BLACKBURN, DE RIJKE AND VENEMA: *MODAL LOGIC*. A REVIEW.

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Modal logic is a very broad field, which has demonstrated its usefulness mostly through applications in philosophy, linguistics, and computer science, and occasionally also to mathematics. Yet, the variety of textbooks available for the would–be student is not as rich as in other disciplines of logic. The choice is between Goldblatt [2], Hughes and Cresswell [3] and Chagrov and Zakharyaschev [1]. Now Patrick Blackburn, Maarten de Rijke and Yde Venema, who at one stage have worked together in Amsterdam, have added another one to this list. It is a book of seven chapters and more than 500 pages, a book that is highly readable and informative but not without problematic aspects.

It starts with a chapter introducing the basics of modal logic: normal modal logics, consequence relations, frames, models, and general frames. The authors introduce both the plain modal language based on a single operator as well as abstract similarity types, allowing for any number of operators of any arity. Though the book clearly focusses on the basic case (one unary operator), many examples are provided showing the usefulness of the more abstract approach (tense logic, propositional dynamic logic, since and until logic, and multi-dimensional logics). Chapter 2 brings the model theory of modal logic: simulations, bisimulations, the Standard Translation, the finite model property, modal saturation, ultrafilter extensions, and definability. Chapter 3 continues on that track for frames: it discusses definability, the Goldblatt–Thomason theorem, and then turns to Sahlqvist's theorem, providing a characterization both of the modal axioms as well as the first order formulae defined by it. In Chapter 4, the book turns to completeness. We learn about canonical models, completeness via Sahlqvist's theorem, and via some kind of tableau calculus. Positive as well as limitative results are given; the extensions of S4.3 are classified according to Bull and Fine, the finite model property of PDL is proved, and we learn about noncanonical logics and about incomplete logics.

Chapter 5 provides the key concepts and results of algebraic logic: algebras, general frames, persistence, duality, and the Jónsson–Tarski Theorem. After that follows a chapter on complexity of modal logics. After defining the basic notions, some methods for proving decidability and undecidability are given. Then follows a discussion of modal logics that are in NP, in PSPACE and in EXPTIME. Chapter 7 closes the book. It contains a selection of diverse topics such as since and until logic, hybrid logics, the guarded fragment and a Lindström type characterization of modal logic. There are many exercises, there are chapter overviews and various 'roadmaps', and three so–called *toolkits*, one on logic, one on algebra, and one on computability. Each chapter also contains a fairly extensive historical and bibliographical overview. Finally, there is a guide to the literature and even a web site for the book.

The authors have decided to cover the material that they have worked on themselves, giving the book a bias towards Dutch style modal logic. Here I think they have greatly succeeded. The exposition of correspondence theory is lucid and presents material that is well worth knowing. Also, the connections with first–order logic are very much up to date. The proofs on preservation classes use saturated models, which greatly simplifies the argument. The connections with computer science are worked out well, and there is a lot of relevant material to be found in the book. On the other hand, there are a number of topics that the book does not cover, such as logical deduction, proof methods, interpolation, and intuitionistic logic (to remain with the geographical terminology: there is little Eastern style modal logic). While the authors acknowledge this in part, I am not convinced by the claim that the book would have grown out of proportion if these topics would have been included. For there is other material included that is not of immediate interest for the beginner. The omission of deductive calculi and the proof theory of modal logic severely restricts the usefulness of the book. Even the most basic things are missing here or else presented in a very informal manner. The reader does not learn what a rule is let alone an admissible rule, and s/he does not learn about the Deduction Theorem except in an exercise. This is surprising insofar as the Gabbay–style axiomatization of irreflexivity is discussed, which uses special rules in place of axioms. Moreover, interpolation and related properties are too important to be left unadressed. Notice that in [4] the reviewer has proposed a method for showing decidability of modal logics that is quite easy to apply and allows to deduce interpolation for the basic logics in one line. Thus, it would have cost the authors very little to include these topics and derive valuable results

about them. This leaves the reader with a somewhat unbalanced view of modal logics. Given this, the novice is well advised to consult Chagrov and Zakharyaschev [1] in addition to the present book, if s/he wants to get a complete picture of the field. The survey of the literature that this book contains also suffers from this defect. While the authors praise books of questionable quality, their comments on [1] are relatively speaking unfavourable. No mention is made of books that discuss deductive calculi (for example, Rybakov [5] or Wójcicki [6]).

In sum, the book is recommended as an introduction to the more applied modal logic, especially Dutch–style modal logic. However, the student who is more interested in the theory of modal logic will find the book too uninformative given that its title suggests that this is a book about modal logic as a whole.

References

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[3] George E. Hughes and Max J. Cresswell. *A New Introduction to Modal Logic*. Routledge, London, 1996.

[4] Marcus Kracht. *Tools and Techniques in Modal Logic*. Number 142 in Studies in Logic. Elsevier, Amsterdam, 1999.

[5] Vladimir V. Rybakov. *Admissibility of Inference Rules*. Number 136 in Studies in Logic. Elsevier, Amsterdam, 1997.

[6] Ryszard Wójcicki. *Theory of Logical Calculi*. Kluwer, Dordrecht, 1988.

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