting $E \wedge \langle \prec \rangle S$

Slightly more complicated is the picture (3) of event time E with R inside it.

(3) $\boxed{E_\circ}\boxed{E_\circ,R}\boxed{E_\circ}$ (segmented $E_\circ$, whole R)

Whereas E in (16) is unbroken and whole, the "differentiation" in (3) puts E through a universal grinder $\cdot_\circ$ described in section 2, alongside notions of $\mathfrak{A}$-whole and $\mathfrak{A}$-segmented fluents, the contrast between which surfaces in pairs such as (17) and (18).

(17) Ernest was explaining $\not\models$ Ernest explained

(18) Ernest was laughing $\models$ Ernest laughed

The non-entailment (17) is clear from (19).

(19) Ernest was explaining when he was made to stop.

To extract a rigorous account of (17) versus (18) from the assumption that explaining is whole and laughing is segmented (as fluents) would require stepping beyond lexical/internal aspect (considered in sections 2 and 3 above) to grammatical/external aspect, hinted at in (3), as well as tense. Some details compatible with the present approach can be found in (Fernando, 2008).[6] Suffice it to say that additional temporal parameters from tense and aspect enlarge the set X of fluents that, under the inverse limit $\mathfrak{IL}_\pi(\Phi)$ in section 4, refines granularity. While we have taken pains to show how to interpret a string of subsets of Φ in a segmentation (essentially, a finite, ordered partition of an interval from a Φ-timeline), no Φ-timeline is used to define $\mathfrak{IL}_\pi(\Phi)$, resulting in a notion of time that branches (away from any single segmentation or timeline). There is sure to be junk in $\mathfrak{IL}_\pi(\Phi)$ to throw out; but what use tense and aspect might have for timelines not represented in $\mathfrak{IL}_\pi(\Phi)$, I fail to see (apart from linking temporality up with other linguistic mechanisms such as quantification). Work on tense and aspect has led to extensions of ordinary temporal logic in three directions.

(20) a. addition of temporal parameters (e.g. R)

b. expansion of points to intervals

c. recognition of events and states

Stringing together finite sets of fluents, we attend to (20c) in sections 2 and 3 above, and to (20a) in section 4, putting the distinction (20b) between points and intervals down to the set X of fluents under consideration.[7]

[6] An alternative would be to follow along (Galton, 1984; Galton, 1987). There are likely to be many ways to fill in the details. In the case of the perfect, for instance, the basic approach outlined here is, as far as I can tell, neutral between extended now accounts (Pancheva, 2003) augmented with (7)

(7) a. $\boxed{V_\circ}\boxed{E,V_\circ}\boxed{V_\circ}$ (depicting $E \sqsubset V$)

b. $\boxed{E_\circ}\boxed{E_\circ,V}\boxed{E_\circ}$ (depicting $V \sqsubset E$)

and consequent-state approaches (Moens and Steedman, 1988; Kamp and Reyle, 1993; Pulman, 1997) that might be augmented with inertia (Dowty, 1986) and forces (Fernando, 2008).

[7] *Added in haste*. The literature on interval temporal logic is vast, and the present paper has doubtless failed to do it justice. In particular, (Nishimura, 1980) and (Moszkowski, 1986) deserve to be mentioned properly in this paper, which I hope to do in a revision.

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