The Fine Structure of Spatial Expressions

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In this paper I argue that the meaning of locative PPs is obtained in several stages, more than hitherto assumed. Each of these stages corresponds to a layer of the PP. This is in line with recent developments indicating that the PPs have more than just two heads (Svenonius (2006)). The main innovation of this paper consists in a new way in which the semantics of location denoting PPs is conceived. The idea is that these PPs induce a momentary coordinate frame into which the space is coded. This coding, called *aspect*, is also compositionally defined. Therefore, in order to understand this process properly one needs to look not only at the meaning of the elements but also at the accompanying aspect they induce on the space around us.

1. Introduction. This paper is a continuation of Kracht (2002).[1] Recently, new work has appeared or come to my awareness that has made it necessary to rethink large parts of the earlier work. I am referring here especially to O'Keefe (1996, 2003), Svenonius (2006), and Zwarts (2003, 2005). The novelty of the present paper is that it uses in addition to the meaning of an expression also the *aspect*, which is an encoding of the space surrounding us.

My main interest is in finding the exact nature of meanings of the various parts involved in a locative expressions and how they work together to form the overall meaning. Much of what I am going to say is not all that new. However, it seems to me that there has been little interaction between—especially—semantic field work and linguistic theory, notably formal semantics. For example, there are studies on direction in language (see the collections (van der Zee and Slack, 2003), (Senft, 1997), (Bennardo, 2002), (Levinson and Wilkins, 2006), and the monograph (Levinson, 2003) among others), yet formal semantics seems to be untouched by it. I am trying to rephrase what has been said in other work in terms of conventional (= Montague style) formal semantics in order to bridge this gap. In doing so, I also wish to get a tighter grip on the conceptual issues raised by expressions for location.

2. The structure of Local PPs. The structure of locative PPs is traditionally thought to be as follows (see Jackendoff (1983)):

(1) $[\mathrm{P}_{\mathrm{Dir}}\ [\mathrm{P}_{\mathrm{Stat}}\ \mathrm{DP}]]$

Here, $\mathrm{P}_{\mathrm{Stat}}$ is a static preposition[2] and $\mathrm{P}_{\mathrm{Dir}}$ is a preposition denoting a path or a movement pattern, depending on conceptualisation. An exemplification of this structure is

(2) [from [under the bridge]]

'the bridge' denotes a particular object; 'under' selects a location on the basis of this object. Finally, 'from' describes a motion in space on the basis of the location. Recently, it has emerged that there is more structure to locative PPs. Based on data of Kham and other languages, Svenonius (2006) concludes that inside $\mathrm{P}_{\mathrm{Dir}}$ we also find a projection of $\mathrm{P}_{\mathrm{AxPart}}$, describing axial parts of an object (see also Pantcheva (2006)). Caponigro and Pearl (2006) argue for the existence of a silent location creating preposition, which I call here $\mathrm{P}_{\mathrm{Loc}}$. Taking this together we arrive at the following structure.

(3) $[\mathrm{P}_{\mathrm{Dir}}\ [\mathrm{P}_{\mathrm{Stat}}\ [\mathrm{P}_{\mathrm{AxPart}}\ [\mathrm{P}_{\mathrm{Loc}}\ \mathrm{DP}]]]]$

An example, analysed in detail in Section 7, is

(4) $[_{\mathrm{Dir}}$ to $[_{\mathrm{Stat}}$ in $[_{\mathrm{AxPart}}$ front of $[_{\mathrm{Loc}}\ e\ [_{\mathrm{DP}}$ Schumacher's car$]]]]]$.

The present paper will fill this structure with semantic detail. It will be seen that each of these projections can independently be motivated and that they are type theoretically different. (For the notation on types see Appendix B.) This provides a convergence of purely semantic analysis on the one hand and syntactic/morphological analysis on the other, which is so often lacking. Part of the reason that the relationship is so tight is in my opinion the fact that the structure of PPs is less obscured due to movement than that of a clause.

3. Aspects of Space. This paper will be incomprehensible without getting clear about certain fundamental distinctions. To make this more accessible to a linguistic audience, I have moved the technical discussion to the end (see Appendix A). Here I shall deal with the conceptual issues. One important issue is the distinction between *points*, *vectors* and *coordinates*. On the one hand there is an ontological distinction between them (see also Appendix A). On the other hand the same space can be looked at essentially in three ways, using points, using vectors, and using coordinates. These ways of looking at the same object (the space) I call an *aspect*. The space consists of points (the aspect is the identity). Points are given to us as *locations* of things. Based on some fixed givens we can name the locations (or points) via some scheme. The first scheme is by means of *vectors*. These are sets of pairs of points (see Appendix A). A vector indicates *motion*. (The name comes from Latin *vehere* 'to carry, to drive'.) Given a set of cardinal points we can set up a coordinate frame and associate with each point a tuple of numbers, called the *coordinates*.

The distinction between these three notion is relevant even in the Euclidean space $\mathbb{R}^3$, where points *are* triples of numbers, and vectors can be (and often are) seen as members of the space itself. Even here the identification between points and vectors is not as innocent as it appears because the coordinates of an object change as we change the cardinal points of our coordinate frame. Thus, coordinates are not absolute; they are established with reference to some other points. This makes them impractical in daily life. We shall return to this issue below.

However, the distinction is perhaps best seen by looking at the nature of physical spaces. Spaces are sets of points that have a particular topological structure. They constitute what is called a *differentiable manifold*: if you have an object moving in it you can define the derivative of motion, the *motion vector*. The motion vector, unfortunately, cannot always be identified with a vector of your manifold. Consider living, as we do, on the surface of a ball. Then if some object, say a car, is in motion along a street, the motion vector points straight ahead. If you follow that vector you are actually moving away from the surface. The car, however, does not actually follow that vector even if it appears to go straight; rather, it will constantly readjust its motion vector so as to stay on the surface. (This is a consequence of the earth's gravity. If that did not exist, the car would really leave the surface.) One may check in fact that any motion vector for some object has the same property: though it points in the direction of motion, it can only be followed for a small distance until it needs readjustment. One would like to dismiss these facts as hair-splitting since the curvature is so small. But I plead caution: we shall argue that many Oceanic systems are exactly like our cardinal directions; but because the singularity of the system is quite close to the speakers, the anomaly of such systems is clearly felt. Also, when sailing with a ship, these problem really do matter. Vector fields arise naturally (cardinal directions are a case in point) and we shall look at them below.

Thus vectors arise as the directions of momentary motion. How can we actually represent them? The idea is this. Manifolds are subsets P of the Euclidean space $\mathbb{R}^n$ for some n. Vectors are not objects of P, they are equivalence classes of pairs of points. At each point $\underset{\cdot}{\mathrm{x}}$ of P there is a so-called *tangent space* of vectors tangential to the surface of P. (Intuitively, it is the space of all vectors from $\underset{\cdot}{\mathrm{x}}$ to $\underset{\cdot}{\mathrm{z}}$, where $\underset{\cdot}{\mathrm{z}}$ is infinitesimally close to $\underset{\cdot}{\mathrm{x}}$.) A *vector field* is a function f from P into the vectors of $\mathbb{R}^n$ such that for each $\underset{\cdot}{\mathrm{x}}$, $f(\underset{\cdot}{\mathrm{x}})$ is a member of the tangent space at $\underset{\cdot}{\mathrm{x}}$. Vectors can be added but points or pairs of points cannot (see Appendix A). Typically, sentences do not talk about vectors directly. Mostly, they qualify their length (by talking about speed). For example, the following does not talk about a motion vector, it only gives us begin and end points of a motion event.

(5) Jack was driving from Paris to Bordeaux.

A further distinction to make is that between points and their coordinates. It is often taken for granted that points in space simply are triples of numbers. The fact that this identification is possible in theory does not mean, however, that it is practically speaking helpful. There is namely a difference between talking abstractly about coordinates and actually calculating with them. There are good arguments to reject the view that humans store locations of objects by assigning absolute coordinates to them (a view, that has to my knowledge never been explicitly proposed anyway):

(1) it is very difficult if not impossible to establish these coordinates in the first place, and
(2) since we are ourselves moving in space we would either have to constantly update our own position or else recalculate all coordinates of the objects.
(3) there are no agreed upon coordinates of universal validity.

Instead, we establish coordinates—when needed—relative to some objects. And so we express location relative to an object (the *landmark*) and all positions are calculated relative to this object. Thus whether or not an object is seen as 'moving' depends on its relative position with respect to the landmark, and not on its own movement.[3]

To see the difference, consider two racing cars, one steered by Schumacher and the other by Alonzo. Alonzo is staying exactly 50m behind Schumacher throughout the first lap. One way of saying this is as follows.

(6) Alonzo stayed behind Schumacher during the first lap.

It is clearly not the case that Alonzo's car was at rest. It was, however, at rest relative to the other car. To express this simple fact in absolute coordinates—anchored, say in Greenwich—would be a painful exercise, and useless, too. Instead, we think of Schumacher's car as the origin of a coordinate frame relative to which the trajector's location is defined. Now, if there is anything we must take home from physics it is this: it does not matter where we choose the origin of our coordinate system. All that changes is the triple of numbers for each point.

The point ultimately is what actually is cognitively speaking the most convenient way to establish or find a location. We would expect on that basis that absolute coordinates are rarely used as opposed to positions relative to known landmarks (mountains, buildings etc). Moreover, we use polar coordinates, separating distance and orientation, rather than Cartesian coordinates. This has been established also experimentally (see Landau (2003) and O'Keefe (2003) for just two sources).

However, as much as the present paper is informed about the cognitive research on space, it is not the primary goal to go into any details of it. Rather, I want to take the opportunity to develop a framework inside formal semantics that takes as much advantage as it can from cognitive and foundational work in order to present a coherent picture. Again, Joost Zwarts in the work cited above has taken important steps in this direction. What is missing so far is a complete, bottom-up picture of how spatial meanings are being built.

4. Orientation. We shall walk the reader through the three steps: from objects to axial parts, from there to locations, and finally to directions. (As will become apparent later, some of these steps are optional.) The reader should think about the example (4). At the bottom we have an object, from which we extract a location. Using the object we establish some target location using polar coordinates: they consist in a direction combined with a distance. While the distance is given by the metric of the space, the direction is established with reference to some cardinal axes. Here is an example.

(7) We live 1/4 mile to the west of UCLA.

The reference point is (the location of) UCLA, the direction is 'west', and the distance is 1/4 mile. We occasionally draw attention to the fact that statements such as (7) must be understood with reference to some chosen 'centre' of the region under discussion. For it is evident that distance from a region is not well defined if that region is large, as in this case.

An example involving motion is this.

(8) They sailed north for 1000 miles.

The reference point is (the location of) the ship, direction is 'north', and the distance covered is 1000 mile. There are significant differences between these examples, as we shall see.

4.1. Origin. The calculations begin at the bottom, with the element *e* of (4). It is usually empty. It derives a *location* $\underset{\cdot}{\mathrm{o}}$ from an object. Moreover, as it does so, it provides us with a new aspect to our space: from now on we may actually look at each of the space points as a vector. Namely we associate to $\underset{\cdot}{\mathrm{x}}$ the vector $\overrightarrow{\underset{\cdot}{\mathrm{o}}\underset{\cdot}{\mathrm{x}}}$.

The DP provides us with the *landmark* (or *ground*, depending on usage). From this we derive a region as follows. We assume that there is a function loc', which, given an object x and a time point t returns the region that x (or more exactly the *solid* of x) occupies at t.

(9) $$\mathsf{loc}' : e \times \tau \to r$$

To define the origin of the coordinate frame, we can make several choices, for example, we may choose the volumetric centre (see Appendix A for the function $c(\cdot)$):

(10) $$\mathsf{origo}' : e \times \tau \to p : (x, t) \mapsto c(\mathsf{loc}'(x, t))$$

4.2. Directions. Having established the origin, we now turn to coordinate frame. The coordinate frame is the basis for defining any directions. It needs three points that define what I call the *cardinal directions*, which we may, loosely speaking, identify as 'up', 'front' and 'left', in that order.

Definition 1. *A **coordinatiser** is a function from points of the space (and additional parameters) into the set of coordinate systems such that $f(\underset{\cdot}{x})$ is a coordinate system anchored at $\underset{\cdot}{x}$.*

Thus, coordinatisers are mappings from $P \times S$ into P^4, such that $f(\underset{\cdot}{o}, \vec{q}) = \langle \underset{\cdot}{o}(\vec{q}), \underset{\cdot}{p}_1(\vec{q}), \underset{\cdot}{p}_2(\vec{q}), \underset{\cdot}{p}_3(\vec{q})\rangle$ is a coordinate system (where $\vec{q} \in S$ is a possibly empty sequence of extra parameters). The intuitions behind this is that the coordinate system represents an ideal viewer positioned at $\underset{\cdot}{o}$, facing $\underset{\cdot}{p}_2$, head directed towards $\underset{\cdot}{p}_1$. Our definition therefore restricts the range of coordinatisers to right handed orthonormal systems. This can be relaxed; we may allow vectors of any length for the coordinate systems (applying some rescaling, so that the metrical structure is not affected) and we may even allow skew coordinates (where the axes are not orthogonal to each other). Nothing will affect the setup that I sketch here, so I shall remain with a more conservative choice. Once a coordinate frame has been set up, our aspect on the space changes again; now points are no longer assigned vectors, but rather triples of numbers, called *coordinates* (Cartesian or polar).

Alternatively, a coordinatiser is a triple of maps

(11) $$(\underset{\cdot}{x}, \cdots) \mapsto \langle u(\underset{\cdot}{x}, \cdots), f(\underset{\cdot}{x}, \cdots), \ell(\underset{\cdot}{x}, \cdots)\rangle$$

where $u(\underset{\cdot}{x}, \cdots)$, $f(\underset{\cdot}{x}, \cdots)$ and $\ell(\underset{\cdot}{x}, \cdots)$ form a right handed system of vectors. Here the dots represent additional parameters on which the functions depend. This allows to reduce coordinatisers to maps from P to vectors, as we shall from now on assume. As Levinson (2003) explains, there are mainly three systems, *absolute*, *intrinsic* and *relative*. The key difference between them is the number of parameters that go into their definition.

4.2.1. Absolute Frames. The easiest case is provided by functions needing no additional parameters. These are the absolute systems. The standard absolute system derives from a coordinatiser which returns for every point on earth three directions: up, north, and west. However, there is more flexibility.

A *absolute system* creates a coordinate system based on only one input: the origin.[4] Given that we need three vectors, it is easy to see that an absolute coordinatiser is a triple of vector fields.

Recall that a vector field is a function that returns for every point a vector. Examples of vector fields are: the direction and string of wind on earth. There is a theorem known as the *Hairy Ball Theorem*, which says that for every continuous vector field over the earth there must be at least one point for which the value of the vector field is the zero vector. Thus, there must always be one point on earth where the wind is not blowing. There is an application for absolute systems: no matter how an absolute system is defined, it is either discontinuous or is undefined at certain points. Consider for example the vector field $\lambda \underset{.}{x}.\mathsf{north}'(\underset{.}{x})$ denoted by 'north', which returns for every point a unit vector pointing north. By the Hairy Ball Theorem this is impossible. Indeed, if you are standing on the north pole, there is no direction north any more, while if you are standing on the south pole, any direction could be called north. Similar systems are found in many Oceanic languages: these generally have two directional terms, one meaning 'seaward' and the other 'landward' (see part 4.2 in the survey (Palmer, 2002)). The 'landward' vector points from the location towards some specific location $\underset{.}{\ell}_c$ on the island, say, the peak of some mountain. (Oceania consists basically of plenty of relatively small islands.) 'Seaward' points opposite to 'landward'. Obviously, when standing at that location $\underset{.}{\ell}_c$, neither 'landward' nor 'seaward' are defined any more. If we were to extend the system over the entire surface of earth, there would be analogous point (the antipode of $\underset{.}{\ell}_c$).[5]

The Oceanic systems work well on a small scale, while they become unusable on a large scale. Our cardinal directions are somewhat hard to use in general, and people vary in their ability to use them consistently. Levinson (2003) has argued that speakers of Guugu-Yimidhirr manage to keep track of the direction of north, and has argued that this is due to the lack of intrinsic or relative frames. Such language is certainly extreme. Moreover, more than just the influence of language seems to be needed. Ed Keenan (p.c.) tells me that conversations in Malagasy can only be successful if the participants know where north is. In contrast to speakers of Guugu-Yimidhirr, who seem to manage this orientational task with considerable ease, one does find Malagasy speakers enter in a conversation with people in a foreign village and, before starting the actual conversation, ask the other one "Tell me where north is!". Thus, setting up coordinate frames (and therefore coordinates) is a considerable task, one that has been somewhat neglected in the work in formal semantics of spatial expressions.

4.2.2. Intrinsic Frames. Coordinatisers that are not absolute are *relative*. A subpart of the relative coordinatisers are the *intrinsic coordinatisers*. An *intrinsic* coordinatiser is one that uses the properties of the landmark alone. The additional parameters are thus the landmark plus its orientation and posture. One example is 'in front of', which defines an axis depending on the canonical front of the object. I will not go into much detail; the front is as much shape dependent as it depends on use, for example. Suffice it to say that 'in front of' is certainly not absolute since on the same spot objects can be oriented in different ways. A distinct version of intrinsic frames is exemplified by 'ahead' or 'behind'. The direction 'front' is here specified using the direction that the landmark is taking at reference time. This is one use of 'behind', which is distinct from another and which uses the canonical orientation rather than the direction. This can lead to ambiguities. Consider the following sentence

(12) Alonzo is right behind Schumacher.

Imagine the situation where Schumacher is actually driving backwards. Then there is a conflict between an understanding of 'behind' as deriving from 'towards the rear of the car' and another understanding which is 'against the direction of movement'. Typically these are aligned: if someone is driving forward, in order to face the people behind him he has to turn his head, that is, look against the direction that he is driving. But when he is driving backwards, he is facing the people that are driving 'behind' him, in one sense of 'behind'; but in another he is not. This ambiguity is pervasive. 'Top' can mean among other: 'against gravitational force' (absolute), or 'in direction of its top' (intrinsic).

In Niikanne (2003) it is reported that Finnish has postpositions that can only be used when the landmark is in motion.

(13) Buick on Volvon edellä.
'The Buick is driving such that it stays in front of the Volvo.'

(14) Buick on Volvon edessä.
'The Buick is in front of the Volvo.'

I analyse this as follows. The postposition simply requires motion of the landmark. It is not clear to me whether this is because it establishes the coordinate frame on the basis of the movement direction (since this would require motion). That will have to be established.

4.2.3. Viewer Oriented Frames. There is a third kind of coordinatiser, called *relative* by Levinson; since the previous was also relative, let me call this one *viewer oriented*. This one involves yet another point, namely the point of a *viewer*. The latter is left undefined; under normal circumstances it will be 'speaker's eyes', but that is shiftable. One situation where this might occur is in indirect speech or speech or attitude report. The viewer contributes a canonical system in the following way. 'Up' is aligned with 'top' (so, if the viewer is standing upside down, that may be what other people call 'down'). 'Front' is aligned with the vector from viewer to landmark. (And here, as in all cases, directions are established from centre of the solid to centre of the solid, or, in the case of humans, using the 'third eye' in place of the volumetric centre as origo.) 'Top' presents this third kind: I can use it with viewer centred coordinates. When I am standing upside down then what I call top is what other people call bottom. Let me note that viewer centred directions are subject to the same dichotomy as tenses: we can imagine them to pick as viewer the one producing the utterance, or the one holding an attitude in an attitudinal report or issuing the utterance in a speech report. In Pima (Uto-Aztecan), verbs are obligatorily accompanied by particles that indicate whether the situation happens in front of the speaker or on the side or elsewhere.[6] These particles respond to the place of the person issuing the utterance; when used in reported speech, they respond to issuer of the utterance in which they occur. Thus, they are *relative* or *shiftable*.

I will briefly mention a subtlety concerning orientation. I am consistently speaking of vectors. This means that given a point as origin and a line through it, there are two unit vectors originating at this point, and they go in opposite directions. This matters, though not always to the same degree. There is a noticeable difference between 'up' and 'front' on the one hand and 'left' on the other. The first two typically are distinguished form their antipodes 'down' and 'back', while many languages do not differentiate 'left' and 'right'. This happens in some Oceanic languages (see (Palmer, 2002), or (Hyslop, 2002) for specific cases). Psychologically, confusion between 'left' and 'right' is not uncommon, while the other directions are never confused with their antipodes. This is important insofar as the idea that a preposition such as 'behind' needs an entire coordinate system to be defined will strike one as an overkill: it simply needs only one direction (direction of movement) and is otherwise rotationally symmetric. Yet, it makes a (psychological) difference whether that axis is the 'up' or 'front' axis as opposed to the 'left axis' (see work cited in Niikanne (2003)), and so I opted for the more elaborate analysis.

Before closing this section, let me add some remarks on the life span of a coordinate frame. In principle, any preposition can use its own coordinate frame. Thus, when we say something is south of UCLA and to my right I am issuing descriptions based on two different coordinate systems. Furthermore, one is intrinsic and the other is absolute. Thus, whether at a given instance we apply our own coordinate system or an intrinsic one or an absolute one depends—in part—on the preposition. Thus part of the meaning of the preposition is contained in the coordinatiser. (This is why I have decided to gloss 'in front of' as containing the parts 'in (direction)' and 'front of'. The second is the coordinatiser.) Not all prepositions need a coordinatiser; I mention 'in' as an example. True body part expressions are another case in point. In the latter case we talk about the close region projected from the volumetric centre through some specified region of the solid. This part—it seems to me—is specified independently of the coordinate frame. A case in point is provided by 'back' in Zapotec, see (Lillehaugen, 2006). It has two meanings: 'behind' and 'at or on the back of'. These are distinct in the case of a deer, for example. It seems, if only for historical reasons, beneficial not to force the true body part nominals through the process of coordinatisation, and have them establish the region directly through projection from the volumetric centre.

This does throw up questions about the entire system of prepositional meanings in a language and its interconnections because we need to know when and how it is that a body part nominal is coerced into a purely spatial expression that functions by means of coordinatisation. The empirical side is well documented (see (Svorou, 1993)). What I wish to provide here is a formal framework within which these questions can be raised to sufficient degree of precision.

Also, if a rocket is flying vertically upwards, then to fly behind the rocket is the same as to fly upwards. The 'front' axis then seems to be aligned with the 'up' axis. This, however, is a misunderstanding. Each spatial expressions projects its own coordinate system; sometimes some of the expressions are used with the same coordinatiser, for example, 'south' and 'west'; and similarly 'front' and 'left' in their intrinsic readings. Also, many coordinate frames that appear to be skewed result from a conflict between the definitions of the axes. Some axes are defined by the sun, others by the direction of winds (e. g. the Monsun), yet others by the shape of the environment (the coastline, rivers), see (Palmer, 2002) for details. In all these cases, it is generally assured that different adpositions are based on the same coordinate frame and therefore do not constitute evidence for skew coordinate systems.

4.3. Summary. The origo is defined as above. The 'up' axis is typically defined to be 'against gravitational force'. (Exceptions exist.) So, it remains to specify 'front' and 'left'. Here, considerable variation occurs.

① **Absolute type**. The directions are fixed by the origo $\underset{\cdot}{o}$ alone. Typical representatives: 'north', 'west'.

② **Intrinsic type**. The directions are fixed by properties of the landmark x alone.
  - (a) 'front' is defined to be: 'in direction of x's motion'. This obviously requires x to be in motion. Typical example: Finnish 'edessä' (see (Niikanne, 2003)); English 'behind'.
  - (b) x has an inherent axis of the required kind (people, animals, cars, chairs, buildings). Typical example: 'in front of'.

③ **Viewer oriented type**. The directions are fixed with reference to a second entity (typically the viewer). The front/back axis is determined by drawing a line from viewer to the origo. Typical examples: 'behind'. (In Hausa, 'in front of' is used in this situation just like English 'behind', as if the object faces the same way as the viewer rather than facing the opposite direction.)

It is noted that the Hausa preposition 'in front of' is classed as intrinsic, but from a type theoretic point of view it must be seen as involving the viewer; it is therefore viewer oriented.

Notice the qualifying phrase 'of the right kind' in ②(b). A flagpole, for example, has an axis, but is significantly less felicitous with 'in front of' since its canonical orientation is upward. It is interesting to note that if that same flagpole was lying on the ground, still the phrase 'in front of' is not felicitous simply because the flagpole has no intrinsic front-axis. One should bear in mind, though, that this is a convention of the phrase ‘in front of’ and the English language.

It thus emerges that the sentence ‘the car turned left’ can have various distinct meanings. If the car is moving we may consider ourselves aligned with its motion vector and determine the direction of ‘left’ using the direction of our left arm. This is independent of which way the car or the driver is facing. If you are sitting in the car facing opposite, ‘left’ then means exactly the other direction. Still different is the meaning of the sentence uttered by an outside observer, who may take his own perspective on the matter. Crucially, the distinctions have nothing to do with the direction of ‘left’ per se. They have to do with the way the coordinate frame is established.

5. Location. When we are done with setting up the coordinate frame, our space can be seen as fully coordinatised. Regions can be coded as subsets of $\mathbb{R}^3$, with the coordinates established using the origin and the axis system. We have therefore reached a third level of abstraction, where the spatial schema is now represented using triples of numbers.

The location of the figure is determined on the basis of the location of the landmark. This is typically done by giving a direction and a distance, both of which are optional.

(15) The library is 100m to the west from here.

(16) The squirrel is in front of the car.

(17) Hamburg is less than 300 km away from Berlin.

The distance is determined by the metric and shall interest us no further; the direction however makes use of the coordinate frame just established. A preposition such as ‘in front of’ determines the direction where the figure is found by comparing it with the direction of the front axis. This is in turn the y-axis of the intrinsic landmark system.[7] This is the idea behind *templates* (from (Logan and Sadler, 1996)). A template is a function from $\mathbb{R}^3$ to $[0, 1]$ (of type $\xi \to d$). It measures the degree to which a given point is in the described location (see also O’Keefe (1996, 2003)). A point will fit the description to a certain degree and so there are not just two answers to the question ‘is x in front of y’ but rather a continuum; I shall refer to the elements of $[0, 1]$ as *truth degrees*.

5.1. Spatial templates. A spatial template tells us how well the trajector is located, based on its coordinates. It takes advantage of the coordinate frame. O’Keefe (2003) gives specific formulae to calculate the value. The formula correlates distance with aberration from the ‘front’ axis. For example, O’Keefe (2003) uses the formula for the so-called *boundary vector cells* (BVC), which predicts the strength of the stimulus in a cell (I have simplified the formula setting $d_i = \varphi_i = 0$; $\propto$ means 'is proportional to').

$$g(r,\theta) \propto \exp[-r^2/2\sigma_r^2]/\sqrt{2\pi\sigma_r^2} \times \exp[-\theta^2/2\sigma_a^2]/\sqrt{2\pi\sigma_a^2} \tag{18}$$

These templates have the shape of a raindrop. The idea here is that there is a two way normal distribution across distance and aberration. From this we can get a template $\varphi(x)$ calculating the polar coordinates for x before submitting this to $g(r,\theta)$.[8] Therefore, the meaning of 'in front of' does not simply establish two sorts of regions, those that *are* in front of and those that are *not*. Rather, the judgement is graded. An object is not merely 'in front of' another object; it is 'in front of' that object to a certain *degree*. The degree of fit is added through a process that is governed by cognitive processes which are independent of the language in question.[9] An additional consequence of formulae like (18) is that the acceptability of 'in front of' decreases with distance, which is desired. All directionals have the tendency to become less good at great distances.

If this template derives from the physiology of the visual apparatus, then from a linguistic perspective it is enough to know that 'in front of' denotes a certain axis. Linguistically, the meaning of 'in front of' resides entirely in the coordinatiser (rather, the second component thereof). This at least is language dependent. In Zapotec, for example, many objects have intrinsic parts (see Levinson (2003)), and in Hausa trees have intrinsic fronts. We may learn that 'front' is determined by shape, use, direction to different degrees. However, once we know all this there still is a residue that is encoded in the template and seems to be quite independent of language, and more or less cognitively determined. However, it is important to point out that the fact that 'in front of' can be taught simply by saying which axis it establishes does not mean that the axis alone is its real meaning. The addition of templates definitely changes the extensional content of 'in front of' and must therefore be counted in. It is the fact that language is taught from humans to humans that allows us to omit certain details of meaning. Part of the meaning is simply 'subliminal'; but that does not mean it is not there.

In order to judge whether an object is 'above' or 'in front of' a landmark, several factors must be taken into account. These are: the aberration from the principal axis that the localiser is based on, the distance of the trajector, and the shape of both trajector and landmark. As O'Keefe (2003) points out, there is a correlation between distance and aberration. The larger the aberration, the closer the trajector must be in order to still be in the relation. All this is a matter of the precise formulation of the relation. What is however substantial is that the functions proposed in (O'Keefe, 2003) do not define precise regions. Rather, they are continuous functions from coordinates to $[0,1]$. This affords us two interpretations: we consider a prototypical notion of 'in front': no aberration, and at a certain distance. We then assign all other points in space a goodness of fit value (from $[0,1]$) which declares how well the location of that point fits the description 'in front of'. Or, we declare the function itself to be the meaning of 'in front of' (which we have to do in an extensional setting).

5.2. Graded Neighbourhoods. A template assigns values to points. However, the location of an object is a region. Therefore, we need to see how that point based template can be upgraded to regions. This could be done in several ways. One is to reduce the region to a point, say, the volumetric centre; another is to average over the entire region; the third is to take the maximum value over the region. I am inclined to dismiss the last option. The reason is twofold. Consider a ring around the landmark. If we take the maximum value we would get that the ring is a perfect fit for 'in front of', but this is intuitively not right. I am unable to choose between the other two options. If we choose the first, we now have constructed the core meaning of 'in front of' as

$$\xi(r) := \lambda r.\varphi(c(r)) : \rho \to d \tag{19}$$

(Recall that ρ ranges over connected subset of $\mathbb{R}^3$, not to be confused with actual regions. As with ξ, a path connected set of triples may denote many regions; exactly which one it denotes depends on the coordinatisation.) More serious problems arise with the landmark itself. We have so far pretended that the landmark is a point at the origin. But how about expressions such as 'in front of the house'? Here, different proposals have been made, see (Carlson et al., 2003). Overall, the idea is to reduce region based relations to point based relations, the difference in the proposals are mainly with respect to the formula that is being employed.

Now take again the phrase

(20) in front of the house

The meaning we have just arrived at for this expression is that of what I call a *graded neighbourhood*. It tells us for any region r to what degree it fits the description. The limiting case of such a neighbourhood is provided by a function that has only two values: true and false. For that function, a region either receives 1 (is in the neighbourhood) or 0 (is out of the neighbourhood). This is precisely the definition of neighbourhood in Kracht (2002). Here we use degrees of membership.

5.3. Parametrised neighbourhoods. Since both the landmark and the trajector may be in motion there is a complex interaction between the motion of the trajector and the actual description as its motion has to be coordinatised through the position of the landmark. To stay behind Schumacher is not to stand still; in fact, the spectators cannot be said to stay behind Schumacher. However, another pilot, say Alonzo, can stay behind Schumacher, though he in fact is also moving very fast. Thus, to uphold a stationary spatial relationship with respect to a moving landmark one has to move as well.

Fortunately, matters are not that complex. Since the landmark is the centre of the coordinate frame, the motion of landmark is already accounted for. Only the relative motion is visible. Let us see how this goes. The ingredients into the meaning of 'in front of' are—at least—twofold. First, we are given a coordinatiser $\kappa(x, t)$, which yields a coordinate frame based on the landmark x and time point t. The second ingredient is the template χ.

$$\sigma(x, t) := (\chi(\mathsf{loc}'(x, t)_{\kappa(x,t)}))^{\kappa(x,t)} : p \to d \tag{21}$$

This function depends on the object and time point. To remove the time dependency we abstract:

$$\sigma := \lambda x.\lambda t.(\chi(\mathsf{loc}'(x, t)_{\kappa(x,t)}))^{\kappa(x,t)} : e \to t \to p \to d \tag{22}$$

So, the meaning is a function from objects to functions from time points to functions from points to values. It gives acceptability ratings for space points, given a landmark and a time point.

5.4. Distance. Distance to the landmark is given in terms of phrases like 'half a mile', '3mm' and so on. The reflex of a syntactician is to label these phrases as specifiers of the PP, which means they would semantically compose with the P′. The meaning of the P′ as we have just indicated is a parametrised region. To apply the meaning 'half a mile' to the parametrised neighbourhood 'behind Schumacher' requires considerable sophistication, though. This is because it factually states the distance of the trajector from the landmark (the r in polar coordinates), and such information is only indirectly present in the parametrised neighbourhood, if at all. On the other hand, in and of itself it can be construed as a relation between regions, and so there is a natural interpretation of it as a modifier for P.

6. Directionality. We have now finished the semantics of the inner layers: the meaning of $\mathrm{PP}_{\mathrm{Stat}}$ is a function from time points to functions from space points to the unit interval. There is a time dependency, which monitors the position of the trajector relative to the landmark. The evaluation metric is determined by the spatial template, which is part of the meaning of $\mathrm{P}_{\mathrm{Stat}}$.

Now we shall look at the outer layer. Directionality specifies change of the trajector with respect to the landmark. Fong (1997) has analysed directionals as phase quantifiers. These are special functions from I to $\{0, 1\}$, the set of truth-values. By contrast, Zwarts (2005) claims that directionality is best described in terms of canonical paths. In (Kracht, 2002) directionals were classed as event modifiers, and directionality was taken to be a phase quantifier, though the definition of a phase quantifier was slightly more liberal. By generalising the idea of phase quantification even more, we obtain a synthesis of these views.

Let us call a *path* a function π from $[0, x] \to \mathbb{R}^3$, where $x \geq 0$.[10] The *velocity* is defined by the length of $\frac{d}{dt}\pi(t)$. (This requires that π is differentiable.) π is *canonical* if its velocity is 1.

Lemma 2. *Let $\pi : J \to \mathbb{R}^3$ be a differentiable path. Then there is a positive real number x, a monotone, bijective function $\sigma : J \to [0, x]$ and a canonical path $\gamma : [0, x] \to \mathbb{R}^3$ such that $\pi = \gamma \circ \sigma$. Both x and σ are unique.*

The number x is the length of the path π. Intuitively, γ represents the motion pattern. It shows us in what order the object visits the space points. σ grounds the abstract motion pattern in real time: it maps an interval of time points onto the unit interval and shows how fast the movement is. Since $\frac{d}{dt}\gamma = 1$ we have

(23)
$$\frac{d}{dt}\sigma = \left|\frac{d}{dt}\pi\right|$$

Since $\sigma(0)$ is given, this completely defines σ.

Zwarts (2005) observes that movement patterns can be characterised as certain sets of canonical paths. We shall be content with a brief list.

① **stative**: the path stays inside the location;
② **cofinal**: the path is moving into the location;
③ **coinitial**: the path is moving out of the location;
④ **approximative**: the path moves closer to the location (but not necessarily reaching it);
⑤ **recessive**: the path moves away from the location.

The motion can in fact be recoded as a change in goodness of fit. Recall that rather than giving us truth values, the spatial template actually supplies real numbers between 0 and 1. These numbers depend—holding aberration constant—directly on distance: the closer an object, the better the fit. It follows that a path is static if the goodness of fit remains constant and close to 1 throughout event time; cofinal if it improves (is upward monotone) during event time and reaches 1 (or some sufficiently close neighbourhood); coinitial if it decreases and approximative if it is monotone increasing. We can now attempt a synthesis of these views in the following way.

Definition 3. *A generalised phase quantifier is a function from* $[0, 1]$ *to* $[0, 1]$ *(of type* $\tau \to d$*).*

The codomain is the domain of truth values, while the domain is actually a unit time interval. Thus a generalised phase quantifier describes a change in truth value. We expect this change to be continuous, but no such condition has been put in. Directionality of movement can now be described entirely by means of the (generalised) phase quantifier that it uses.

A movement is *approximative* if the phase quantifier is monotone increasing, *recessive* if it is decreasing. Movement is *cofinal* if the quantifier is monotone increasing and reaches 1, *coinitial* if it is monotone decreasing and starts at 1.

7. The syntactic structure revisited. I shall now revisit the structure (3) and retrace the types associated with the phrases. Our target phrase is the somewhat stilted (4), repeated here as (24).

(24) $[_{\text{Dir}}$ to $[_{\text{Stat}}$ in $[_{\text{AxPart}}$ front of $[_{\text{Loc}}$ e $[_{\text{DP}}$ Schumacher's car]]]]].

As we walk up the structure, not only do we get different objects, but also our space gets structured differently; the aspect on it changes.

Definition 4. *An **aspect** on the space P is an injective function $\alpha : P \to Q$ for some Q. The type of Q is the type of the aspect α.*

We start with the identity aspect $\alpha : P \to P$. on the space P. P is a manifold embedded into some space $\mathbb{R}^n$. The constant $\underline{\text{sc}}$ denotes Schumacher's car.

7.1. $\mathbf{P_{Loc}}$. This element can be identified with loc'. Applied to an object (denoted by the DP) and a time point (to be supplied), it yields a *region*. Additionally, the region is compressed to a point, the gravitational centre. This point serves as the origin of the coordinate frame. The space becomes structured as a vector space (the tangent space to the manifold at the origin).[11] The empty preposition argued for in (Caponigro and Pearl, 2006) is of the kind $\mathrm{P_{Loc}}$.

Spatial points can be given as vectors. These vectors code the points as follows: given $\vec{x}$ and the origin $\underset{.}{\mathrm{o}}$, the space point denoted by $\vec{x}$ is the unique u such that $(\underset{.}{\mathrm{o}}, \underset{.}{\mathrm{u}}) \in \vec{x}$. If $\underset{.}{\mathrm{o}}$ depends on time, so does $\underset{.}{\mathrm{u}}$. The aspect is the function $\underset{.}{\mathrm{x}} \mapsto \underset{.}{\mathbf{o}}$, with $\underset{.}{\mathbf{o}}$ the origin. It is of *vector type* (v). Thus, the contribution of e is both in having a meaning and giving us an aspect. The phrase $[_{\mathrm{Loc}}e\ [_{\mathrm{DP}}$Schumacher's car]] denotes the location of Schumacher's car at t; its aspect is the function assigning to each space point the momentary vector from Schumacher's car to that point.

Assume an object x. Assuming a time point t, we get an origo $\underset{.}{\mathrm{o}}(t) := c(\mathsf{loc}'(x, t))$ and an aspect. Let $[\![\cdot]\!]$ denote the meaning assignment. Thus we get three objects, the meaning, the origo, and the aspect:

$$\begin{aligned} [\![e]\!] &:= \lambda x.\lambda t.\mathsf{loc}'(x,t) \\ \underset{.}{\mathrm{o}} &:= \lambda t.c([\![e]\!](\underline{\text{sc}}, t)) \\ \alpha_e &:= \lambda t.\lambda \underset{.}{\mathrm{x}}.\overrightarrow{\underset{.}{\mathrm{o}}(t)\underset{.}{\mathrm{x}}} \end{aligned} \tag{25}$$

α_e is a parametrised aspect. The entire phrase $\mathrm{PP_{Loc}}$ is of type $\tau \to r$ (parametrised region).

$$[\![e \text{ Schumacher's car}]\!] = \lambda t.\mathsf{loc}'(\underline{\text{sc}}, t) \tag{26}$$

The origo is the volumetric centre of Schumacher's car. This is a time dependent location, so of type $\tau \to p$.

7.2. $\mathbf{P_{AxPart}}$. This element establishes a coordinate frame; more precisely, it returns the three unit vectors. After this step, the space (more precisely the tangent space) is fully coordinatised. We can now regard our space as isomorphic to $\mathbb{R}^3$. A triple $\langle x_1, x_2, x_3 \rangle$ codes the vector

$$\vec{x} = x_1 e_1 + x_2 e_2 + x_3 e_3 \tag{27}$$

Let $\gamma(\underset{.}{\mathrm{x}}) := \langle u(\underset{.}{\mathrm{x}}), f(\underset{.}{\mathrm{x}}), \ell(\underset{.}{\mathrm{x}}) \rangle$ be the canonical coordinatiser. (I am suppressing the additional parameters on which it depends.) We assume that 'front' is aligned with the y-axis, that is, with $f(\cdot)$. This is to say with more precision: we assume that 'front' uses the coordinatiser γ and projects the result onto the second component. In that case γ defines also the *aspect change* α_{front}

contributed by the word 'front'. This aspect change is not an aspect; it merely takes us from vectors to triples, so it will be composed with α_e to yield the current aspect α_g given further below.

(28)
$$\begin{aligned} [\![\text{front}]\!] &= \lambda \underset{.}{x}.\lambda t.f(\underset{.}{x}(t)) \\ \alpha_{\text{front}} &= \lambda t.\lambda \underset{.}{x}.\lambda \vec{y}.\begin{pmatrix} \langle \vec{y}, u(\underset{.}{x}(t))\rangle \\ \langle \vec{y}, f(\underset{.}{x}(t))\rangle \\ \langle \vec{y}, \ell(\underset{.}{x}(t))\rangle \end{pmatrix} \end{aligned}$$

(Notice that $\underset{.}{x}$ ranges over parametrised locations: $\tau \to p$). The aspect is now a (time dependent) function from P to $\mathbb{R}^3$. In the present case the aspect is calculated using the origo, giving us a the contribution to the aspect by the new word:

(29)
$$\alpha_n := \lambda t.\lambda \vec{y}.\alpha_{\text{front}}(t)(\underset{.}{o}) = \lambda t.\lambda \vec{y}.\begin{pmatrix} \langle \vec{y}, u(\underset{.}{o}(t))\rangle \\ \langle \vec{y}, f(\underset{.}{o}(t))\rangle \\ \langle \vec{y}, \ell(\underset{.}{o}(t))\rangle \end{pmatrix}$$

The meaning of the entire phrase is obtained through the mediation of the aspect. Let

(30)
$$\text{red} := \lambda r.\lambda t.c(r(t))$$

This is a reduction from time dependent regions to time dependent points. It should not be confused with the origo.

(31)
$$\begin{aligned} &[\![\text{front } e \text{ Schumacher's car}]\!] \\ =&[\![\text{front}]\!](\text{red}([\![e \text{ Schumacher's car}]\!])) \\ =&[\![\text{front}]\!]((\lambda r.\lambda t.c(r(t)))(\lambda t.\mathsf{loc}(\underline{sc}, t))) \\ =&(\lambda \underset{.}{x}.\lambda t.f(\underset{.}{x}(t))(\lambda t.c(\mathsf{loc}'(\underline{sc}, t))) \\ =&\lambda t.f(\lambda t_0.c(\mathsf{loc}'(\underline{sc}, t_0))) \\ =&\lambda t.f(c(\mathsf{loc}'(\underline{sc}, t))) \end{aligned}$$

The type of this expression is $\tau \to v$, a time dependent vector. Turning to the aspects, we have $\alpha_e(t) : p \to v$ and $\alpha_n(t) : v \to \xi$. The overall aspect is the functional composition:

(32)
$$\alpha_g := \lambda t.\alpha_n(t) \circ \alpha_e(t) = \lambda t.\lambda \underset{.}{x}.\begin{pmatrix} \langle \overrightarrow{\underset{.}{o}(t)\underset{.}{x}}, u(\underset{.}{o}(t))\rangle \\ \langle \overrightarrow{\underset{.}{o}(t)\underset{.}{x}}, f(\underset{.}{o}(t))\rangle \\ \langle \overrightarrow{\underset{.}{o}(t)\underset{.}{x}}, \ell(\underset{.}{o}(t))\rangle \end{pmatrix}$$

7.3. $\mathbf{P_{Stat}}$. This element picks out a location using a spatial template. This yields a goodness of fit function: a function $\eta : \mathbb{R}^3 \to [0, 1]$, telling us for each coordinate triple how well it fits.[12] Using the aspect, this can be translated into a goodness of fit for space points.

The phrase "front of Schumacher's car" has type $\tau \to v$ and aspect α of type $\tau \to p \to \xi$. This aspect is time dependent. It is used in establishing the meaning of the element "in" (in the sense of "in direction of"):

(33)
$$[\![\text{in}]\!] = \lambda y.\eta(y)$$

However, we do not want 'in' to act on triples but rather to locations. The mediation is though the aspect. Thuse we want to get

$$\lambda\alpha.\lambda\underset{.}{z}.\lambda t.\xi(\alpha(t)(\underset{.}{z})) \tag{34}$$

The way to get this is by definition 'aspectual encoding'. It returns for a template i a recoding of the template in terms of the parametrised aspect:

$$\mathrm{enc} := \lambda i.\lambda\alpha.\lambda t.\lambda\underset{.}{x}.i(\alpha(t)(\underset{.}{x})) \tag{35}$$

The aspect remains the same. Notice that this takes a space point (the location of the trajector), transforms it into a coordinate triple using the aspect and then measures its goodness of fit via η. The meaning of the entire phrase is obtained by "aspect driven application". The meaning of the head is applied to the aspect of its complement using aspectual encoding.

$$\begin{aligned} &[\![\text{in front of } e \text{ Schumacher's car}]\!] \\ &=\mathrm{enc}(\eta)(\alpha_g) \\ &=(\lambda\alpha.\lambda\underset{.}{z}.\lambda t.\eta(\alpha(t)(\underset{.}{z})))(\alpha_g) \\ &=\lambda\underset{.}{z}.\lambda t.\eta(\alpha_g(t)(\underset{.}{z})) \end{aligned} \tag{36}$$

which is of type $p \to \tau \to d$.

7.4. $\mathbf{P_{Dir}}$. The element describes a change of goodness of fit through time. Its input is an interval (typically the event time) and an object (the trajector, the mover of the event, see Kracht (2002)). Based on these two and the denotation δ of $\mathrm{PP_{Stat}}$ we define the following generalised phase quantifier:

$$\pi(x, I) := \lambda t.(1/|I|)\delta(c(\mathsf{loc}(x, t)), t) \tag{37}$$

$\pi(x, I)$ tells us how well the location of x fits the description within I. To minimise distraction we assume that $I = [0, 1]$. Otherwise, we would have to introduce a temporal aspect as well, in order to renormalise the time points. This reduces the previous to

$$\pi(x) := \lambda t.\delta(c(\mathsf{loc}(x, t)), t) \tag{38}$$

The meaning of "to in front of Schumacher's car" is the claim that the goodness of fit is increasing and reaches a certain threshold. So, define

$$\mathsf{to} := \lambda q.((\forall xy)(0 \leq x < y \leq 1 \to q(x) < q(y)) \wedge q(1) > \theta) \tag{39}$$

where $\theta \in [0, 1]$, but closer to 1 than to 0. to is a property of generalised phase quantifiers saying that they are monotone increasing and reach some threshold.

Now, let e be an event. Then $\mathsf{time}(e)$ returns the interval of e, which we suppress, since it is $[0, 1]$) and $\mathsf{mov}(e)$ the trajector of e.

$$[\![\mathrm{to}]\!] = \lambda\pi.\lambda e.\mathsf{to}(\pi(\mathsf{mov}(e))) \tag{40}$$

Then, taking (24) to be an event modifier, we propose the semantics

$$
\begin{aligned}
&[\![\text{to in front of } e \text{ Schumacher's car}]\!] \\
&= [\![\text{to}]\!]([\![\text{in front of } e \text{ Schumacher's car}]\!]) \\
&= (\lambda\pi.\lambda e.\mathsf{to}(\pi(\mathsf{mov}(e))))([\![\text{in front of } e \text{ Schumacher's car}]\!]) \\
&= \lambda e.\mathsf{to}(\lambda t.[\![\text{in front of } e \text{ Schumacher's car}]\!](c(\mathsf{loc}(\mathsf{mov}(e)(t), t))))
\end{aligned} \tag{41}
$$

This is of type $\varepsilon \to t$, a property of events. It says that the degree to which the mover of e is in the location of in front of Schumacher's car during event time increases and goes beyond θ.

8. Resilient cases. There are some expressions that do not fit the scheme as outlined above. If the parallelism between syntax and semantics is perfect, as I believe it is, they raise the question whether the projections postulated in (3) are always present. A problematic case, already mentioned, is 'in'. There seems to be no need to establish coordinate systems in order to form an opinion whether an object is in another object. There is no contradiction in this; we may simply say that the semantics works without a coordinate system. A second case is provided by 'along' and 'around'. Here the decomposition into an inner, static part and an outer, directional part is called into question. For the static meaning of 'along' is the same as 'near', but the movement pattern is not described using a generalised phase quantifier. Similarly for 'around'. In this case, the movement must be described in different terms. We are led to say that the directional part describes a certain kind of path in the vicinity of the landmark.

9. Conclusion. The present paper has given a more fine grained picture of how spatial meanings are built up from the basic meanings of words. In particular, the addition of the coordinatiser has led to considerable change in the way this process is seen. For we must note that when people issue or process spatial expressions there is no coordinate frame to begin with, or at least none that is readily available. Hence, in many cases the first step must be to define an origo and coordinate axes. The choice of axes turns out to be subject to many factors, as research has shown, and provides a fascinating topic in itself. Once they are set, the meaning of axis directed Ps is actually in the point case a single formula expressing a correlation between aberration from the axis and distance from the landmark. This formula is called a *template*. It takes values in $[0, 1]$ and therefore allows to classify change in location using a change in value. This feeds the final element of the locative expression, the one that expresses movement with respect to the landmark.

The notion of aspect seems to be useful also for other areas of semantics. We found the need to recode time points. The mechanics seems to be analogous, but a rigorous formulation still needs to be worked out. This must be left to another occasion, though.

Appendix A. Mathematical Preliminaries. I start with the following geometrical structure. $\mathbb{E}$ is a three-dimensional Euclidean metric space. This means the following. $\mathbb{E}$ consists of a set P or points, a set $G \subseteq \wp(P)$ of lines, and a set $H \subseteq \wp(P)$ of planes, and a function $d : P \times P \to \mathbb{R}_+$, the distance function. d must be invariant under translations and rotations. Points are henceforth denoted by a dot, like this: $\underset{\cdot}{x}$. The postulates are the typical postulates for three dimensional geometry: for any two different points $\underset{\cdot}{x}$ and $\underset{\cdot}{z}$ there is exactly one line (denoted by $\underset{\cdot}{x}\underset{\cdot}{z}$) that contains them, for any three points $\underset{\cdot}{w}$, $\underset{\cdot}{x}$ and $\underset{\cdot}{z}$ not on a line there is exactly one plane—denoted by $\underset{\cdot}{w}\underset{\cdot}{x}\underset{\cdot}{z}$—that contains them, and so on.

Before I continue, I need to briefly discuss the status of these primitives. It has been claimed repeatedly (see Gambarotto and Muller (2003) and references therein) that the Euclidean geometry is not basic from a cognitive point of view; and that it should be replaced by an approach based on regions. I am sympathetic to these claims; however, as Howard (2004) explains, vision is organised into several interacting layers of different degrees of abstraction, and that shape recognition is only one such layer. Given that we are perfectly able to recognise points within a region, I am not sure it is profitable to abandon the Euclidean space altogether. Ultimately, we can do something similar to what has been done in research on time: there is a duality between an ontology based on time points and an ontology based on intervals. One can be defined from the other. I should point out here one major difference between my terminology and the one employed in mereotopology. A *region* is for me not an arbitrary open subset of the space but rather a path-connected set (like intervals on the real line). This is because I maintain that individuation of objects crucially employs a notion of path-connectedness.

Let me now continue and explain how we derive the necessary geometric concepts in $\mathbb{E}$. Angles are measured by the length of the corresponding segments on the unit circle. Distance moreover introduces a topology via the open balls. The open ball of radius δ around $\underset{\cdot}{x}$ is the set

$$K_\delta(\underset{\cdot}{x}) := \{\underset{\cdot}{z} : d(\underset{\cdot}{x}, \underset{\cdot}{z}) < \delta\} \tag{42}$$

A set is *open* if and only if it is a union of finite intersections of unit balls. The topologies on the lines and planes are introduced in such a way as to make the map $(\underset{\cdot}{x}, \underset{\cdot}{z}) \mapsto \underset{\cdot}{x}\underset{\cdot}{z}$ (from pairs of different points to lines) and the intersection $(\ell_1, \ell_2) \mapsto \ell_1 \cap \ell_2$ (from pairs of nonparallel lines to points) continuous with respect to the topologies.

Given this, we can introduce the *half line* $\overset{\mapsto}{\underset{\cdot}{x}\underset{\cdot}{z}}$ as the following set of points:

① $\overset{\mapsto}{\underset{\cdot}{x}\underset{\cdot}{z}} \subset \underset{\cdot}{x}\underset{\cdot}{z}$; and $\underset{\cdot}{x}, \underset{\cdot}{z} \in \overset{\mapsto}{\underset{\cdot}{x}\underset{\cdot}{z}}$;
② for every δ there is exactly one point in $\overset{\mapsto}{\underset{\cdot}{x}\underset{\cdot}{z}}$ that has distance δ from $\underset{\cdot}{x}$;
③ if $\underset{\cdot}{z}$ and $\underset{\cdot}{w}$ are such that $d(\underset{\cdot}{x}, \underset{\cdot}{z}) > d(\underset{\cdot}{x}, \underset{\cdot}{w})$, then

$$d(\underset{\cdot}{x}, \underset{\cdot}{z}) = d(\underset{\cdot}{x}, \underset{\cdot}{w}) + d(\underset{\cdot}{w}, \underset{\cdot}{z})$$

Having half lines, we can define the line segments by

$$\overline{\underset{\cdot}{\mathrm{x}}\underset{\cdot}{\mathrm{z}}} := \overset{\mapsto}{\underset{\cdot}{\mathrm{x}}\underset{\cdot}{\mathrm{z}}} \cap \overset{\mapsto}{\underset{\cdot}{\mathrm{z}}\underset{\cdot}{\mathrm{x}}} \tag{43}$$

Given two points x̣ and ẓ, the ordered pair (x̣, ẓ) defines a *vector* in the following way. We say (x̣, ẓ) $\sim$ (ỵ, ẉ) if and only if (a) the lines x̣ẓ and ỵẉ are parallel, and (b) the lines x̣ỵ and ẓẉ are parallel. In this case, the four points form a parallelogram and opposing sides (the line segments) are of equal length. A *vector* is an equivalence class of $\sim$. Vectors are denoted by arrows, like this: $\overrightarrow{\underset{\cdot}{\mathrm{x}}\underset{\cdot}{\mathrm{z}}}$ (for the vector defined by the pair (x̣, ẓ)). Given a point x̣ and a vector $\vec{v}$, there is a unique point ẓ such that (x̣, ẓ) $\in \vec{v}$.

Having introduced vectors, we can also introduce the scalar product. To start, the *norm* of a vector $\overrightarrow{\underset{\cdot}{\mathrm{x}}\underset{\cdot}{\mathrm{z}}}$ is simply d(x̣, ẓ). This is simply the *length* of any of the line segments that are defined by pairs in the vector. Given two vectors $\vec{x}$ and $\vec{y}$, we pick three points ạ, ḅ and c̣ such that (ạ, ḅ) $\in \vec{x}$ and (ạ, c̣) $\in \vec{y}$. Let ḍ be the point obtained by drawing a line through ḅ perpendicular to ạc̣. (This is the ‘shadow projection’.) Then define

$$\langle \vec{x}, \vec{y} \rangle := \frac{d(\underset{\cdot}{\mathrm{a}}, \underset{\cdot}{\mathrm{d}})}{d(\underset{\cdot}{\mathrm{a}}, \underset{\cdot}{\mathrm{c}})} \tag{44}$$

This number is independent of the representatives, and it also does not change when the roles of $\vec{x}$ and $\vec{y}$ are interchanged. The form $\langle -, - \rangle$ is symmetric, and linear in both components. Two vectors are *orthogonal* if $\langle \vec{x}, \vec{y} \rangle = 0$. That $\mathbb{E}$ is three dimensional means that every sequence of pairwise orthogonal vectors has length 3. Now fix any such sequence, $\langle \vec{e}_1, \vec{e}_2, \vec{e}_3 \rangle$. We require that these vectors are of length 1. Given a vector $\vec{x}$, we can now assign coordinates, x_1, x_2 and x_3, as follows

$$x_1 := \langle \vec{x}, \vec{e}_1 \rangle, x_2 := \langle \vec{x}, \vec{e}_2 \rangle, x_3 := \langle \vec{x}, \vec{e}_3 \rangle \tag{45}$$

It is important to stress two things: these definitions apply to vectors, not to points; and secondly, the coordinates are dependent on the coordinate vectors. Both facts need close attention. The resulting triple is called a *coordinate vector*.

First, three vectors alone do not allow to issue coordinates for points. In order to do this, we must choose an *origin*. Given an origin ọ, a given point x̣ is now assigned the (unique) vector $\overrightarrow{\underset{\cdot}{\mathrm{o}}\underset{\cdot}{\mathrm{x}}}$, and given the coordinate vectors this latter vector is finally assigned a set of three coordinates. Coordinates depend on the origin as follows. If we choose a different origin ṇ, the new vector is obtained by adding (ṇ, ọ) to (ọ, x̣), a vector that is independent of x̣. Coordinates also depend on the chosen vectors. First of all, the coordinate vectors themselves also have coordinates. These are

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{46}$$

The coordinate vectors are written vertically. The coordinate vectors are obtained by

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \langle \vec{e}_1, \vec{x} \rangle \\ \langle \vec{e}_2, \vec{x} \rangle \\ \langle \vec{e}_3, \vec{x} \rangle \end{pmatrix} \tag{47}$$

Let me briefly mention the following fact about coordinates. We may arrange any three vectors $\vec{x}$, $\vec{y}$ and $\vec{z}$ into a matrix, by computing their coordinate triples and arranging them in a sequence. If we do this with the unit vectors we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{48}$$

Now take three vectors $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$. These three vectors have three coordinates each, called a_{ij} for vector i, and so the sequence of these three vectors form a matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \tag{49}$$

Now, the following evidently holds for any coordinate vector x (on condition that A is invertible):

$$x = AA^{-1}x \tag{50}$$

This is standardly seen as follows: the matrix A gives us the new coordinate vectors, expressed in the old ones (so that we have numbers). Therefore, $A^{-1}x$ gives us the vector *x in the new coordinate system A*. One says that coordinates are *contravariant*, since you find them by applying the inverse of A.

Generally, any invertible matrix can be used to recoordinatise the space. However, for our purposes we can restrict attention to the following. A matrix A is said to be *special orthogonal* if $A^{-1} = A^T$, and A has determinant 1. The group of special orthogonal matrices of $\mathbb{E}$ is denoted by SO($\mathbb{E}$). The matrices describe either coordinate sets or transformations, just as triples stand for points and vectors. We shall see how the ambiguity can be resolved. A special orthogonal matrix describes a rotation in space. Thus, new coordinate vectors A forming a special orthogonal matrix can be obtained by rotating the original vectors.

Definition 5. *A* ***coordinate system*** *is a quadruple $(\underset{\cdot}{o}, \underset{\cdot}{e}_1, \underset{\cdot}{e}_2, \underset{\cdot}{e}_3)$ such that*

① *the vectors $\overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_i}$ have length 1;*
② *$\overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_i} \perp \overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_j}$ if and only if $i \neq j$;*
③ *the system $\overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_1}, \overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_2}, \overrightarrow{\underset{\cdot}{o}\underset{\cdot}{e}_3}$ is right handed.*

There are several definitions of right handedness. One is that the determinant must be positive (in whatever coordinates, since it is independent of them); the other is that we can align them with thumb, index finger and middle finger of the right hand. Given a coordinate system C, we write $\underset{\cdot}{x}_C$ for the triple of coordinates that identify $\underset{\cdot}{x}$ in the coordinate frame C. Given a triple $\vec{x}$ of numbers, let $\vec{x}^C$ denote the point that it defines in C.

A *path* is a continuous function $\pi : I \to P$, where I is an interval. A *region* is a path-connected subset of $\mathbb{R}^3$. Here, a set S is *path-connected* if for any $x, y \in S$ there is a path $\pi : [0,1] \to S$ such that $\pi(0) = x$ and $\pi(1) = y$. Reg denotes the set of regions. Given a distance function for the space P, we define the distance for regions R and S by

$$d(R, S) := \inf\{d(\underset{\cdot}{x}, \underset{\cdot}{z}) : \underset{\cdot}{x} \in R, \underset{\cdot}{z} \in S\} \tag{51}$$

The volume of a region is defined as follows. Let C be a coordinate frame, and $R_C := \{\underset{\cdot}{x}_C : \underset{\cdot}{x} \in R\}$.

$$V(R) := \int_{R_C} dxdydz \tag{52}$$

Notice that this is independent of C (for this we require that the distance function is invariant under translations and rotations). The *volumetric centre* is now defined by

$$c(R) := \left(\frac{1}{V(R)} \int_{R_C} d\vec{x}\right)^C \tag{53}$$

Notice that we have to first move to coordinates to do our calculations, and then return to the points. The definition is independent of the actual coordinates chosen.

Appendix B. Types. The semantics below makes use of type theory. Here are my conventions. e is the type of objects; p is the type of space points, v the type of vectors, ξ is the type of triples of reals; r the type of regions; τ is the type of time points, i is the type of time intervals. p ranges over P, our space, while v ranges over the set $P \times P/\sim$ and ρ over the set of path connected subsets of $\mathbb{R}^3$. Finally, d is the type of truth degrees and t the type of truth values.

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Notes

[1]I wish to thank Marcus Smith and Brook Lillehaugen and two anonymous referees for insightful discussions. Thanks also to the audience of "Syntax and Semantics of Spatial P" for their comments. Errors and omissions are my own responsibility.

[2]In this paper, I shall blur the distinction between pre-, post- and adpositions, as the exact nature of the form does not matter. Also, I am not interested in whether what is a preposition in one language is a case in another or something else.

[3]The reader may be aware that there is actually no such thing as an absolute coordinate frame, so that even in physics it is pointless to ask which object is moving and which one is not. In fact, constant motion cannot be differentiated from rest; only the change of velocity can. Our earthly frame of coordinates, on which 'north' and 'west' and so on are defined looks perfectly absolute, but as the earth is revolving around the sun the 'absoluteness' of this frame only holds with respect to points on earth.

[4]I ignore here some fascinating complications. Some local expressions can be used only within a certain range. For example, body parts may very often only be used if distances are small. This is true even for English: you normally don't say that Hamburg is to your left when standing somewhere in Berlin facing north. The islands of Polynesia have a coordinate system based on a seaward-landward axis as 'front'. Such systems tend to be local, they are confined to the (more or less) immediate surroundings of the island. For a specific example of dependence on distance see (Florey and Kelly, 2002).

[5]It was Ben Keil who brought it to my attention that the Oceanic system is analogous to our north-south system.

[6]Thanks to Marcus Smith for bringing this to my attention.

[7]More precisely, the way I'd like to think about it is to align 'front' with the x-axis, 'left' with the y-axis and 'up' with the z-axis. But this is a minor issue. Also notice that a language can use several coordinatisers, and Ps select for one or the other. The details of this are straightforward to implement.

[8]This is just a suggestion.

[9]It may depend on the number of oppositions the language provides, a point that I shall suppress here for simplicity.

[10]I refrain from discussing the possibility of open or half open intervals. They are only relevant in connection with aspect. Also, since we are now assuming that we have a coordinate system, we treat space points as triples of reals. In general, paths are functions into the point set.

[11]See (Kracht, 2002) for a discussion as to why objects and locations are conceptually distinct in language.

[12]In the discrete case the function becomes a subset of $\mathbb{R}^3$, which must be a region. This conflicts the claim in (Kracht, 2002) that we get a set of regions. However, to be accurate, we must lift the goodness of fit function to regions!