Gnosis

Marcus Kracht
Fakultät Linguistik und Literaturwissenschaft
Universität Bielefeld
Postfach 10 01 31
33501 Bielefeld
marcus.kracht@uni-bielefeld.de

Abstract

The transition from form to meaning is not neatly layered: there is no point where form ends and content sets in. Rather, there is an almost continuous process that converts form into meaning. That process cannot always take a straight line. Very often we hit barriers in our mind, due to the inability to understand the exact content of the words just heard. The standard division between formula and interpretation (or value) should therefore be given up when talking about the process of understanding. Interestingly, when we do this it turns out that there are 'easy' formulae, those we *can* understand without further help, and those we cannot.

[...] we must recall what we discovered about the Cartesian doctrine of the mind, namely that the mind is not a distinct container of thoughts, but a temporally ordered structure of thoughts. To say, therefore, that one has a thought is to say that that thought is a part of a certain totality of thoughts. When I say, [...] "I think (that *p*)," I say that the thought that *p* occupies the "present-slot" in a temporally ordered system of thoughts [...]; but when I conclude "therefore I exist," I say that there is a temporally ordered system of thoughts, the present-slot of which is occupied by that thought.
Zeno Vendler: Res Cogitans

1 Introduction

This paper is the first of two papers that unite several independent issues that currently find no real home in logic and linguistic theory. One is the fact that building a representation is a process which goes beyond mere unpacking of definitions, a process that is intertwined with another process that I call "gnosis", translated roughly as "understanding"; second, that semantics does not just consist in denotational meanings, but that there also is a part which guides the process of arriving there; and third, that topic and focus articulation provides part of this "process meaning". It is not new to argue that meanings go beyond what is represented in the mind, Putnam has argued this before (see [9]). However, while he argued that often we do not know enough about the exact meanings of particular words, I claim that even when we do know exactly what the words mean we still may be unable to grasp the meaning of what is said in its entirety. And this is because not everything that can be said can also be understood in a direct way; its meaning must therefore remain opaque unless further tools come to rescue. Even understanding logical formulae may be impossible despite the fact that nothing about their meaning is unclear.

The limits of our mind stem in part from a particular design of our reasoning system, in part they have to do with the problem of abstractness of content: logical notions do not have counterparts in ordinary experience except in the most trivial way: truth is presence of some fact, falsity (or better, non-truth) its absence. Logical languages allow to express facts about the world through concepts that seem to reach beyond it. Examples of such notions are consequence and implication. They are as I see it purely conceptional, and a proper semantics must treat them as that. If that is right, then the meaning of these entities have more to do with our conceptualisations than with the facts themselves. Atomic propositions represent facts of the world, but one level up, at the complex propositions, there is no physical counterpart; complex propositions point to abstract concepts. This is reflected in the standard notion of a model. The problem that we face is to show how we can actually introduce these abstract concepts that the complex propositions point at. Part of the story can be told using logical theory: we lift ourselves up using the dialectics between proof and formula. Once we know what a proof is we can successfully establish higher and higher logical concepts. This is what the present paper is about. Moreover, as we shall see, the fact that complex propositions live at a higher level of abstraction introduces a tension in us; for to be able to really apply knowledge of the higher type we must know how to descend to the ground level. This tension is not easily resolved. Some propositions resist our attempts to explicate them while others are readily understood. Moreover, it is a matter of how they are presented that determines how well we can 'ground' them. This is where the linguistics comes in. As I shall show in a sequel to this paper, topic and focus (or theme and rheme) can in part be understood as providing a different 'grounding schema' to a proposition.

The present proposal is an attempt to give a somewhat more realistic picture of the translation process from text to meaning in order to shed light on certain logical and linguistic problems. Other attempts in that direction have been made in update semantics ([12], [4]) and dynamic semantics ([5]). Also, [14] has investigated the structure of proofs as texts.

All the previous attempts in semantics have however seriously understimated two problems: the first is that understanding is an active process, one which uses the limited capabilities of our brain, and the second is that human processing limitations have real effects on the structure of language. The first of these problems have long been studied in cognitive science (see [11] and references therein). One of the interesting outcomes is that recall of sentences depends on the way they can be grounded; if hearers can construct a model they recall only the content, not the form. If they cannot ground them, they recall the form rather than the content (see [3] for early results in that direction). Semantic theory has nothing to offer here: all sentences can be translated into their meaning, so no such effects should ever exist. On the other hand, to this day it is unclear what the exact nature of the representations is, what can and cannot be represented.

The present paper is a logician's attempt to provide such a psychological basis for the process of understanding. It is argued that from a theoretical point of view the human symbolic processor is limited, and that that limitation does not so much lead to errors but to difficulties in understanding the content of certain utterances even if their content is in principle absolutely clear. The central notion is the act of *judgement*. Humans judge sentences or representations as true, or false, or nonsensical, etc. This act of judgement is immediate and unconditional. The possibility of what is effectively a conditional judgement is possible only because of the ability to represent the content of implication symbolically.[1]

[1]The idea to this paper reach back as far as 1988. At that time I was trying to provide a semantics for theme and rheme, which I learned about from [15]. Unfortunately, at that time few people in formal linguistics were really interested in the matter. That has meanwhile changed. Although topic and focus are now a big issue in linguistics I find the approaches so far too conservative in that they are stuck with standard truth conditional semantics in one or the other form. This paper has been the result of rethinking my earlier attempts in [7] and [6], which remained unpublished. I have benefitted greatly from discussions with Albert Visser and Kees Vermeulen during my stay

2 The Deduction Theorem

Our prime example is the definition of implication via the Deduction Theorem. Before I state it, let me recall that in logic we distinguish between "$\rightarrow$", which is a binary operator on formulae, and "$\vdash$", which denotes a relation between a set of formulae and a formula (or a set thereof). Thus, "$\varphi \rightarrow \chi$" is a formula, while "$\varphi \vdash \chi$" is a statement.[2] I shall have a lot to say about the difference between these two. The most obvious is of course that "$\rightarrow$" is a member of the language itself, while "$\vdash$" is not. Consequently, its syntax is entirely different from that of "$\rightarrow$". "$\vdash$" combines a set of formulae and a single formula. The result is no longer a formula, whence it cannot be nested.

Recall that the Deduction Theorem (DT) states the following

(1) $T \vdash \varphi \rightarrow \psi$ iff $T; \varphi \vdash \psi$

In a standard Hilbert Calculus the DT holds iff $p \rightarrow (q \rightarrow p)$ and $(p \rightarrow (p \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ are theorems (see [10]).

My proposal is that the DT is not a theorem. It is the definitional scheme for "$\rightarrow$". It allows to introduce "$\rightarrow$" into a formula. This has interesting consequences. For let the following formula be given:

(2) $\varphi = \delta_0 \rightarrow (\delta_2 \rightarrow (\cdots \rightarrow (\delta_{n-1} \rightarrow \delta_n) \cdots))$

Suppose that φ holds. In symbols, we may write this as.

(3) $\vdash \varphi$

Using the DT (3) can be rendered into this form:

(4) $\delta_0; \cdots; \delta_{n-1} \vdash \delta_n$

The arrows of the original formula have now disappeared. If the δ_i are basic formulae, they denote concrete propositions, like "that the earth is flat" or "that the sun is shining" and so on. In that case (4) is free of logical constants. We say it is grounded, or of level 0. By contrast, if δ_0, say, contains an occurrence of

in Utrecht in 1992-3 and with Christa Hauenschild, David Pearce, Carla Umbach, Gerd Wagner, Frank Wolter and Heinrich Wansing in the Gruppe Logik, Wissenstheorie und Information.

[2]For formulae and symbols, the quotes "$\cdots$" are used to denote the syntactic object as opposed to the meaning that it has. They may be superfluous when the context makes matters clear anyway.

$\rightarrow$, there is no way to eliminate this occurrence through the use of the Deduction Theorem.

However, the definition as given makes use of the judgement sign "$\vdash$", and thus cannot be used to reduce any of the δ_i should they contain occurrences of $\rightarrow$. In other words: the definition is such that it places limits on reducibility which are not matched by syntactic restrictions. We have not excluded leftward nesting of the arrow, but the arrows can then not be fully eliminated via DT.

This tension exists also in human sentence processing and reasoning. We try to use a definition by simply unpacking it and see where we can take things. But we may get stuck: we may be unable to do all the unpacking at once. To get around the limitation, we need additional tools. What comes to rescue is that we may actually introduce correlates to the linguistic objects. Functions and sets are notions that arise through reification of this sort, and can be used to overcome the restrictions in expressing our thoughts. An example is the notion of a continuous function. The standard definition uses two quantifier alternations. f is continuous in x_0 iff

(5) $$(\forall\epsilon > 0)(\exists\delta > 0)(\forall y)(|y - x_0| < \delta \rightarrow |f(y) - f(x_0)| < \epsilon)$$

In topology we learn that a function is continuous if the preimage of an open set contains an open set. This can be represented as

(6) $$(\forall O)(\exists U)(f[U] \subseteq O)$$

Here quantification is over open sets. So, f is continuous in x_0 iff

(7) $$(\forall O)(\exists U)(f(x_0) \in O \rightarrow x_0 \in U \wedge f[U] \subseteq O)$$

The move to open sets reduces the quantifier complexity by one at the expense of introducing a new and more complex notion of an open set.

It is helpful in this respect to look at computer languages. A function is generally defined by saying what its values are on the inputs. Languages with type systems may allow to use functions as inputs to functions. Once defined, a function may then also be applied to a function as long as it matches the type requirement of the input variable. The type of a function that uses inputs of type δ_i, $i < n$, is exactly as given in (3). Unless arguments are themselves understood to be functions (as the definition of the function may require), the types of each δ_i is actually primitive. The complex types not covered by (3) can come into existence only through reification by means of functions. Notice, however, that these functions are purely symbolic. The functions do not exist, so I claim, in the same way as a chair exist. Their correlate is abstract; at best you may see it as a particular program for a computer.

The main interest below is actually neither in type theory nor in meaning. It is in the idea that the limitations of the sort explained above have consequences for semantics of natural languages. They suggest that there is something fundamentally different between logical and nonlogical notions in that logical notions may require reification of higher order concepts, but that this reification comes at high costs. In mathematics we get trained to reason effectively with notions that are conceptually very difficult: one example is the notion of a continuous function. It takes most people a very long time until they even understand what it says—not to mention the problem of applying the notion in proofs.

3 Understanding What Is Said

In Montague Grammar, as in logic textbooks, the meaning of the connective "→" is defined casewise, via its truth table. The introduction rules for logical connexives are however syncategorematic, that is, are not lexical. Thus, Montague style semantics uses a binary mode of composition as follows.

(8) $$F(\langle\varphi,\tau\rangle,\langle\chi,\tau'\rangle) := \langle(\varphi \to \chi), \tau \supset \tau'\rangle$$

Here, φ and χ are formulae, and $\supset$ is the following function:

$\supset$	0	1
0	1	1
1	0	1

Thus, this calculus generates pairs $\langle\varphi,\tau\rangle$ where φ is a formula and τ its truth value.

Montague's solution is fine as far as objective meanings are concerned. However, subjectively there may be problems in applying the rules. One of them may simply be that we do not know the value of a formula. The second may be that the formula is not intended to have a specific value—for example when it is a variable. In these cases we are stuck. Montague Grammar does not allow for symbolic computations. Yet humans seem to be able to understand that the following Stoic reasoning is true:[3]

> If the first then the second. The first. Therefore the second.

[3]This is due to Chrysippos, see [8]. The text contains the ordinals α′ and β′ in place of the Greek words, so "1." and "2." in place of "first" and "second" would be a little more exact.

Here, “the first” and “the second” are variables, and the first two sentences are assumptions. Thus, this is nothing but (the enacting of)

(9) $p; p \to q \vdash q$

Notice that we see that this is valid without knowing anything about p and q.

But with Peirce’s Law we have trouble understanding what it tells us:

(10) $((p \to q) \to p) \to p$

Other than being a formula of a syntactic kind, what does it actually *say*? Can we find out whether it is true? In principle yes, we just check the truth values under all assignments. Yet, most students of logic (at least initially) find this revolting. Clearly, in their mind implication is not to be treated that way. Their intuitive understanding of implication is a different one.

The problem to be addressed here is that there are laws of logic that can be understood with ease while others are almost impenetrable. No matter how hard we try, we cannot make sense of them. If on the other hand we understand well what the primitives mean (here: implication), where does the difference come from? And what is it that we understand in one case but not in the other?

My answer concerning “$\to$” specifically is this. Our intuitive understanding of “$\to$” is that of a language internal equivalent of “$\vdash$”. We can use it to encode the fact that if the premiss is true so is the conclusion. This definition does not use truth values, only the notion of truth. We may also think that the meaning of “$\to$” is rather $\supset$. However, this meaning of “$\to$” is different in kind. It commits us to particular truth values, and it requires combinatorics. To find out whether a formula is true we need to run through a series of assignments. This qualifies $\supset$ as an ‘off line’ meaning, something we use when nothing else works. However, an ‘online’ meaning must work differently. It must allow us to follow the argumentation in real time.

This still does not really explain why we cannot (or rather: do not) use truth tables. Something is missing. That something is a psychological theory of reasoning that shows why certain steps are harder than others. It will among other things claim that there is a distinction between subconscious (or implicit) knowledge and conscious (or explicit) knowledge; and that we cannot picture the absence of truth. We can only picture that something is true. Finally, one must we realise that humans are not interested in facts as such. If they were, it would be enough to just add the representation of an utterance to our knowledge base. That is not what is going on. Rather, the minute we hear something we try to understand what it says. Understanding is something we do. It takes time, and we may not be able to reserve enough time and resources to see the consequences that an utterance has. Also, while certain acts involved in understanding are automatic and straightforward, others involve some sort of conscious bookkeeping, yet others involve reflection on acts of reasoning, and it is these that are problematic and come at high costs. In what is to follow I shall expose a theory that explains understanding as a process that involves a stream of elementary acts, the most prominent of which are *conversion*. Conversion is the process of trading some piece of form for its meaning (and back). It means unpacking the parcel into which the message has been wrapped to recover the content. Reasoning is the step that applies certain rules to arrive at conclusions that are strictly speaking not 'in the message'. The mental process of understanding can be reflected on; humans are able to retrace it, if only partially. The reflection gives rise to concepts which denote the kinds of mental acts just talked about. In turn, since these concepts denote mental acts, they can be used to steer the process of understanding in the listener. This is what makes some ways of saying the same thing more digestible than others.

4 Enacting Meanings

There is an interpretation of the conditional attributed by Quine to Rhinelander (see [1]), which goes as follows. Suppose I say

(11) `If Paul is a raven, he is black.`

Rather than reading this as an assertion about Paul, this interpretation says it is not an assertion at all until the premiss is satisfied. It turns into an assertion if Paul is a raven. And what it says is that Paul is black. This interpretation is akin to Ramsey's interpretation of conditionals: a conditional statement $p \to q$ means that q on condition that p. Ramsey also thought that the conditional probability $p(A|B)$ is not a probability assigned to a pair of events, but to the event A alone, but its use is restricted to situations where B obtains. Similarly, a conditional obligation is an obligation that is enforced only when the condition is met on which it is conditional; a conditional promise is void if the condition is not met. If I promise you a meal when you repair my bicycle, as long as you do not repair my bicycle there is no promise not even an empty one. It is as if nothing had been said. The clearest case is perhaps provided by obligations. Suppose you are obliged to stop at a red light. Now is it the case that there is an obligation to see to it that either the light is not red or to stop, or is it rather an obligation to stop, which comes into effect when the light is red? I think it is clearly the latter.

For (11) the matter is however not so clear. For it could just as well be seen as a statement, the statement that Paul is either not a raven or black. So who is to prevent me from seeing it as a statement simpliciter? How do we judge the matter?

My answer is sibyllinic: I think that the implication forms a single claim, but we can only understand it by viewing it as a corresponding disposition. Let me explain. I consider both the arrow "→" and the words /`if....then`/ as linguistic objects (of different languages). If asked to defend the claim that $p \to q$ we do the following. We ask the listener to suppose p. Then we proceed to a demonstration of q. (Similar proposals are due to Ramsey and Lorenzen.) Similarly, the way we see whether or not $p \to q$ holds, we first suppose p and see whether or not q. We symbolically describe the fact that q follows after supposing p by

(12) $\quad p \vdash q$

The notation is an objectified description of a subjective disposition. In logical theory (12) means "there is a proof of q from p". Subjectively speaking it means, for some reasoner a, "if a consents to p then a will also consent to q". Such dispositions can be attributed even to animals. We say that (12) in turn is the *meaning* of $p \to q$. The equivalence between $p \vdash q$ and $\vdash p \to q$ is valid only in the external sense as a description of someone's behaviour; this is because "$p \vdash q$" is not a representation, hence it is not stored verbatim.

Now, suppose I want to find out whether $p \to q$ holds. Even if the question is about objective validity, I have to do the reasoning myself, trusting that I can perform the necessary steps in the right way. As we shall see below, the pure calculus actually is transsubjective because it requires the application of Modus Ponens alone and requires neither complex reasoning nor empirical facts.[4] Now, to check whether or not $p \to q$ holds, i.e. whether or not $\vdash p \to q$, I cannot simply undo it to become $p \vdash q$. The latter I cannot represent in my head; it says that there is a certain disposition of mine—and I might not know about that. Yet, what I *can* do is to find out that I do posses the disposition of holding q true after holding p true; but the way to see that is indirect. This is because the disposition is

[4]It does presuppose mental resources to perform the reasoning as well as a certain mental stability to correctly perform the reasoning. This is not at all easy nor trivial, and it means that the transsubjective character is only an idealisation. But without this idealisation nothing of substance can be said.

not something that I can directly see myself (and in fact anyone else) as having.[5] The way to find about my disposition is to *enact it*. I suppose p and then see for myself. The enacting is what makes me see whether or not (12) is true. Notice that dispositions are subjective: you and I may have different ideas of what it means for one thing to follow from another. In that case it seems more straightforward to parametrise $p \vdash q$ for the agent that is performing the reasoning. This introduces a parameter that the phrase /`if...then`/ does not display. Humans are aware of the problem and are (with limitations) able to perform acts of reasoning as if being someone else. I have chosen to refer here to use the 'mathematical mode', which is assumed to be independent of the impersonator, to avoid getting into deep water before having stated my point.[6]

The point is that I really do consider "$\vdash$" a symbol to denote someone's judgement dispositions. It is external and not internal. The agent himself does not use "$\vdash$" for that. To see the difference consider Pavlov's dog. Recall that Pavlov had trained his dog by always giving it food after he rang a bell. After doing that a number of times the dog had developed an expectation. When Pavlov rang the bell, the dog expected food. We might describe this as saying that the dog believes this: "if the bell rings there will be food". But as far as we know, dogs do not put their beliefs into words. The dog does not believe it as that. In particular, it has no notion of implication. Rather, what it has internalised is the connection between the bell ringing and there being food. In other words, the correct ascription to the dog is [7]

(13) `The bell rings.` $\vdash$ `There is food.`

The difference is that the latter does not lead to any expectations in case that the bell does not ring. This is a rule that only fires when the premiss is satisfied. No bell, no food, not even the expectation of food. In particular, it is incorrect to

[5]To see whether I possess the disposition "$p \vdash q$" one would have to identify a specific pattern of my mind that corresponds to it. In principle, an agent can only know that he possesses the disposition "$p \vdash q$" by trying out on himself. It is like reverse engineering: if you do not know whether the chip makes it function in a particular way (because the manufacturer refuses to tell you what hardware he uses) you run a few tests and then decide.

[6]To be fair, it must be said that "$\vdash$" in (12) should rather be written "$\vdash_T$", with some parameter theory T, since in logical theory there is not one universal deductive relation but a continuum of them. Thus, effectively, the lack of intersubjectivity of $\vdash$ is already present in logic.

[7]To be exact, the dog does not know about English, so I would have to replace the English strings by something else. However, this would result in overly pedantic notation and obscure matters at hand.

ascribe to the dog the following:

(14) `There is no food.` $\vdash$ `There is no bell ringing.`

And this is not just because there is no way for a dog to represent negative beliefs. It is also incorrect because the schema underlying it cannot be enacted. If there is no food, there is no expectation of bell ringing. Modus Tollens is not a mode of reasoning that dogs use...

Now, the statement

(15) `If the bell rings there is food.`

uses a linguistic object, here the words /`if`/ and /`then`/. It puts to words what "$\vdash$" says. Thus, once we have have grasped (13) we are able to understand (15) because all the difference is in the introduction of a linguistic object whose meaning is clear to us. In turn, the rule (13) is assessed through enacting; in this way (15) expresses a reflection on an internal process. For its meaning is (indirectly) the enacting scheme sanctioning (13). A human subjected to Pavlov's experiment would not only be able to put words to the thought "the bell rings" and "there is food". He will inevitably at some point stop and think this way: "whenever the first holds, then the second holds as well". In symbolic terms, he has realised that his habit is described by (13) (through using his own concepts and judgements as carriers of the thought). He can say:

(16) `Suppose the bell rings. Then there is food.`

And, knowing what /`if`$\cdots$`then`/ means he may *phrase* this as (15).[8] Thus, a human observer is able to look back at his own reasoning, and moreover can use it to deduce quite complex patterns either of himself or of the world.[9]

The difference between conditional speech acts and their enacting is very important and often overlooked. But the importance cannot be overestimated. The

[8]To be able to reason this way, one will also have to be able to represent the fact that the bell is ringing out of context. This requires symbolic capacities that presumably only humans have. On the other hand, note that the fact that the dog actually comes to acquire (13) through learning means that facts are somehow represented and the dog actually remembers them. The connection between the ringing of the bell and food in the plate is reinforced in stages. A moth is unable to learn that way. It acts on light always in the same way.

[9]One may think that he has learned a connection between facts of the world. But that is not exactly true. If he learns the connection between bell ringing and food it will first be in the form of the disposition (13). He can then rationalise on his own acquired disposition.

first consequence is that there are things that differ not in truth conditions but in what one may call ‘packaging’. Suppose I say

(17) `If people continue to drive cars then oil prices will rise.`

Was I saying (17) to claim that either people will not continue to drive or else the oil price will rise? Or was it to say that the oil price will rise, but I added a condition that I really only say this if people continue to drive cars? I think I would have a hard time telling you. Similarly in logic. There is no real difference in the following two claims.

❶ $p \vdash q$

❷ $\vdash p \to q$

In fact, as I have said above, I am claiming that ❶ is what ❷ actually means. So, when people hear me saying (17) they are given a choice: to represent this in ‘logicalese’ either as (18) or as (19).

(18) $\vdash$ `People continue to drive cars` $\to$ `Oil prices will rise.`

(19) `People continue to drive cars.` $\vdash$ `Oil prices will rise.`

However, as matters stand, (19) is not a way that things can be represented in someone’s head.[10] Thus, while I can have an attitude towards (18), I cannot have an attitude towards (19). What I can do to see how matters stand, is enact it: assume the premiss holds and check my stance on the conclusion. Enacting is a way to understand. So, if you want to *understand* what I am saying when I say (17) then (18) is one step further from the goal than (19). Because understanding means unpacking the meaning of $\to$ (or /`if...then`/), it leads you straight into enacting (19).

The upshot is this: you may either refuse to look at the meaning of “$\to$” (you listen, but you don’t understand) or you do want to understand it. Then you use the Deduction Theorem[11] and subsequently enact the conditional claim. You suppose

[10]It is important to understand the difference between coding and representing. (19) can be coded, as the expectation of food after bell ringing is coded in the head of Pavlov’s dog. But representations are symbolic. “$\vdash$” is not part of the representation. For that, its correlate “$\to$” must be used.

[11]You may use anything that you have learned to be equivalent to it. I am not claiming that the Deduction Theorem is the only way to unpack the arrow. But it seems to me to be the canonical way.

that people will continue to drive cars and then see for yourself whether the oil price rises. The enacting can actually be directly invoked by linguistic means in the following way.

(20) `Suppose that people continue to drive cars. Then oil prices will rise.`

The meaning of the latter clearly is not a statement. The first half is an imperative (/`suppose`/) even though the sentence is not ended by an exclamation mark. It asks you to picture the situation in order to enact the rule.

5 The Calculus of Judgements and Propositions

I will present a formal account of the various notions presented so far. This will be a model of what is going on in the mind of a person $\mathcal{P}$. To start, we distinguish between **acts** and **dispositions**. Dispositions are timeless; acts on the other hand happen in time. If there is a disposition to perform an act $\mathfrak{a}$, $\mathfrak{a}$ may be performed on different occasions. Dispositions may change, but this is a long term process and not within the scope of this paper. We are studying here the way dispositions are enacted and how this can account for complex reasoning behaviour.

We fix a language $\mathcal{L}$ of propositions or formulae, with certain syntactic rules. $\mathcal{L}$ may use various natural languages, but most importantly it uses certain internal symbols, such as "$\to$", "$\wedge$". The syntax is considered a public object, the meaning private. Signs of various languages may be mixed. (This requires that they share the same type system.) Thus, if /`baker`/ is a word of English, it is admissible to write "`baker`(x)" to say that it applies to x. It is equally admissible to write "baker′(x)", where baker′ is one's own concept of /`baker`/. This mixing of languages allows to replace parts of one language by parts of another bit by bit. It also allows to leave certain words unresolved. If you have never heard some word you do not simply reject the sentence that contains it but rather work around it as much as you can. Also, we do not need to double a public concept by a private one. You may or may not form your own concept of a baker. If you don't, you simply work with the word /`baker`/ instead, which you use as a unary predicate in the same way.

A **judgement** is a pair $\succ\varphi$, where $\succ$ is **phematic sign** and φ a member of $\mathcal{L}$. Notice that a judgement applies to φ verbatim. That means that it is directed towards the exponent, not the meaning. This is because in thinking about some subject matter I have to represent it in my head and I can only consent to my mental representation, not to the intended meaning. At present the only phematic signs are "$\vdash$" and "$\dashv$", which stand for **acceptance** and **supposition**, respectively. A judgement has a preparatory phase of **apprehension**. This is the moment when the formula is brought into focus for judgement. Since judgement is an act we should rather introduce a time point t and write "$t : \succ\varphi$" to say that the judgement occurred at time t; we may consider adding more details to the conditions under which the judgement occurred. In fact, when stored in memory, such things will often be added. We typically remember under what circumstances we came to know a certain fact or reached a particular conclusion. Often enough, however, we do not remember such detail and only recall the conclusion itself. An apprehension need not yield a judgement; there may be formulae whose truth value we do not know. But nevertheless, in order to find out, we have to apprehend them first. The result of apprehension is what I call a **phematic act**. There are several types of phematic acts, judgemental (accept, reject) or non-judgemental (suppose, unsuppose). We shall assume that at any given moment of time there can be at most one phematic act; once it is performed it becomes history.

A **conditional judgement disposition** has the form $\Delta \succ \varphi$ where Δ is a set of formulae and $\succ\varphi$ is a judgement. A **theory** is a set of conditional judgement dispositions. Our theory of the world is thus described by a set of conditional judgement dispositions. There is no condition on this set; it may even be inconsistent. The **state** of a reasoner is a triple $\langle T, S, A\rangle$ such that T is a theory, S a (structured) set of formulae called **slate**, and A is empty or contains one judgement. Reasoning proceeds by passing from one triple to the next. All three components of the triple are time dependent, though we shall keep T fixed throughout, to keep matters simple. When T is empty, it will be dropped. Slates are short term devices to keep track of one's acts. Once we have made a judgement we need to store it in the slate or else it will be lost. Hence we proceed from $\langle T, S, \vdash \varphi\rangle$ to $\langle T, S^\frown\varphi, \varnothing\rangle$ and not to $\langle T, S, \varnothing\rangle$. This would be a rather pointless procedure, though logically entirely correct. The fact that S now contains φ means that we can retrieve φ for further use.

For the construction of slates we have a constructor "$\ulcorner$". "$\ulcorner\varphi$" indicates that φ is an assumption. This is reminiscent of line-by-line deductive systems where vertical lines are drawn to symbolise the area of validity of a particular premiss. In a slate a premiss is valid to its right but not to its left. Thus, if the slate has the form $S^\frown\ulcorner\varphi^\frown S'$, then φ is valid throughout S'. The judgement window A is thought to be placed at the end of the slate. Thus all assumptions are visible for A. We shall give some basic rules of transitions. The arrow $\rightsquigarrow$ is used as follows. If $\mathfrak{s} \rightsquigarrow \mathfrak{t}$ then a reasoner can pass from state $\mathfrak{s}$ to state $\mathfrak{t}$. (See Definition 1.)

① **Assumption** $\langle T, S, A\rangle \rightsquigarrow \langle T, S, \dashv\!\exists\ \varphi\rangle$.

② **Conversion** $\langle T, S, \dashv\!\exists\ \varphi\rangle \rightsquigarrow \langle T, S^\frown \ulcorner\varphi, \emptyset\rangle$.

③ **Conversion** $\langle T, S, \vdash \varphi\rangle \rightsquigarrow \langle T, S^\frown \varphi, \emptyset\rangle$.

④ **Activation** $\langle T, S^\frown\varphi^\frown S', A\rangle \rightsquigarrow \langle T, S^\frown\varphi^\frown S', \vdash \varphi\rangle$.

⑤ **Activation** $\langle T, S^\frown\ulcorner\varphi^\frown S', A\rangle \rightsquigarrow \langle T, S^\frown\ulcorner\varphi^\frown S', \vdash \varphi\rangle$.

⑥ **Phatic Enaction** $\langle T, S, A\rangle \rightsquigarrow \langle T, S, \succ\varphi\rangle$, provided that $\Delta \succ \varphi \in T$ and for all $\delta \in \Delta$ either δ or $\ulcorner\delta$ occurs in S.

⑦ **Reflection** $\langle T, S^\frown\ulcorner\varphi, \vdash \chi\rangle \rightsquigarrow \langle T, S, \vdash \varphi \to \chi\rangle$.

⑧ **Firing** If $\varphi \to \chi \in S$ or $\ulcorner\varphi \to \chi \in S$ then $\langle T, S, \vdash \varphi\rangle \rightsquigarrow \langle T, S, \vdash \chi\rangle$.

⑨ **Forgetting** $\langle T, S^\frown\varphi^\frown S', A\rangle \rightsquigarrow \langle T, S^\frown S', A\rangle$.

Some comments are in order. The rule of Conversion is very important: it allows to convert a judgement into a proposition that can be stored in the slate. If the phematic sign is "$\vdash$" this is somewhat trivial. But it could be something else like rejection "$\dashv$" (see below). In this case, it is not the proposition itself that is being stored but its negation. Thus, in general we have rules of conversion that proceed from $\langle T, S, \succ\varphi\rangle$ to $\langle T, S^\frown O_\succ(\varphi), \emptyset\rangle$, where $O_\succ$ is a propositional operator corresponding to $\succ$. Below we shall also look at structural correlates for judgements.

The opposite of conversion is activation (it might therefore be called "deconversion"). Again, in the more general setting it takes the form

(21) $$\langle T, S^\frown O_\succ(\varphi)^\frown S', A\rangle \rightsquigarrow \langle T, S^\frown O_\succ(\varphi)^\frown S', \succ\varphi\rangle$$

Furthermore, there are rules of structure elimination, as we shall discuss below.

There are two rules concerning the behaviour of implication. Reflection is a principle that relates the structure building operation "$\ulcorner$" with "$\to$". Firing allows to use the implication to reason forward with a premiss. Lastly, there is a rule that allows to drop intermediate results.

Definition 1 (Derivation) *A **derivation** is a sequence S_i, $i < n$, such that each S_i is a state $\langle T_i, S_i, A_i\rangle$, and for every $i < n$, S_i is an axiom or follows from S_{i-1} via any of the above rules.*

Notice that in contrast to standard calculi we can only check the previous state, the others are inaccessible.

Of particular interest are the conditional judgement dispositions of a reasoner. They can be immediate or derived. If $\Delta \succ \varphi \in T$ then the disposition is immediate, otherwise derived. It matters how many steps it takes to derive the disposition for the more steps the more time is needed to execute it.

Definition 2 *Write* $\Delta \succ_T \varphi$ *if* $\langle T, \ulcorner\Delta, \succ\varphi\rangle$ *is derivable, where* $\ulcorner\Delta := \{\ulcorner\delta : \delta \in \Delta\}$. *If* $\Delta = \varnothing$ *or* $T = \varnothing$ *it can also be dropped.*

Proposition 3 *Suppose* $T \subseteq T'$. *Then if* $\Delta \vdash_T \varphi$ *then also* $\Delta \vdash_{T'} \varphi$.

We say that an agent consents *unconditionally* to φ if $\vdash_T \varphi$, and that he consents to φ *a priori* if $\Delta \vdash_\varnothing \varphi$. The formulae that an agent unconditionally and a priori consent to have a special status; anyone can in principle find out that they are true.

6 Examples

We shall now look at some examples in more detail. We shall show that any agent can be brought to consent a priori and unconditionally to an intuitionistically valid formula in $\to$—and only those.

The lines in these examples show the state of a person at a given time. Time is implicit (not represented) and is thought to proceed top to bottom. The column labelled S shows the slate at the given moment; the column labelled A shows us the phematic act at that time. We assume T to be empty, though any other T will do, by Proposition 3. It is therefore not represented here. The rightmost column shows the rule that has been used; it is *not* part of the state.

(22)

S	A	
	$\dashv\!\exists\ \varphi$	①
$\ulcorner\varphi$	$\varnothing$	②
$\ulcorner\varphi$	$\vdash \varphi$	⑤
	$\vdash \varphi \to \varphi$	⑦

(23)

S	A	
$\varnothing$	$\dashv \varphi \to (\chi \to \psi)$	①
$\ulcorner\varphi \to (\chi \to \psi)$	$\varnothing$	②
$\ulcorner\varphi \to (\chi \to \psi)$	$\dashv \varphi \to \chi$	①
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi$	$\varnothing$	②
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi$	$\dashv \varphi$	①
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi$	$\varnothing$	②
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi$	$\vdash \varphi$	⑤
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi$	$\vdash \chi \to \psi$	⑧
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi^\frown\chi \to \psi$	$\varnothing$	③
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi^\frown\chi \to \psi$	$\vdash \varphi$	④
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi^\frown\chi \to \psi$	$\vdash \chi$	⑧
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi^\frown\chi \to \psi$	$\vdash \psi$	⑧
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi^\frown\ulcorner\varphi$	$\vdash \psi$	⑨
$\ulcorner\varphi \to (\chi \to \psi)^\frown\ulcorner\varphi \to \chi$	$\vdash \varphi \to \psi$	⑦
$\ulcorner\varphi \to (\chi \to \psi)$	$\vdash \varphi \to \chi \to (\varphi \to \psi)$	⑦
	$\vdash \varphi \to (\chi \to \psi) \to$ $(\varphi \to \chi) \to (\varphi \to \psi)$	⑦

We begin with some remarks on derivations. If there is a derivation of $\langle S, A\rangle$ then there also is a derivation of $\langle U^\frown S, A\rangle$ for any U. Simply prefix every intermediate state with U.

Lemma 4 (Leftward Monotonicity) *Let $\langle S, A\rangle$ be derivable. Then so is $\langle U^\frown S, A\rangle$.*

A set S of formulae is said to be closed under modus ponens (MP) if whenever $\varphi \to \chi, \varphi \in S$, also $\chi \in S$.

Theorem 5 *The set of propositions that a person $\mathcal{P}$ unconditionally accepts contains intuitionistic logic for $\to$ and is closed under modus ponens.*

Proof. Intuitionistic logic for $\to$ is characterised by two axioms and one rule: the axioms are (FD) (derived above) and $\varphi \to (\chi \to \varphi)$ and the rule is modus ponens

([10]). We derive the second formula as follows.

(24)

S	A	
$\varnothing$	$\dashv\varphi$	1
$\ulcorner\varphi$	$\varnothing$	2
$\ulcorner\varphi$	$\dashv\chi$	1
$\ulcorner\varphi^\frown\ulcorner\chi$	$\varnothing$	2
$\ulcorner\varphi^\frown\ulcorner\chi$	$\vdash\varphi$	5
$\ulcorner\varphi$	$\vdash\chi\to\varphi$	7
	$\vdash\varphi\to(\chi\to\varphi)$	7

Now we show closure under MP. Suppose that $\mathcal{P}$ unconditionally accepts $\varphi\to\chi$. Then it derives $\langle\varnothing,\vdash\varphi\to\chi\rangle$ and from there $\langle\varphi\to\chi,\varnothing\rangle$. Now suppose that $\langle\varnothing,\varphi\rangle$ is derivable. By Leftward Monotonicity, so is $\langle\varphi\to\chi,\vdash\varphi\rangle$. Then $\langle\varphi\to\chi^\frown\varphi,\varnothing\rangle$ and so $\langle\varphi\to\chi^\frown\varphi,\vdash\chi\rangle$. Then $\langle\varphi,\vdash\chi\rangle$ and also $\langle\varnothing,\chi\rangle$ are derivable. □

Classical logic, however, is *not derivable*. To see this, let us define the following translation for sequences.

(25a) $\sigma(\varphi^\frown S) := \varphi\wedge\sigma(S)$

(25b) $\sigma(\ulcorner\varphi^\frown S) := \varphi\to\sigma(S)$

Lemma 6 *Suppose that $\langle S,A\rangle$ is derivable. Then for every decomposition $S = S'^\frown\varphi^\frown S''$, φ is intuitionistically derivable from $\sigma(S')$.*

Proof. By induction on the length of a proof. □

Theorem 7 *$\Delta\vdash\varphi$ iff $\bigwedge\Delta\to\varphi$ is intuitionistically valid.*

Notice that this does *not* apply to $\dashv\varphi$; that is, we cannot show $\sigma(S)\to\varphi$ if $\langle S,\dashv\varphi\rangle$ is intuitionistically valid. For the latter is always derivable. Now, if $\langle\varnothing,\vdash\varphi\rangle$ is derivable, so is also $\langle\varphi,\varnothing\rangle$. Hence, every unconditionally a priori accepted formula is intuitionistically valid. It follows that Peirce's formula is not accepted unconditionally a priori.

Another useful property is the deduction theorem (DT). In this calculus the DT comes almost for free.

Theorem 8 (Deduction Theorem) *$\langle S,\vdash\varphi\to\chi\rangle$ is derivable iff $\langle S^\frown\ulcorner\varphi,\vdash\chi\rangle$ is derivable.*

Proof.

(26)

S	A	
U	$\vdash \varphi \to \chi$	
$U^\frown \varphi \to \chi$	$\varnothing$	②
$U^\frown \varphi \to \chi$	$\dashv \varphi$	①
$U^\frown \varphi \to \chi^\frown \ulcorner \varphi$	$\varnothing$	②
$U^\frown \varphi \to \chi^\frown \ulcorner \varphi$	$\vdash \varphi$	④
$U^\frown \varphi \to \chi^\frown \ulcorner \varphi^\frown \varphi$	$\varnothing$	③
$U^\frown \varphi \to \chi^\frown \ulcorner \varphi^\frown \varphi$	$\vdash \chi$	⑩
$U^\frown \ulcorner \varphi$	$\vdash \chi$	⑩

Conversely, using (Reflection) we get back $\langle S, \vdash \varphi \to \chi \rangle$ from the last line. □

We get the following exchange property for free:

Corollary 9 *Let S be a permutation of S'. Then $\langle S, A \rangle$ is derivable iff $\langle S', A \rangle$ is derivable.*

Notice also that since we may drop all premisses of the form φ from S, it is enough to study only derived dispositions of the form $\Delta \vdash \varphi$.

So, what then happens to Peirce's formula? We can try to get it by using the DT. In that case, the agent assumes $(\varphi \to \chi) \to \varphi$ and sees whether he can consent to φ. The assumption is an implication, so he may try to assume the premiss, $\varphi \to \chi$. This gets him the conclusion but leaves him with showing $\varphi \to \chi$. We know of course that no matter how he turns matters he cannot do it. But it is enlightening to try it out anyway.

7 Different Forms of Knowledge

One of the advantages of this model is that it allows to represent the difference between the following.

(27) `If` φ `then` χ`.`

(28) `Suppose` φ`. Then` χ`.`

(24) consists in the single judgement "$\vdash \varphi \to \chi$", while (25) consists in the sequence of judgements "$\dashv \varphi$" followed by "$\vdash \chi$". For notice that every natural language sentence contains in addition to a proposition also a phematic sign. In

this way the utterance of a sentence becomes a phatic act. We do not simply make a noise or denote a proposition, we also endow it with a force. A thetic sentence endows it with the force of the affirmative judgement sign “$\vdash$”. Thus, by uttering /`If` φ `then` χ`.`/ we claim the truth of the propositional content. That is why its representation is “$\vdash \varphi \to \chi$”. The sequence of two sentences allows to make two phatic acts. The first of them, /`Suppose` φ`.`/ is actually an imperative: it asks the listener to assume φ. The representation is “$\dashv \varphi$”. The second says /`Then` χ`.`/. It claims that under this condition χ holds.

(29) $\vdash \varphi \to \chi$

(30) $\dfrac{\dashv \varphi}{\vdash \chi}$

Additionally, it is possible to represent knowledge in different ways. The theory of $\mathcal{P}$ may contain either the disposition $\varphi \vdash \chi$ or the disposition $\vdash \varphi \to \chi$. Though by the results obtained above these two representations are identical in what can be deduced from them, there are differences in how easy this is.

(31)

T	S	A
$\varphi \vdash \chi$		$\dashv \varphi$
$\varphi \vdash \chi$	$\ulcorner\varphi$	$\varnothing$
$\varphi \vdash \chi$	$\ulcorner\varphi$	$\vdash \chi$

T	S	A
$\varphi \vdash \chi$		$\dashv \varphi$
$\varphi \vdash \chi$	$\ulcorner\varphi$	$\varnothing$
$\varphi \vdash \chi$	$\ulcorner\varphi$	$\vdash \chi$
$\varphi \vdash \chi$		$\vdash \varphi \to \chi$

(32)

T	S	A
$\vdash \varphi \to \chi$		$\dashv \varphi$
$\vdash \varphi \to \chi$	$\ulcorner\varphi$	$\varnothing$
$\vdash \varphi \to \chi$	$\ulcorner\varphi$	$\vdash \varphi \to \chi$
$\vdash \varphi \to \chi$	$\ulcorner\varphi^\frown\varphi \to \chi$	$\varnothing$
$\vdash \varphi \to \chi$	$\ulcorner\varphi^\frown\varphi \to \chi$	$\vdash \varphi$
$\vdash \varphi \to \chi$	$\ulcorner\varphi^\frown\varphi \to \chi$	$\vdash \chi$
$\vdash \varphi \to \chi$	$\ulcorner\varphi$	$\vdash \chi$

T	S	A
$\vdash \varphi \to \chi$		$\vdash \varphi \to \chi$

In (28) the theory contains the simple disposition $\varphi \vdash \chi$, while in (29) it contains the ‘coded’ version $\vdash \varphi \to \chi$. While the simple disposition $\varphi \vdash \chi$ can be used directly, the disposition $\vdash \varphi \to \chi$ must be recalled and then used for reasoning via the rule of firing. However, if used on the sentence (24) the reasoning is much faster. This is because the target of judgement is known to $\mathcal{P}$ *verbatim*.

In the other cases, however, phrasing it like (25) makes it easier for $\mathcal{P}$ to follow. He has to do less work for each individual claim. He can asses the validity by looking at A almost step by step.

It is an interesting question to ask how someone comes to acquire $\varphi \vdash \chi$ as opposed to $\vdash \varphi \to \chi$ into his theory. Basically, as we observed earlier, $\varphi \vdash \chi$ corresponds to a disposition learned by experience. Unlike the latter it does not use any linguistic sign to encode the relation between φ and χ. However, $\varphi \to \chi$ does just that and is typically the result of explicit instruction or thought process.

8 Conversion

A sign has at least two faces: a **signifier** and a **signified**, also known as **exponent** and **meaning**, respectively. The exponent is a tag, and is an arbitrarily chosen entity that is coupled with the meaning in a sign. Only in the sign does the exponent get its meaning, and only by virtue of the sign is the meaning the meaning of the exponent. A language is constituted by a set of signs. Not knowing a language means not knowing what signs it actually has. The word /`telephone`/ denotes some class of objects, or maybe a concept.

Using the knowledge of the signs we should in principle be able to understand the sentences of a language. For example, knowing the words of English will allow me to understand what this sentence means:

(33) `John has two telephones.`

In Montague Semantics this is exactly what happens; each word is translated into its meaning and the meanings are combined according to the rules of the grammar. At the end we get a formula like this

(34) $(\exists X)(\mathrm{card}(X) \geq 2 \wedge (\forall y \in X)(\mathsf{own}'(\mathsf{john}', y) \wedge \mathsf{telephone}'(y)))$

There are several reasons why this is not everything that is going on. One aspect of this is that the translation into λ-calculus (31) may not be much better than (30). It is maybe more precise but may for some be just as difficult to understand. However, if our goal is not not represent the content but to understand it then we are no wiser. (I am deliberately ignoring the obvious advantage of a logical language of being unambiguous.)

For concreteness, let's return to "$\to$". As such it is just a piece of code as is its natural language counterpart /`if`$\cdots$`then`/. I may be told that the meaning of $p \to q$ is the same as that of $p \vdash q$, for example. The latter, however, is not a representation. It describes a disposition to answer q when p. If you like, it is a program inside of me that is activated by p and returns q. In fact, it is difficult even in natural language to express that very meaning. Typically, we explain the meaning by enacting it in the form (25). Now, when we are given an implication, be it in the form of natural language or in formal languages, the process of understanding what it says is the same: if we do not have other means (like knowing the answer by heart) then we must proceed by enacting it. And we have seen that enacting does not resolve all arrows, only some. There are other cases like this. Consider the sentence

(35) `It is true that it is raining.`

What does it take to understand this? My proposal is to say that "φ is true" is nothing but "$\vdash \varphi$". Thus, the formula describes a judgement (or a disposition, whichever). Notice, however, that then we cannot represent the idea that (32) is true. This would require writing

(36) $\vdash\vdash$ `It is raining.`

However, "$\vdash$" cannot be used that way. The judgement sign attaches only to a proposition, not to a judgement. What it technically says is "there is a disposition to consent to there being a disposition to consent to that it is raining" or something of that sort. It is a second order notion. I claim that it does not exist inside my head. And this is because the disposition of myself to consent to the truth of p is not what I can apprehend. Recall that apprehension requires a formula. Facts are not apprehended. You don't judge seeing something; you only judge seeing something as something, in other words, when you categorise the experience using concepts. Thus all I can do is consent to p and see myself doing that. The disposition has to be enacted to become visible. Once I have observed myself giving consent to p I can express that in the thought: "p is true", or formally "$\mathsf{T}(p)$", with T the exponent of my own truth predicate. Thus, the rule of judgement for $\mathsf{T}(p)$ is this: judge $\mathsf{T}(p)$ true if φ. Since φ in turn must be assessed, we get this:

(37) $\langle T, S, \vdash \varphi\rangle \rightsquigarrow \langle T, S^\frown\mathsf{T}(\varphi), \varnothing\rangle$

Notice that this can be iterated:

(38)

T	S	$\vdash \varphi$
T	$S^\frown\mathsf{T}(\varphi)$	$\varnothing$
T	$S^\frown\mathsf{T}(\varphi)$	$\vdash \mathsf{T}(\varphi)$
T	$S^\frown\mathsf{T}(\varphi)^\frown\mathsf{T}(\mathsf{T}(\varphi))$	$\varnothing$

Namely, if I consent to p, I am entitled to consent to $\mathsf{T}(p)$; once that is done I can now apprehend $\mathsf{T}(\mathsf{T}(p))$. Since $\mathsf{T}(p)$ is there (in the slate, representing my memory), I judge $\mathsf{T}(\mathsf{T}(p))$ to be true. What is crucial is to understand that "is true" is a concept and "T" is a sign. Every layer of "T" buries the content of the thought one level down; my original consent to p is history. Just now I have given my consent to $\mathsf{T}(\mathsf{T}(p))$. The fact that this was because I gave consent to p is something I may recall from memory. It is not something that I at this very moment can apprehend and judge. I can however always return and rethink my judgement, but no two judgements can be done at the same time.

At this point we may understand why paradoxes are not such a big problem for natural language semantics. Suppose I meet the following inscription in a classroom.

(39) `This sentence is false.`

Then in order to understand it I do not simply translate it into a formula and then get a truth value in a model (which is impossible). Instead, the approach is in stages. I might say: so this is allegedly true, let's see. I convert /`is false`/ to the disposition to reject the content, which is that very sentence. So I reject it. I enter again, converting the meaning of /`is false`/ into the disposition to accept, and so on. I may continue like a moth spiralling into the light, or else recall that I had reached that point before. I smile and leave. I refuse to do any more work on that.

The logician in me might protest, thinking: how can the same thing be both true and false? And how come you didn't see it coming? Here I wish to answer only the second complaint: because understanding is an act that unfolds in time. It is an act that we may also refuse to perform or put to its proper (?) conclusion. Normally, facts radiate to some degree. Our mind produces conclusions in an instant. The word /`Berlin`/ invokes images in me that the word and its meaning do not support; they are real for me, I have lived there long enough for them to be automatic. But the radiation only goes a certain way; I do not immediately start to picture everything I know about it; only a little bit. And the same for the sentence above. The words it has in it typically do not radiate very much. Since we have no intuitions about the sentence at all, we go the pedestrian's way, converting the words into representations, until we either wake up to the fact that we have been fooled, or give up without result at some point.

9 Structured Slates

So far we have only dealt with implication and consent. The calculus can be enriched to contain conjunction. One way to do this is via conversion rules. The motivation for conversion rules is this: given a sign $\sigma = \langle e, m\rangle$, that is, a pair between some expression e and some meaning m, we may *use* σ to replace e by m and conversely. For example, in knowing that English has a sign $\langle \mathsf{dog}, \mathsf{dog}'\rangle$, we may step from the judgement "$\succ \mathsf{dog}(x)$" to the judgement "$\succ \mathsf{dog}'(x)$". For example, we may pass from "$\vdash \mathsf{dog}'(x)$" to the judgement "$\vdash \mathsf{dog}(x)$". That means, having consented to the proposition that x is a dog' we now consent to the fact that the expression /dog/ can be applied to x. Conversely, once we have consented to the proposition that /dog/ applies to x we may also consent to "$\mathsf{dog}'(x)$". This is something that speakers of English perform quasi automatically. Having consented to something being called /dog/ a speaker of English also consents to it being a dog (or rather it satisfying dog', the speaker's native concept of a dog). I do not require that the parts in a sign be necessarily of the form $\langle L\text{-expression}, L\text{-meaning}\rangle$; they could also be of the form $\langle L\text{-expression}, L'\text{-expressions}\rangle$, for example, $\langle \mathtt{telephone}, \mathtt{puhelin}\rangle$, provided that we know what the signs mediate between (here: expressions of English and expressions of Finnish).

It might now appear that the unfolding of the meaning of sentences is a matter of applying this process in one direction; and that packaging the meaning into sentences is to go in the other. This is essentially the way Montague Grammar proceeds: every overt element gets converted into its meaning and then everything is put together. But things are not that simple. In addition to the question of actually performing the translation (which has to be stepwise), the basic problem that I am troubled with is that as far as I can see some of the words do not have anything of a proper internal correlate. For example, it basically makes no sense to declare that /and/ means $\lambda p.\lambda q.q \wedge p$ or that /$\mathtt{or}$/ means $\lambda p.\lambda q.q \vee p$. And even if it did it would get us no nearer to an intuitive understanding of what is said. For we still have to understand what will make us consent to $q \wedge p$. At that point we feel compelled to say that consenting to $q \wedge p$ means to consent to q and p. And to see whether that is so, one finally has to actually apprehend each of them in turn. Therefore, instead of getting cashed in in terms of some static mental correlate, logical connectives unfold into action sequences. When unpacked they become sequences of noetic acts—and that *is* their meaning, at least normally. This brings to life ideas by Lorenzen about dialogs in logic, though in a rather indirect way. For while Lorenzen thought about *dialogs* as foundations of logic, here we seek the foundation in a *monologue*. There is no attack and counterattack: there is only the question of gathering enough support to be able to make ones own claim.

This back-and-forth usage of signs connects with logic, showing up in the dialectics between a formula and its proof. We shall use natural deduction style format now. The following describe the enaction of conjunction.

$$(40) \qquad \frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

These rules say the following: if you have φ and ψ you may write down $\varphi \wedge \psi$. Conversely, if you have $\varphi \wedge \psi$ you may write down φ, or you may write down ψ. Notice however that rules of enaction do not require deletion of the material, while rules of conversion do. Therefore, rules of conversion must be exact; so there must be something that we trade for the connective "$\wedge$". It could be as simple as the following: it is the set of φ and ψ. This would require that the set is manipulated in the required way.

However, I will go a slightly different path. I shall describe the meaning of the connectives in terms of judgements. Here is a rule for conjunction.

Consent to "$\varphi \wedge \chi$" if you have consented to "φ" and χ".

This can be translated into the following rule:

$\langle T, S, A\rangle \rightsquigarrow \langle T, S, \vdash \varphi \wedge \chi\rangle$ if φ (or $\ulcorner\varphi$) and χ (or $\ulcorner\chi$) is in S.

For since the speaker has a slate S, he can recall whether or not he has consented to φ by looking it up; he can also look into his knowledge base. Thus "have consented" effectively means "exists in S".

In this way the rules factually become asymmetrical: our judgement becomes contingent not on other judgements but on other propositions. This is exactly what I have tried to argue all along. In fact, this is just the opposite of the conversion of a judgement into something that can be entered into S. Notice how we have converted the judgement "$\dashv\!\exists\ \varphi$" into the object "$\ulcorner\varphi$". Similarly, "$\vdash \varphi$" may be converted into "$\mathsf{T}(\varphi)$" when entered into S from A.

Notice that the consent to $\varphi \wedge \chi$ is conditional on the previous consent to φ and χ. However, it is never excluded that there is another way to derive a positive judgement of $\varphi \wedge \chi$. Also, the consent to φ can be derived from consent to a more complex formula:

Consent to "φ" if you have consented to "$\varphi \wedge \chi$".

The rule for the truth predicate is equally simple.

> Consent to "$\mathsf{T}(\varphi)$" if you have given consent to "φ".

The rule for implication is quite complex. Consent to $\varphi \to \chi$ requires assumption of φ and then deriving χ from that assumption. If we succeed, we may eliminate the assumption of φ.

Conjunction has been straightforward. Disjunction however is not. The validity of $\varphi \vee \chi$ cannot be assessed by looking up the validity of φ and χ on their own. (Truth is another matter, but recall that there is no notion of truth sui generis.) Hence we must resort to a construction that allows to represent the disjunctive state. A straightforward way is to allow the slate to branch. Even easier is to allow sets of slates, each member represents a different state. The overall state is the disjunction of the entire set of states. There must be rules governing the behaviour of these sets. A third solution is to implement a notion of proof by exhaustive enumeration.

The structural approach to slates is effectively present in the notation "$\ulcorner\varphi$". This notation suggests that we were assuming φ, so it looks like a judgement. But it is not a judgement. Rather, '$\ulcorner$' is some kind of constructor that produces a subordinate structure ('window') with label φ. To close the window we must introduce an arrow ('reflection'). In this way the logical connectives get structural correlates, similar to display logic ([2]). The juxtaposition is conjunction, splitting is disjunction and opening a subwindow is related to implication.

As for negation, we now need to introduce a new kind of judgement: rejection, denoted by "$\dashv$". Write "$\dashv p$" to say that p is rejected. In the internal calculus there is nothing that corresponds to it, just like conjunction. When we transfer "$\dashv \varphi$" from A into the slate we convert it into the formula $\neg\varphi$.

(41) $\langle T, S, \dashv \varphi\rangle \rightsquigarrow \langle T, S^\frown\neg\varphi, \varnothing\rangle$

Conversely, we may explain rejection (partially) by the following rule.

> Reject "φ" if you have consented to "$\neg\varphi$".

This is exact reversal of the previous. A stronger rule is this.

> Reject "$\neg\varphi$" if you have consented to "φ".

This allows to deduce "$\vdash \varphi \to \neg\neg\varphi$".

(42)

S	A
	$\mathrel{-\!\!\!3}\ \varphi$
$\ulcorner\varphi$	$\varnothing$
$\ulcorner\varphi$	$\vdash \varphi$
$\ulcorner\varphi^\frown\varphi$	$\varnothing$
$\ulcorner\varphi^\frown\varphi$	$\dashv \neg\varphi$
$\ulcorner\varphi^\frown\varphi^\frown\neg\neg\varphi$	$\varnothing$
$\ulcorner\varphi^\frown\neg\neg\varphi$	$\varnothing$
$\ulcorner\varphi^\frown\neg\neg\varphi$	$\vdash \neg\neg\varphi$
$\ulcorner\varphi$	$\vdash \neg\neg\varphi$
	$\vdash \varphi \to \neg\neg\varphi$

The formula $\neg\neg\varphi \to \varphi$ is however *not* deducible. The calculus is asymmetrical. For example, it is not possible to assume that φ fails to be the case. We can only assume "$\neg\varphi$", which is unfortunately always a complex formula unlike φ which can be simple.

10 Conclusion

The present paper has tried to develop a somewhat more realistic model of reasoning. It is based on the idea that reasoners can pay attention only to one formula at a time. When asked whether this formula is true they either know the answer off hand or have to decompose it to see what is inside. If its structure conforms with the unpacking mechanism they can arrive at an answer. If not, difficulties arise that they may not effectively be able to resolve.

It is a significant result of this paper that the logic resulting from this process oriented view on logic is intuitionistic logic and not classical logic. In this connection it is important to emphasise that the rule system above is only a proposal, nothing is sacrosanct about it. Moreover, since it makes empirical predictions about ease of understanding, it would be desirable to check the predictions empirically. Though there is some latitude, it seems to me quite a robust property that the calculus derives intuitionistic logic, not classical logic.

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