

# Notes on the Complexity of Complex Heads in a Minimalist Grammar\*

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## Abstract

The type of a *minimalist grammar (MG)* introduced in Stabler 1997 provides a simple algebraic formalization of the perspectives as they arise from Chomsky 1995b within the linguistic framework of transformational grammar. As known (cf. Michaelis 2001a, 2001b, Harkema 2001), this MG-type defines the same class of derivable string languages as, e.g., *linear context-free (string) rewriting systems* (Vijay-Shanker et al. 1987, Weir 1988). In this paper we show that, in terms of weak equivalence, the subclass of MGs which allow (*overt*) *head movement* but no *phrasal movement* in the sense of Stabler 1997, constitutes a proper subclass of *linear indexed grammars (LIGs)*, and thus *tree adjoining grammars*. We also examine the “inner hierarchic complexity” of this embedding in some more detail by looking at the subclasses canonically resulting from the distinction between left and right adjunction of the moved head to the attracting one. /\* Correction --> \*/ Furthermore, we show that adding the possibility of phrasal movement by allowing at most one “indistinguishable” licensee to trigger such movement already increases the weak generative capacity of at least two of the considered subclasses, while this is not true for the particular subclass of MGs which do not employ any movement at all. The latter define the same class of derivable string languages as *context free grammars*. /\* <-- Correction \*/ On the other hand however, MGs which do not employ head movement but whose licensee set consists of at most two elements, are shown to derive, i.a., languages not derivable by any LIG. In this sense our results contribute to shedding some light on the complexity as it arises from the interplay of two different structural transformation types whose common existence is often argued to be linguistically motivated.

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## 1 Introduction

The type of a *minimalist grammar (MG)* introduced in Stabler 1997 provides a simple algebraic formalization of the perspectives as they arise from Chomsky 1995b within the linguistic framework of a principles and parameter–approach to transformational grammar. As known (cf. Michaelis 2001a, 2001b, Harkema 2001), this MG–type constitutes a *mildly context-sensitive formalism* in the sense that it defines the same class of derivable string languages as *linear context-free (string) rewriting systems (LCFRSs)* (Vijay-Shanker et al. 1987, Weir 1988).<sup>1</sup> In particular, the MG–

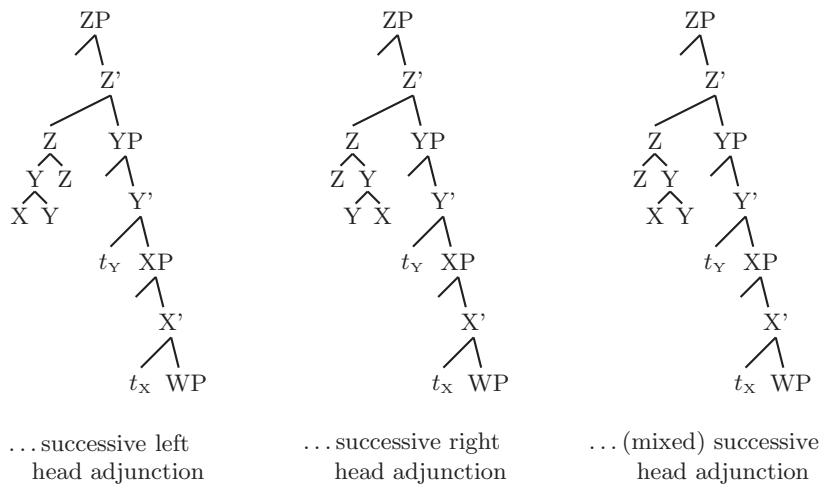


Figure 1: Complex head “Z”<sup>2</sup> resulting from ...

definition permits a type of (*overt*) *head movement* which rather directly reflects the derivation mode of (*successive*) *head(-to-head) adjunction*—which, in the minimalist approach, takes place due to the necessity of feature checking—(successively) creating more complex heads (cf. Figure 1).<sup>3</sup> Nevertheless, there is a further notable property of MGs which—in connection with Michaelis 2001a—follows from Harkema 2001 as well as Michaelis 2001b: each MG can be transformed into a weakly equivalent MG that neither employs head movement nor *covert (phrasal) movement* as allowed by the general MG–definition.<sup>4</sup> In fact it is this MG–subtype which, e.g., is covered in terms of the succinct MG–reformulation in Stabler & Keenan 2000 (reducing MGs to their “bare essentials”), and which the MG–recognizer in Harkema 2000 (working in polynomial time depending on the length of the input string) is defined for. Moreover, the “MG–internal” equivalence result can be seen as providing some technical support to more recent linguistic work which, in particular,

<sup>1</sup> Hence, MGs as defined in Stabler 1997 join to a series of weakly equivalent formalism classes among which, beside LCFRSs, there is, e.g., the class of set-local *multicomponent tree adjoining grammars (MCTAGs)* (cf. Weir 1988). For a list of some further of such classes of generating devices, beside MCTAGs, see e.g. Rambow & Satta 1999.

<sup>2</sup> In terms of an X-Bar representation.

<sup>3</sup> Note that a corresponding derivational description is less immediately accessible within a *tree adjoining grammar* or a *linear indexed grammar*.

<sup>4</sup> The only movement possibly used is *overt phrasal movement*.

tries to completely dispense with any type of movement different from overt phrasal movement (e.g. Koopman & Szabolcsi 2000, Mahajan 2000).

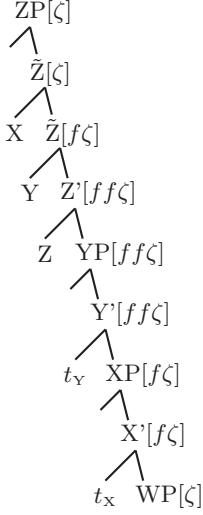


Figure 2: Complex head “Z” resulting from successive left head adjunction as representable in an RLIG.

which allow only right head adjunction derive more languages than CFGs, and MGs allowing right as well as left head adjunction seem to provide a further proper extension. Furthermore, adding the possibility of phrasal movement by allowing the MG’s licensee set to consist of at most one

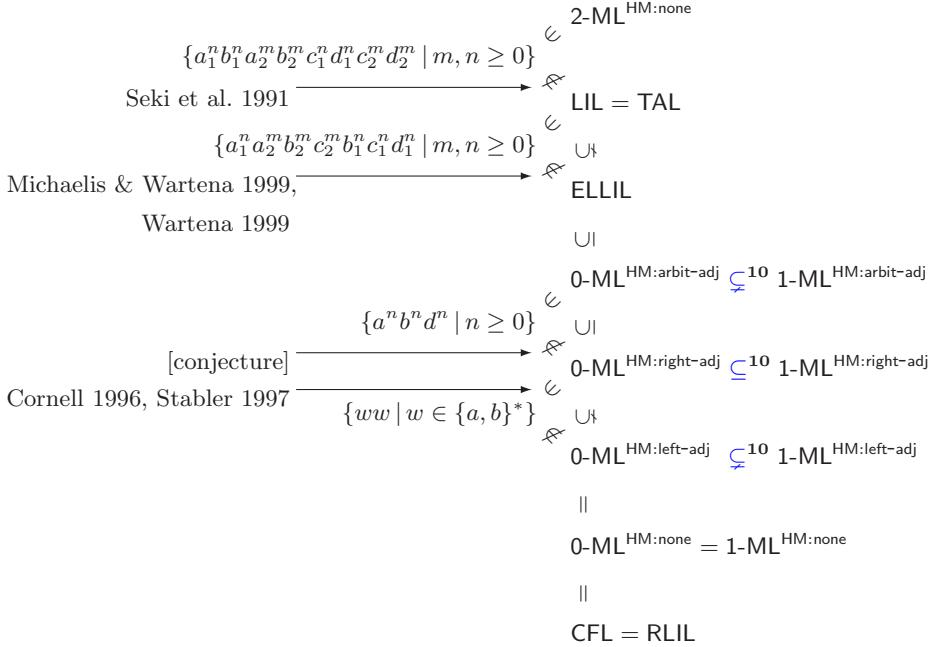
Many linguists working within the transformational tradition, however, believe head movement to be indispensable for an adequate explanation of natural language syntax.<sup>5</sup> How the kind of head movement originally allowed in an MG can be (re)integrated into the succinct MG–definition is shown in Stabler 2001. As indicated there, the recognition complexity w.r.t. such an MG and an input string increases—adapting the methods of Harkema 2000—at most as much as in the case when adding two new distinct *licensees*, i.e. two new distinct features potentially triggering phrasal movement, to the grammar.

Concentrating on questions concerning the increase of generative complexity, we show in this paper that MGs which allow head movement but no phrasal movement in the sense of Stabler 1997, in terms of derivable string languages, constitute a proper subclass of *linear indexed grammars (LIGs)*, and thus *tree adjoining grammars (TAGs)*. This is done by embedding MGs weakly equivalently into *extended left LIGs (ELLIGs)* in the sense of Michaelis & Wartena 1999.<sup>6</sup> Examining the “inner hierarchic complexity” of this embedding in some more detail, MGs which allow only left head adjunction can be shown to define the same class of derivable languages as RLIGs,<sup>7</sup> and thus *context free grammars (CFGs)* (cf. Michaelis and Wartena 1997, 1999). MGs

<sup>5</sup> Even current accounts which argue in favour of overt vs. covert movement do not completely exclude overt head movement (e.g. Kayne 1998, 1999).

<sup>6</sup> In Michaelis & Wartena 1999, ELLIGs were defined as the formal counterpart of *extended right LIGs (ERLIGs)*. An ELLIG (respectively, ERLIG) is an LIG,  $G$ , such that for each nonterminating production  $r$ , the distinguished symbol on the righthand side (rhs) is the leftmost (respectively, rightmost) nonterminal, i.e.,  $r$  is of the form  $A[\zeta \dots] \rightarrow wB[\eta \dots] \alpha$  (respectively,  $r = A[\zeta \dots] \rightarrow \alpha B[\eta \dots] w$ ), where  $w$  is a string of terminals. Thus, applying such an  $r$  to a corresponding object  $A[\zeta \theta]$ , the stack associated with the nonterminal  $A$  is passed on to the leftmost (respectively, rightmost) nonterminal child replacing  $\zeta$ , the upper part of the stack, by  $\eta$ . If, in addition, for each such nonterminating rule  $r$ , no terminal symbol appears to the left (respectively, right) of the distinguished nonterminal symbol of the rhs, i.e., if  $w$  is always the empty string, then  $G$  is simply referred to as an *LLIG* (respectively, *RLIG*).

<sup>7</sup> A corresponding weakly equivalent RLIG can be defined such that complex heads, created by successive head adjunction, are represented much in line with the proposal given in Michaelis & Wartena 1997 independently of the MG–definition (cf. Fig. 2), and therefore, in a way formally comparable to the TAG–analysis of verb raising constructions in West Germanic languages proposed in Kroch & Santorini 1991.

Figure 3: Schematic overview of our results.<sup>11,12</sup>

feature<sup>8</sup> increases the weak generative capacity of at least two of the considered MG–subclasses, while this is not true for the particular subclass of MGs which do not employ any movement at all. The latter as well as MGs which do not employ head movement but allow phrasal movement by means of at most one licensee, define the same class of derivable string languages as CFGs. On the other hand however, MGs which do not employ head movement but whose licensee set consists of at most two elements, are shown to derive, i.a., languages not derivable by any LIG.<sup>9</sup>

The presented results are of interest in at least two respects: first, they contribute in a more general sense to one of the central issues mathematical approaches to linguistics are concerned with, namely, how much strong generative capacity can be squeezed out of a formalism without

<sup>8</sup> Thus, at most one “indistinguishable” type of phrasal movement is available.

<sup>9</sup> A schematic overview is given in Figure 3.

<sup>10</sup> /\* Correction \*/

<sup>11</sup> Here, for  $n \geq 0$  and  $x \in \{\text{none}, \text{left-adj}, \text{right-adj}, \text{arbit-adj}\}$ ,  $n\text{-ML}^{\text{HM}:x}$  denotes the class of all languages derivable by any MG whose licensee set consists of at most  $n$  elements, and which permits only head(–to–head) adjunction of the type indicated by  $x$ .

<sup>12</sup> Michaelis and Wartena (1999) actually show that the language  $\{a_1^n a_3^l b_3^l c_3^l b_1^n c_1^n a_2^m b_2^m c_2^m d_1^n \mid l, m, n \geq 0\}$ , although derivable by some LIG, is not derivable by any—what is called there—*extended unidirectional indexed grammar (EUIG)*. Wartena (1999)—adopting the proof methods used by in Michaelis & Wartena 1999—actually shows that the language  $\{a_1^n b_1^n c_1^n a_2^m b_2^m c_2^m d_1^n \mid m, n \geq 0\}$ , derivable by some LIG, is not derivable by any ERLIG. For reasons of symmetry however, it becomes immediately clear from the corresponding proof details that the language  $\{a_1^n a_2^m b_2^m c_2^m b_1^n c_1^n d_1^n \mid m, n \geq 0\}$ , although derivable by some LIG, is not derivable by any ELLIG.

increasing the weak generative power.<sup>13</sup> Second, since the presented results provide a first narrow step towards an answer to the question of how the specific types of head movement and phrasal movement as defined in MGs are related to each other in terms of generative capacity, they may not only be a promising starting point, when seeking for a lower upper bound on the parsing complexity of MGs, but also shed some light on the structural complexity as it arises from the interplay of two different structural transformation types whose common existence is often argued to be linguistically motivated.

For illustrative purposes we demonstrate how MGs allowing head movement but no phrasal movement can be weakly equivalently embedded into a subclass of TAGs, instead of LIGs, which in its turn is weakly equivalent to ELLIGs (Section 3). Largely skipping formal details afterwards, we subsequently emphasize the crucial points concerning the hierarchy of the corresponding MG–subclasses resulting from the different types of head movement as available in the MG–formalism (Section 3.1–3.4). Then, we turn to simple phrasal movement as allowed by the MG–definition (Section 4) and, finally, present an example of an MG which does not employ any head movement, but phrasal movement “slightly beyond” the simple type, thereby deriving a language not derivable by any LIG (Section 5). But first, since the reader might be less familiar with MGs, we briefly introduce them in their aspects relevant here.

## 2 Minimalist Grammars

An MG is a formal device which specifies a countable set of *expressions* (over  $\mathcal{S} \cup \mathcal{P} \cup \mathcal{I}$ ),<sup>14</sup> i.e., a countable set of finite, binary (ordered) trees each equipped with a leaf–labeling function assigning a string from  $\mathcal{S}^* \mathcal{P}^* \mathcal{I}^*$  to each leaf, and with an additional binary relation, the asymmetric relation of (*immediate*) *projection*, defined on the set of pairs of siblings (cf. Figure 4).

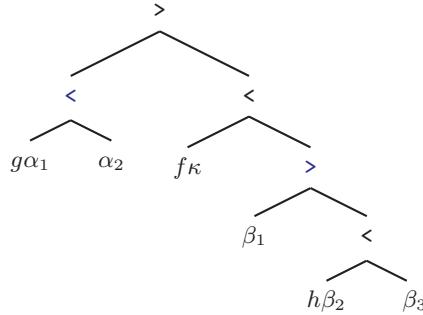


Figure 4: A typical (minimalist) expression.<sup>15</sup>

<sup>13</sup> See e.g. Joshi 2000 and references therein for a more recent discussion.

<sup>14</sup>  $\mathcal{S}$ ,  $\mathcal{P}$  and  $\mathcal{I}$  are assumed to be pairwise disjoint sets, namely, a set of *syntactic*, *phonetic* and *interpretable features*, respectively.  $\mathcal{S}$  is partitioned into *basic categories*, *selectors*, *licensees*, and *licensors*. There is at least the basic category  $\mathbf{c}$ .

<sup>15</sup> Here, “<” (respectively, “>”) as “label” of a non–leaf node means “my left (respectively, right) child projects over my right (respectively, left) child.”

A *maximal projection* within such an expression  $\tau$  is a subtree of  $\tau$  which is either identical to  $\tau$ , or its root is projected over by its root's sibling. The *specifiers*, the *complement* and the *head* of (a maximal projection in)  $\tau$  are determined in the canonical way by means of the projection relation (cf. Figure 5).  $\tau$  is *complete* if its head-label is in  $\{\mathbf{c}\}\mathcal{P}^*\mathcal{I}^*$ , and each other leaf-label in  $\mathcal{P}^*\mathcal{I}^*$ . The *phonetic yield* of  $\tau$  is the string which results from concatenating the leaf-labels in “left-to-right-manner” ignoring all instances of non-phonetic features.

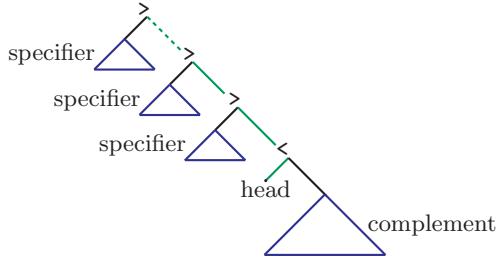


Figure 5: The typical (minimalist) expression structure.

The base of an MG,  $G$ , is formed by a *lexicon* (a finite set of simple expressions, i.e. single node trees in the above sense, also called *heads*) and two structure building functions: *merge* (combining two trees) and *move* (transforming a single tree). Both functions build structure by canceling two particular matching instances of syntactic features within the leaf-labels of the trees to which they are applied. The closure of the lexicon under these two functions is the set of trees specified by  $G$ . The (*string*) *language* derivable by  $G$  is a particular subset of  $\mathcal{P}^*$ , namely, the set of the phonetic yields of the complete expressions within this closure.

The function *merge* is applicable to  $\langle v, \phi \rangle$ , a pair of expressions, if  $\phi$ 's head-label starts with an instance of some basic category  $x$ , and  $v$ 's head-label with an instance of  $\bar{x}$ , the corresponding *weak selector* of  $x$ . Depending on whether  $v$  is simple or not,  $\phi$  is selected as the complement or the highest specifier, respectively. Within the resulting tree,  $\text{merge}(v, \phi)$ , the corresponding instances of  $\bar{x}$  and  $x$  are cancelled (cf. Figure 6). In case  $v$  is a head, its label may likewise start

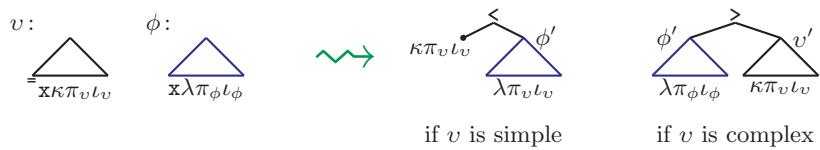
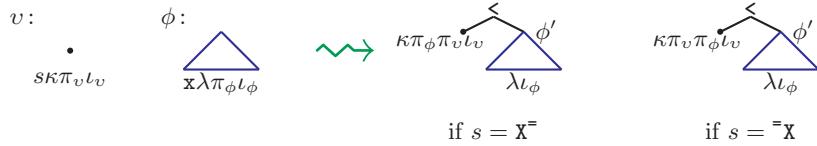


Figure 6:  $\text{merge}(v, \phi)$  — weak selection.

with an instance of a corresponding *strong selector* of  $x$ ,  $\bar{x}$  or  $X^=$ , both additionally triggering (*overt*) *head movement*, i.e.,  $\text{merge}(v, \phi)$  is defined as before, but in addition  $\pi_\phi$ , the string of phonetic head-features of the selected  $\phi$ , is incorporated into the label of the selecting head  $v$ , either immediately to the right (triggered by  $\bar{x}$ ) or immediately to the left (triggered by  $X^=$ ) of  $\pi_v$ , the string of phonetic features within  $v$ 's (unique) label (cf. Figure 7).<sup>16</sup>

<sup>16</sup> In the minimalist approach suggested in Chomsky 1995b the merge-operator applies freely, and head

Figure 7:  $\text{merge}(v, \phi)$  — strong selection.

The function *move* is applicable to an expression  $v$ , if there is exactly one maximal projection  $\phi$  in  $v$  whose head–label starts with in instance of some licensee  $-x$  such that  $v$ 's head–label starts with an instance of  $+x$  or  $+x$ , the corresponding *strong* and *weak* licensor of  $-x$ , triggering *overt* and *covert phrasal movement*, respectively.<sup>17</sup> If  $v$ 's head–label starts with an instance of the corresponding strong licensor then, within the resulting tree  $\text{move}(v)$ ,  $\phi$  is moved into the new created, highest specifier position, while the triggering instances of  $+x$  and  $-x$  are cancelled, and the “original” position of  $\phi$ 's root becomes a single node labeled with the empty string (cf. Figure 8). If  $v$ 's head–label starts with a weak licensor then, within the resulting tree  $\text{move}(v)$ ,

Figure 8:  $\text{move}(v)$  — overt phrasal movement.

the triggering instance of  $+x$  is cancelled, while a copy of  $\phi$  in which the triggering instance of  $-x$  as well as all instances of phonetic features are cancelled, is moved into the new created, highest specifier position, and while another copy of  $\phi$  in which all instances of non–phonetic features are cancelled is “left behind.”<sup>18</sup>

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movement is a separate step following a corresponding application of this operator. As noted by Chomsky (1995a, p. 327), in a strong sense this can be seen as a violation of the “extension condition” on structure building functions. Stabler (1998, p. 78, fn. 5) argues that the implementation of head movement within MGs not only avoids such a violation, but “it [also] explains the coincidence of the selection and head movement configurations.” Note also that the implementation of head movement is in accordance with the *head movement constraint*, demanding that a moving head can never pass over the closest c-commanding head. To put it differently, whenever we are concerned with a case of successive head movement, i.e. recursive adjunction of a (complex) head to a higher head, it obeys *strict cyclicity*. The way in which MGs reflect the “usual” linguistic notion of head adjunction arising from head movement is made explicit in Stabler 1998.

<sup>17</sup> The uniqueness of  $\phi$  provides us with a strict version of the *shortest move constraint*.

<sup>18</sup> For more details on the definition of *merge* and *move* see Stabler 1997. Particular examples of an MG are given below.

### 3 MG–Head Movement in Terms of TAGs

Let  $G$  be an MG which allows head movement but no phrasal movement.<sup>19</sup> A nonterminal in our weakly equivalent TAG,  $G'$ , is either the start symbol,  $S$ , or a pair  $\langle y, t \rangle$  with  $y$  being a basic category from  $G$ , and with  $t \in \{\text{weak}, \text{strong}\}$ , where **weak** and **strong** are two new, distinct symbols.

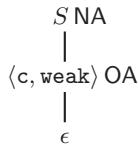


Figure 9: The unique initial tree of  $G'$ .

There is a single initial tree (cf. Figure 9), and for each lexical MG–item  $\alpha$ , there are two elementary auxiliary trees depending on the form of the (unique) label of  $\alpha$ : we generally distinguish the cases  $y\pi\iota$  (cf. Figure 10) and  $s^=x_1 \cdots ^=x_n y\pi\iota$ ,  $s$  being any selector,  $=x_1, \dots, =x_n$  weak selectors for an  $n \geq 0$ ,  $y$  a basic category,  $\pi \in \mathcal{P}^*$ , and  $\iota \in \mathcal{I}^*$ . The latter case divides into three subcases depending on whether  $s$  is of the form  $=x$ ,  $X^=$ , or  $=X$  (cf. Figure 11–13). Hence,  $G'$  only uses auxiliary elementary trees which may be called *extended right auxiliary*, i.e., auxiliary trees in which the foot node is the leftmost nonterminal node on the frontier, and all interior nodes left of the spine are marked for null adjunction.<sup>20,21</sup>



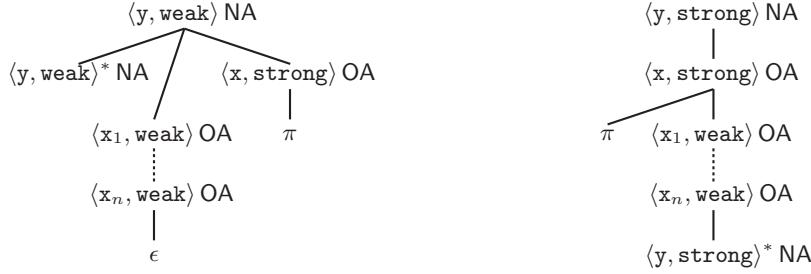
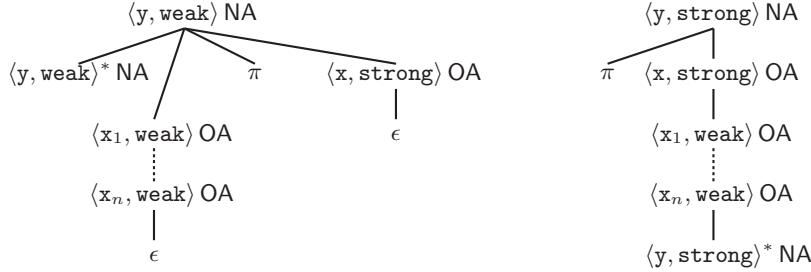
Figure 10: Elementary auxiliary trees of  $G'$  resulting from the lexical MG–item  $y\pi\iota$ .

$G'$  simulates the MG–derivation of an expression  $\tau$  whose head–label starts with a basic category  $y$  by “reversing the top–down order,” i.e., the complement becomes the highest constituent, and the specifiers are successively attached top–down in the sense indicated in Figure 14. Such a derivation, indeed, is simulated by  $G'$  exactly twice in the two outline ways. Vice versa, each TAG–derivable auxiliary tree  $T$  which does not permit (further) adjunction to any of his nodes, corresponds to an MG–derivable expression  $\tau$  whose head–label starts with a basic category  $y$ , in exactly one of the two outlined ways. Thus—ignoring the label of  $T$ ’s foot node— $T$ ’s yield either equals  $\tau$ ’s phonetic yield, or this is true up to the fact that the substring of  $\tau$ ’s phonetic yield contributed by  $\tau$ ’s head,

<sup>19</sup> That is,  $G$  does not employ the function *move* to derive any expression from some instances of the lexical items. Therefore, we may assume that no (label of any) lexical item contains an instance of some licensee or licensor feature.

<sup>20</sup> This TAG–subtype may also be seen as a straightforward extension of a particular subtype of a *tree insertion grammar* (Schabes & Waters 1995).

<sup>21</sup> With the intend of simplifying our presentation,  $G'$  also fits in with the “classical” TAG–definition allowing selective adjunction, but not substitution (see e.g. Vijay–Shanker & Weir 1994).

Figure 11: Elementary auxiliary trees of  $G'$  resulting from the lexical MG-item  $=x=x_1 \dots =x_n y \pi \iota$ .Figure 12: Elementary auxiliary trees of  $G'$  resulting from the lexical MG-item  $X==x_1 \dots =x_n y \pi \iota$ .Figure 13: Elementary auxiliary trees of  $G'$  resulting from the lexical MG-item  $=X=x_1 \dots =x_n y \pi \iota$ .

$yield_{\mathcal{P}}(head_{\tau})$ , is ‘shifted’ to the front within  $T$ ’s yield. If the latter, it is just  $yield_{\mathcal{P}}(head_{\tau})$  which constitutes  $T$ ’s yield left of its spine, and in MG-terms,  $T$  is connected with the expectation that the represented  $\tau$  is inevitable selected strongly in a further derivation step (expressed by the second component of the label of  $T$ ’s root node, and thus foot node). Otherwise,  $T$ ’s yield left of its spine is empty, and the represented  $\tau$  is connected with the demand of being selected weakly in a further derivation step (again coded by means of the second component of the label of  $T$ ’s root/foot node).

### 3.1 Null Head Adjunction

As an immediate consequence of our construction we observe that in case  $G$  does not use any strong selectors (i.e., no head movement takes place deriving an expression belonging to the closure of the lexicon of  $G$ ), only (strictly) right auxiliary trees are effectively used in order to derive a tree in

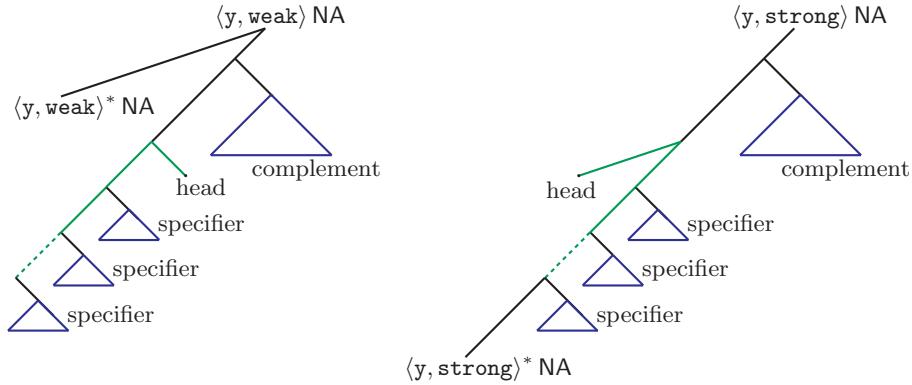


Figure 14: The MG–expression structure simulated by  $G'$ .

the weakly equivalent TAG  $G'$ . Thus, in this case, we are in fact concerned with a particular type of *tree insertion grammar* in the sense of Schabes & Waters 1995 additionally allowing adjunction constraints in the sense of the usual TAG–definition. Since adding the possibility of such constraints to the TIG–type does not increase the weak generative power, and since TIGs are known to constitute a weakly context–free formalism, this yields another proof of the well–known fact that MGs which do neither employ head movement nor phrasal movement only derive context–free languages (CFLs).<sup>22</sup>

### 3.2 Left Head Adjunction

As mentioned in the introduction, it can be shown that MGs which—beside the simple merge–operation—permit only left head adjunction triggered by a corresponding strong selection feature, do only derive CFLs as well. Skipping any formal details here, we just mention that, as far as a complex head of a corresponding MG is concerned, the dependencies between the (phonetic yields of the) single lexical heads incorporated into the (phonetic yield of the) complex head and their respective “traces” are nested. This allows us to use a single stack in order to “correctly redefine” the concept of successive left head adjunction within the weakly equivalent RLIG.<sup>23</sup>

### 3.3 Right Head Adjunction

The crucial difference between successive right head adjunction and successive left head adjunction is constituted by the fact that—within a complex head created by the former derivation mode—the dependencies between the (phonetic yields of the) single lexical heads incorporated into the

<sup>22</sup> Vice versa, it is not hard to verify that each CFG is weakly equivalent to some MG of this kind. This can be proven rather straightforwardly, e.g., by starting with a CFG in Chomsky normal form.

<sup>23</sup> Note that the type of RLIG needed does use the stack in only the following “normalized” way: once an element has been popped from the stack, the stack has to be emptied before a new element can be pushed onto the stack. This, of course, is just a reflex of the successive cyclicity by which a complex head is created.

(phonetic yield of the) complex head and their respective “traces” are cross-serial. This kind of dependencies can be made “visible” by means of a respective specifier being attached right beyond each “trace,” and containing some particular phonetic material; e.g., a copy of the lexical phonetic material of the head by which the specifier is selected as in the case of the MG  $G_{ww}$  deriving the copy language  $\{ww \mid w \in \{a, b\}^*\}$ , and consisting of the following 9 lexical items:<sup>24</sup>

$$\begin{array}{ccccccc} =C =x_a x & a & & =x_a x & a & & =X c \\ & x_a a & & & & & c \\ \\ =C =y_b y & b & & =y_b y & b & & =Y c \\ & y_b b & & & & & \end{array}$$

The MG  $G_{a^n b^k d^n e^k}$  provides a further example of an MG which generates a non-CFL by means of simple applications of the merge-operator and those involving right head adjunction. This MG, which derives the language  $\{a^n b^k d^n e^k \mid k, n \geq 0\}$ , is determined by the following 8 lexical items:

$$\begin{array}{ccccccc} c & & =X_b =x_e x_b & b & & =x_e x_b & b \\ & & x_e e & & & & =X_b c \\ \\ =X_b =x_d x_a & a & & =X_a =x_d x_a & a & & =x_d x_a a \\ & & x_d d & & & & =X_a c \end{array}$$

### 3.4 Arbitrary Head Adjunction

An MG which derives the non-CFL  $\{a^n b^n d^n \mid n \geq 0\}$  by means of mixed successive head adjunction is the MG  $G_{a^n b^n d^n}$  from below. We see that, at the same time, while  $G_{a^n b^n d^n}$  derives cross-serial dependencies between  $a$ 's and  $d$ 's by means of successive right head adjunction analogously to the way in which such dependencies are derived by  $G_{ww}$ ,  $G_{a^n b^n d^n}$  additionally derives nested dependencies between  $a$ 's and  $b$ 's as well as between  $b$ 's and  $d$ 's. Since these additional nested dependencies are derived by “stepwise intervening” left head adjunction this suggests that a language like  $\{a^n b^n d^n \mid n \geq 0\}$  is not derivable by an MG which uses only right head adjunction. The MG  $G_{a^n b^n d^n}$  consists of the following 6 lexical items:

$$X = z y & b & x & a & z & d & =Y x & a & Y = c & c$$

## 4 Simple Phrasal Movement

Assume  $G_{MG}$  to be an MG such that each selection feature that occurs within the label of some lexical entry is weak, and such that the set of licensees is a singleton set. Thus,  $G_{MG}$  does not allow any kind of head movement, but an “indistinguishable” type of phrasal movement. Again, we will skip formal details, when arguing that the language derivable by  $G_{MG}$  is a CFL, and briefly sketch the crucial point here.

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<sup>24</sup> Since, by definition, lexical items are always simple expressions, we usually identify each such item with its (head-)label. Throughout, we also tacitly assume that each of our example MGs consists of an empty set of interpretable features, and that  $\{a, b, d, e\}$  constitutes its set of phonetic features. Note further that, referring to Cornell 1996, the example MG  $G_{ww}$  is also given in Stabler 1997.

Suppose that an expression  $\phi$  is selected by another expression  $v$ , yielding an expression  $\tau = \text{merge}(v, \phi)$  such that the head-label of  $\phi'$ , the subtree of  $\tau$  resulting from  $\phi$ ,<sup>25</sup> starts with an (unchecked) licensee instance. We could additionally “store” this information in the head-label of  $\tau$  with the intend of preventing  $\tau$  from being merged with another expression such that the resulting tree would contain two different maximal projections with an unchecked licensee instance. More generally, we could additionally “store” the head-label of  $\phi$  within the head-label of  $\tau$ ; and within the head-label of each expression subsequently derived from  $\tau$  we could not only store the head-label of  $\phi$ , but also the number of the still unchecked licensee instances introduced by this head-label as long as there is still such an unchecked licensee instance. This would enable us to postpone the “actual insertion of  $\phi$ ” until it has reached its final landing site.

The last considerations, indeed, equip us rather straightforwardly with a method of constructing a weakly equivalent CFG for  $G_{\text{MG}}$ . This, at least, is true if we take into account only expressions being derivable from the lexicon by means of *merge* and *move*, and serving to derive a complete expression: first, it should be mentioned that there is only a finite number of possibilities for an extended head-labeling in the outlined way.<sup>26</sup> But the crucial reason why such a construction becomes finally possible is that each expression  $\tau$  derivable from the lexicon of  $G_{\text{MG}}$ , and serving to derive a complete expression contains at most one maximal projection with an unchecked instance of some licensee feature starting the head-label, since the cardinality of the licensee set is 1.<sup>27</sup> That is to say, whenever we would “develop” a complete expression in a top-down manner and predict a specifier position to be the landing site of some maximal projection  $\tau_1$ , we do not have to worry about the possibility that  $\tau_1$  in its turn contains a “trace” which has arisen from extracting some maximal projection  $\tau'_1$  out of  $\tau_1$ . Such a configuration cannot appear under any circumstances, /\* Correction --> \*/ independently of whether such a  $\tau'_1$  might have been moved out of  $\tau_1$  before  $\tau_1$  itself moves or afterwards. This enables us to transform the MG into a weakly equivalent CFG employing a *slash*-feature-like notation in the sense of GPSG.

Note that the weak generative capacity of each of the considered subclasses of MGs allowing different types of head movement but no phrasal movement is increased if, additionally, the set of licensees is allowed to contain a single element. This, at least, is true w.r.t. the corresponding subclass of MGs allowing only left head adjunction, and the subclass allowing both left as well as right head adjunction. But taking into account our argumentation from above (Section 3.4), we conjecture that the same holds for the subclass of MGs allowing only right head adjunction but no phrasal movement, since by means of right head adjunction and an additional licensee it is possible to derive the language  $\{a^n b^n d^n \mid n \geq 0\}$ . A corresponding MG generating this language is provided by  $G'_{a^n b^n d^n}$  consisting of the following 6 lexical items:

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<sup>25</sup> That is,  $\phi'$  is either the complement or the highest specifier of  $\tau$ .

<sup>26</sup> Recall that the MG lexicon is finite, that each label of a lexical head is a finite string of features, and that an MG builds structure exclusively by canceling particular feature instances introduced in the course of a derivation by means of (the label of) some lexical head.

<sup>27</sup> Recall that, due to the definition of *move*, the implementation of the *shortest move constraint* within MGs allows at most one such maximal projection for each different licensee in order to let an expression occur in the derivation of a complete one.

$$=x_d =x_b +L =x_a x_d -1 \ d \quad x_b b \quad x_a a \quad =x_b =x_a x_d -1 \ d \quad =x_d +L c \quad c$$

An example of an MG which derives the non-CFL  $\{a^n b^n d^n \mid n \geq 0\}$  by means of left head adjunction and a single licensee is the  $G''_{a^n b^n d^n}$ , which results from  $G'_{a^n b^n d^n}$  by replacing the first lexical item with

$$X_d =x_b +L =x_a x_d -1 \ d$$

The MG specified next,  $G_{\text{NON-ELLIL}}$ , employs left as well as right head movement and also a single licensee triggering phrasal movement. This MG derives the language  $\{a^n a^k b^k d^k b^n d^n e^n \mid k, n \geq 0\}$ , and thus a language which is not derivable by any ELLIG (cf. fn. 12).  $G_{\text{NON-ELLIL}}$  consists of all lexical items of the MG  $G'_{a^n b^n d^n}$  and the following 8 additional ones:

$$\begin{aligned} &=Y_e =y_b +L =y_a y_d -1 \ d \quad y_b b \quad y_a a \quad Y_d =y_e e \quad =y_b =y_a y_d -1 \ d \quad =y_e +L c \\ &\quad =y_b =z =y_a y_d -1 \ d \quad =x_d +L z \end{aligned}$$

An MG which also employs left as well as right head movement and a single licensee triggering phrasal movement, and which even derives a language not being derivable by any LIG, namely, the language  $\{a^n b^n a^k b^k d^n e^n d^k e^k \mid k, n \geq 0\}$ , is the MG  $G_{\text{NON-LIL}}$  consisting of the following 15 lexical items:

$$\begin{aligned} &=Y_e =y_b +L =y_a y_d -1 \ d \quad y_b b \quad y_a a \quad Y_d =y_e e \quad =y_b =y_a y_d -1 \ d \quad =y_e y \quad y -1 \\ &=X_b =x_e +L =x_d x_a -1 \ a \quad x_e e \quad x_d d \quad X_a =x_b b \quad =x_e =x_d x_a -1 \ a \quad =X_b +L =y x \quad =y x \\ &\quad =X +L c \\ &\quad /* \text{-- Correction */} \end{aligned}$$

## 5 Beyond simple phrasal movement

We leave it an open problem here, how the language classes respectively determined by the subclasses of MGs permitting head movement of a particular kind and phrasal movement by means of at most one licensee compare in detail to the language class determined by TAGs. We rather conclude by emphasizing that phrasal movement in the sense of the MG-definition which arises from the interaction of two different licensees also permits us to derive languages not derivable by any TAG. As an example of a corresponding MG we finally present the MG  $G'_{\text{NON-LIL}}$  deriving the language  $\{a^n b^n a^k b^k d^n e^n d^k e^k \mid k, n \geq 0\}$ , and consisting of the following 17 lexical items:

$$\begin{array}{cccc}
y_b b & =y_a y_e e & =y_e y d & y -l_1 \\
\\
=y_d +L_1 y_b b & =y_b y_a -l_1 a & =y_a +L_2 y_e e & =y_e y_d -l_2 d \\
\\
x_b b & =x_a x_e e & =x_e x d & x -l_2 \\
\\
=x_d +L_2 x_b b & =x_b x_a -l_2 a & =x_a +L_1 x_e e & =x_e x_d -l_1 d \\
\\
& & & =x =y +L_2 +L_1 c
\end{array}$$

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