DOV GABBAY: *FIBRING LOGICS*. A REVIEW.

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Studying combined logics is a growing trend in applied logic. The general question is this: Suppose language $\mathfrak{L}_3$ has been formed by fusion of two languages $\mathfrak{L}_1$ and $\mathfrak{L}_2$. Let two logics L_1 and L_2 be defined over $\mathfrak{L}_1$ and $\mathfrak{L}_2$, respectively. What can be said about conservative extensions L_3 in $\mathfrak{L}_3$? This is a fairly broad problem. Conservative extensions are not unique, and they may have different properties. In modal logics, several extensions have been studied in detail. For example [1] studied bimodal modal logics which are the minimal conservative extensions of their monomodal fragments (called fusions). Another example are the products of modal logics, to which the author of the present book has made important contributions. Numerous questions arise: what are the properties of the combination and can they be predicted from the properties of the fragments? Is there a general recipe by which we can obtain combined systems?

The present book promotes *fibring* as a general model construction for combined systems. The general idea is this. Let the languages $\mathfrak{L}_1$ and $\mathfrak{L}_2$ and two classes of models be given. Fibred models can be thought of as models for one of the languages enriched with an oracle that answers about the truth of formulae of the other language. This oracle takes the form of a function F, which, given a model $\mathfrak{M}$ (which typically contains a distinguished world or state), returns a model $F(\mathfrak{M})$ of the other language in which φ is evaluated. F is called the *fibring function*. If φ actually contains operators of the first language, this construction thus toggles between the two model classes until φ is totally decomposed.

The possibilities for combination are galore. We can try to combine logics whose signature is more or less similar (like two predicate logics with different but possibly nondisjoint signature), modal logics with different sets of operators, but we may also try something more radical by combining logics that are basically dissimilar. Two paradigm examples are intuitionistic modal logic (where the modal operator with its modal semantics is added on top of the intuitionistic connectives) and the temporalisation of systems. The latter is interesting insofar as it shows that it might be useful to study systems that do not use the full power of the combined language. Suppose that we have an arbitrary logical language L, say that of predicate logic. Now we study the language that arises by issuing temporal statements whose primitive clauses are constituted by members of L. Thus the connectives of L may not take temporal connectives in their scope. A similar, no less important example comes from programming. It is often desirable to import programs from a different programming language. Some implementations allow this. However, the languages may be very different (say, Prolog and C). One therefore does not wish to fuse the languages; one simply wishes to be able to import the results of a query issued in another language. At times the book actually suggests that just about any consequence relation in the joint language $\mathfrak{L}_3$ is a 'fibring' of the consequence relations induced on the fragments $\mathfrak{L}_1$ and $\mathfrak{L}_2$ (see Pages 327 – 328).

Fibring is supposed to be a universal construction that allows to construct a semantics for combined systems. The fundamental insight on which it rests is that basically any language with operations that are either monotonic or antitonic with respect to their arguments are complete with respect to Kripke–like structures. This allows to use an idea of fibring that derives from the 'fig cactus' models, in which the fibring function is a function which assigns a model of the other language to each world of the structure of the first language. The fusion of the modal logics L and M, denoted by $L \otimes M$, is Kripke–complete if L and M are. (Actually, [2] has provided a construction even in case that the logics are incomplete.) Moreover, it inherits many joint properties of the fragments. Unfortunately, the intuition behind fibring suffers from a defect that is never really addressed. The fibring function effectively glues Kripke–frames for one language onto Kripke–frames of the other. This is done in parallel for each of the worlds, and each of the models added is an oracle not only for the formulae we are interested in but for all others as well. However, take the example of $\mathsf{K.B} \otimes \mathsf{K.B}$, where $\mathsf{K.B} = \mathsf{K} \oplus \Diamond\Box p \to p$, the logic of symmetric Kripke–frames. The two fragments use the same boolean connectives and a modal operator, $\Box$ for the first and $\blacksquare$ for the second fragment. Then if $w \; R_\Box \; x$ and $x \vDash \blacklozenge\blacksquare\Box p$ then $w \vDash p$, because $x \vDash \Box p$ and $x \; R_\Box \; w$, by symmetry. This is independent of the way in which the models are combined; it follows from the properties of the logics alone (namely, that $\Diamond\blacklozenge\blacksquare\Box p \to p$ is a theorem of the fusion). This can make the construction break down because we might be adding the wrong model at another world. (This is why the construction of [1] is rather involved.) The book briefly glosses over the problem without proving the innocence of the construction in general.

The book is a tour de force of applied logic. Every area is touched on: classical, nonclassical, modal, fuzzy and substructural logic, logic of programs, propositional and predicate logic. Certainly, for some readers this will be a plus because the method is amply exemplified. Unfortunately, in an attempt to show that fibring is *the* solution to the problem of combining logics, the book rarely goes beyond defining the model structures in the various cases. This however leaves little space to discuss the general theory of fibring itself. Instead, one example after the other is presented, and often it is not clear what contribution fibring actually makes. For example, in making the logic fuzzy more has to be done that just fibring the models. First, the semantics of the nonmodal language has to be fuzzified (which is not done using fibring). Second, the interpretation of the box is actually not unique despite expectation. (We could make $R_\Box$ fuzzy, too, which effectively means that we combine a fuzzy modal logic to begin with rather than *making* it fuzzy.) Also, it sometimes turns out that the logic of the fibred structures is not even conservative over its fragments. Strangely enough, in the view of the author this does not seem to disqualify them.

There are other deficiencies, too. One is the relaxed use of terminology. There is practically no distinction between 'structure' and 'model', for example. The notation has not been given much thought and occasionally changes without warning. In general, proofs are not really worked out. One example is the completeness proof of fibring for polymodal logics, another is the claim on Page 61 that necessitation is admissible in $\mathbf{T}_I^F$ if the individual logics have the disjunction property. This is however not sufficient (a counterexample is the combination of $\mathsf{K.T}$ with itself, a fact that is noted on Page 54).

In sum, even though the topic is certainly interesting and the technique of fibring deserves attention, this book is not the kind of book that will easily promote it. It has obviously been produced in a hurry. At numerous places the book iterates material, and sometimes it even contradicts itself. The benefits of fibring are not demonstrated beyond doubt, especially since the construction needs to be carried out with care.

References

[1] Marcus Kracht and Frank Wolter. Properties of independently axiomatizable bimodal logics. *Journal of Symbolic Logic*, 56:1469 – 1485, 1991.

[2] Frank Wolter. Independent fusions of modal logics revisited. In Marcus Kracht, Maarten de Rijke, Heinrich Wansing, and Michael Zakharyaschev, editors, *Advances in Modal Logic*, pages 361 – 379, Stanford, 1997. CSLI.

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