

# An Introduction to Minimalist Grammars:

## *Formalism*

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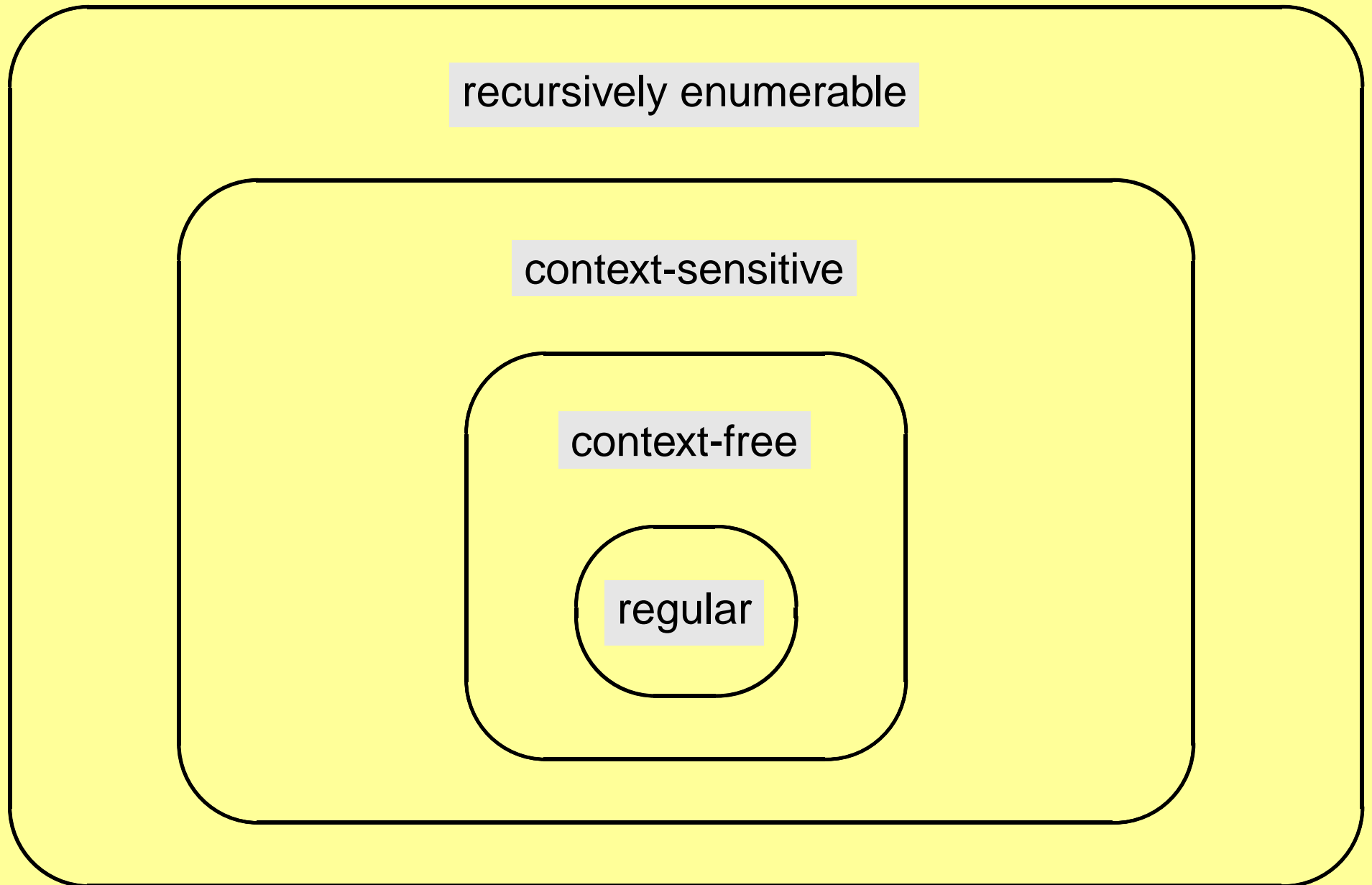
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# Introduction

- Research on natural language syntax in terms of **transformational grammar (TG)** has always been accompanied by questions on the complexity of the individual grammars allowed by the general theory.
- From the **perspective of formal language theory**, special emphasis has more generally been placed on two specific aspects:
  - a) the **location within the Chomsky hierarchy** of any grammars supposed to be adequate models for natural languages,
  - b) the **complexity of the parsing problem** for such grammars.

## Chomsky hierarchy



# Introduction

- Peters and Ritchie (1971, 1973) proved the *Aspects*-model of TG (Chomsky 1965) to be Turing equivalent.  
⇒ For every recursively enumerable set (i.e., type 0-language), there is a particular *Aspects*-grammar deriving it.
- Subsequently, **locality conditions (LCs)** — established in Ross 1967 and Chomsky 1973, 1977 — were studied intensively in work by many others searching **for ways to reduce expressive power**.
- ◆ See, e.g., Huang (1982), Chomsky (1986), Rizzi (1990), Cinque (1991), Manzini (1992), Müller & Sternefeld (1993), Szabolcsi & Zwarts (1993).

# Introduction

- Complexity results, however, have been largely absent for those grammars with LC-add-ons. (Notable exception: Rogers 1998.)

The picture changed with **minimalist grammars (MGs)** (Stabler 1997, 1999) as a formalization of “minimalism” (Chomsky 1995).

MGs in that format constitute a **mildly context-sensitive grammar formalism** in the sense of Joshi 1985 (Michaelis 1998, 2001).

- ◆ Two crucial features of MGs helped achieving this result:
  - the **resource sensitivity** (encoded in the checking mechanism),
  - the implementation of the **shortest move condition (SMC)**.

- A **concept** motivated by the intention of characterizing a narrow class of formal grammars which are
  - “**only slightly more powerful than context-free grammars,**”
  - **nevertheless** allowing for natural language descriptions in a **linguistically significant** way.

- A concept motivated by the intention of characterizing a narrow class of formal grammars which are
  - “only slightly more powerful than context-free grammars,”
  - nevertheless allowing for natural language descriptions in a linguistically significant way.
  
- A **mildly context-sensitive grammar (MCSG)** fulfills **three criteria**, understood as a “rough characterization” (cf. Joshi 1985, p. 225).
  - 1) **Parsing** problem is solvable **in polynomial time**.
  - 2) Language has the **constant growth property**.
  - 3) Finite **upper bound on** the number of **different instantiations of factorized cross-serial dependencies** occurring in any sentence.

# MCSG-landscape

• Indexed Grammar

• Lexical Functional Grammar

**MCSG**

Linear Context-Free  
Rewriting Systems

Linear Indexed Grammar

• Tree Adjoining Grammar

• Combinatory  
Categorial Grammar

Context-Free Grammar  
(GPSG)

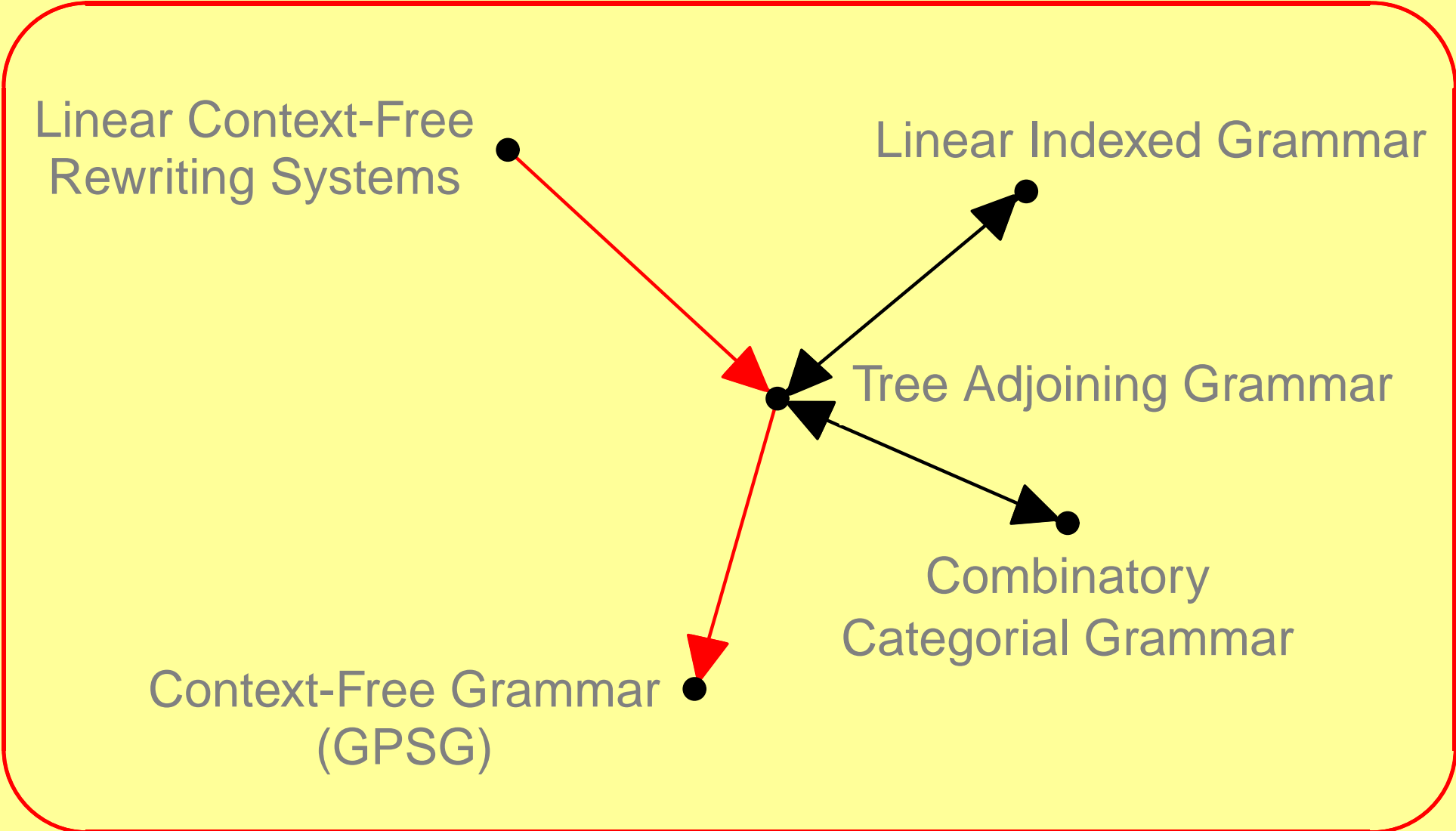


# MCSG-landscape

- Indexed Grammar

- Lexical Functional Grammar

**MCSG**



# Minimalist grammars (1)

- **Minimalist grammars (MGs)** provide an attempt at a rigorous algebraic formalization (of some) of the perspectives adopted in the minimalist branch of generative grammar.

Work on MGs defined in this sense can be seen as having led to a realignment of “grammars found ‘useful’ by linguists” and formal complexity theory.

## Two types of locality conditions (LCs)

- **In particular**, a study in terms of MGs can enhance our understanding of the **complexity/restrictiveness of LCs**.

In fact, such a study shows that, though **the addition of an LC** may reduce complexity in an appropriate and intuitively natural way, it does not necessarily do so, and **may even increase complexity**.

- One can formally distinguish **two types of LCs**.

# Two types of locality conditions (LCs)

## ■ Intervention-based LCs (ILCs)

- often in terms of minimality constraints, such as minimal link, minimal chain, shortest move, attract closest etc.

in MGs: **shortest move condition (SMC)** (Stabler 1997, 1999)

## ■ Containment-based LCs (CLCs)

- often in terms of (generalized) grammatical functions, such as adjunct islands, specifier islands, subject island etc.

in MGs: **specifier island condition (SPIC)** (Stabler 1999)

in MGs: **adjunct island condition (AIC)**

(Frey & Gärtner 2002, Gärtner & Michaelis 2003)

# Two types of locality conditions (LCs)

## ■ Intervention-based LCs (ILCs)

- often in terms of minimality constraints

essential structure:  $[\dots \alpha \dots [\dots \beta \dots \gamma \dots]]$

## ■ Containment-based LCs (CLCs)

- often in terms of (generalized) grammatical functions

essential structure:  $[\dots \alpha \dots [\beta \dots \gamma \dots]]$

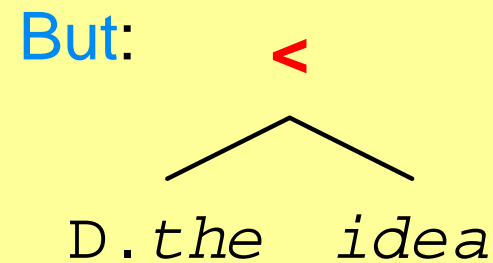
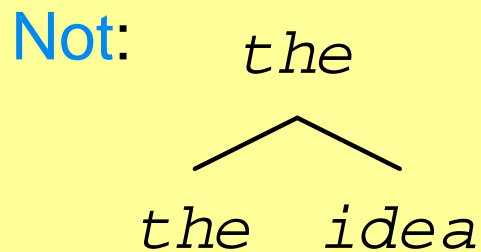
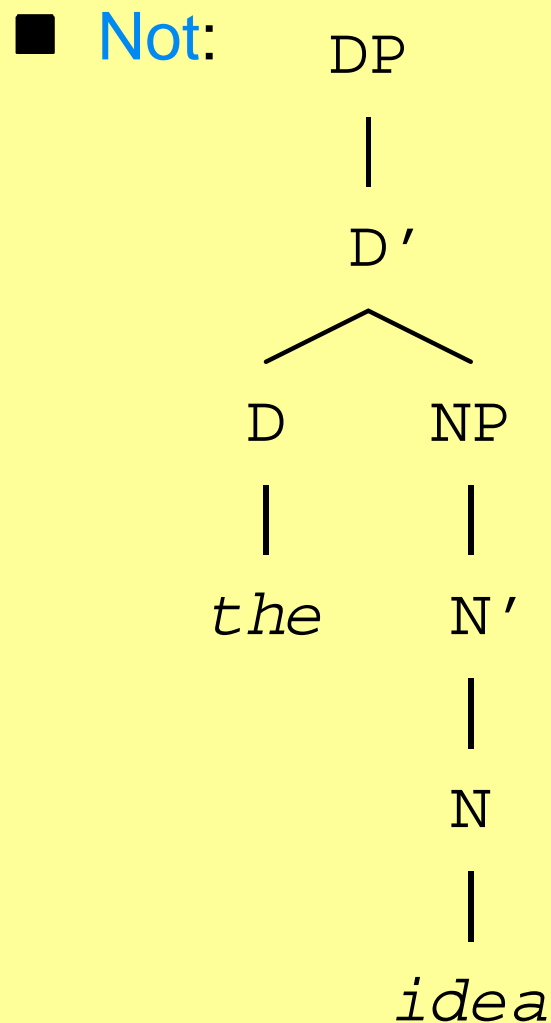
# Minimalist grammars (1)

- **More generally**, MGs are capable of integrating (if needed) a variety of (arguably) “odd” items from the syntactician’s toolbox such as:
  - **head movement** (Stabler 1997, 2001)
  - **(strict) remnant movement** (Stabler 1997, 1999)
  - **affix hopping** (Stabler 2001)
  - **adjunction** and **scrambling** (Frey & Gärtner 2002)
  - **late adjunction** and **extraposition** (Gärtner & Michaelis 2003)  
— to some extent without rise in generative power
  - **copy-movement** (Kobele 2006)
  - **wh-clustering** (Gärtner & Michaelis 2007)

# Minimalist expressions

- The objects generated by an MG are called **minimalist expressions**.

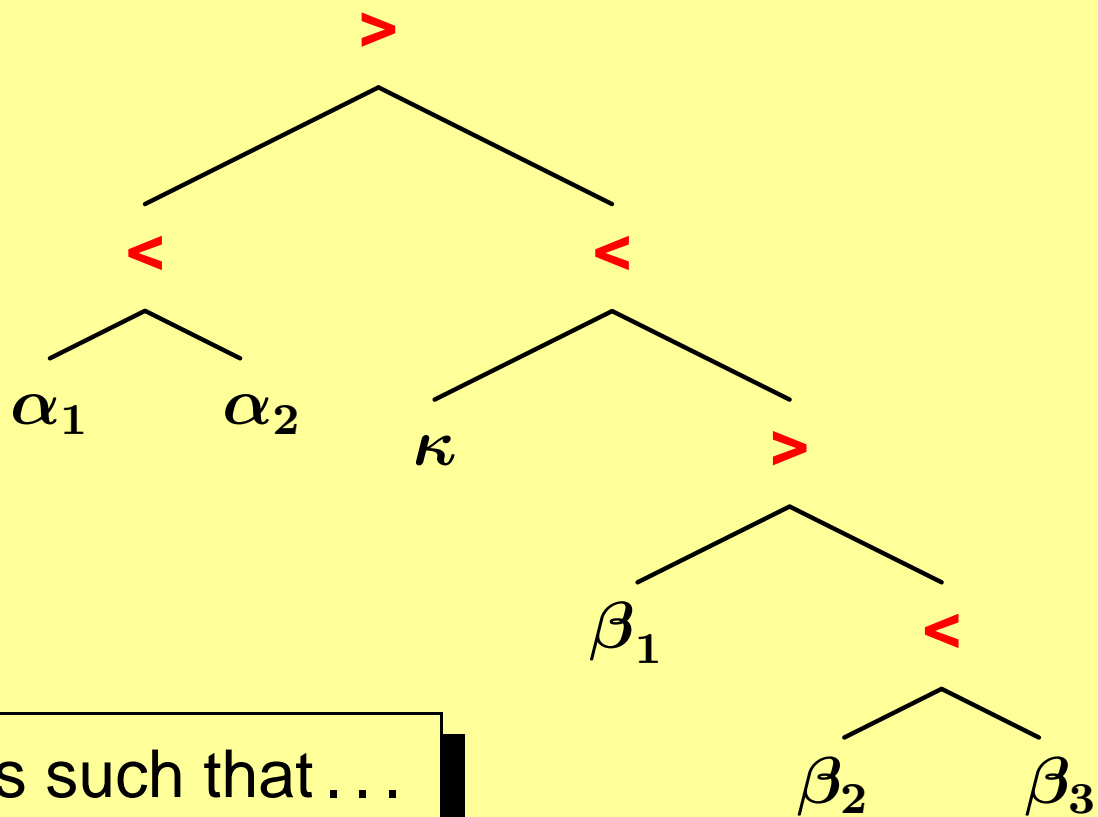
# Minimalist expressions



The **<** “points towards” the **projecting daughter**, and thus — by means of transitivity — towards the **head** of the phrase.



# Minimalist expressions

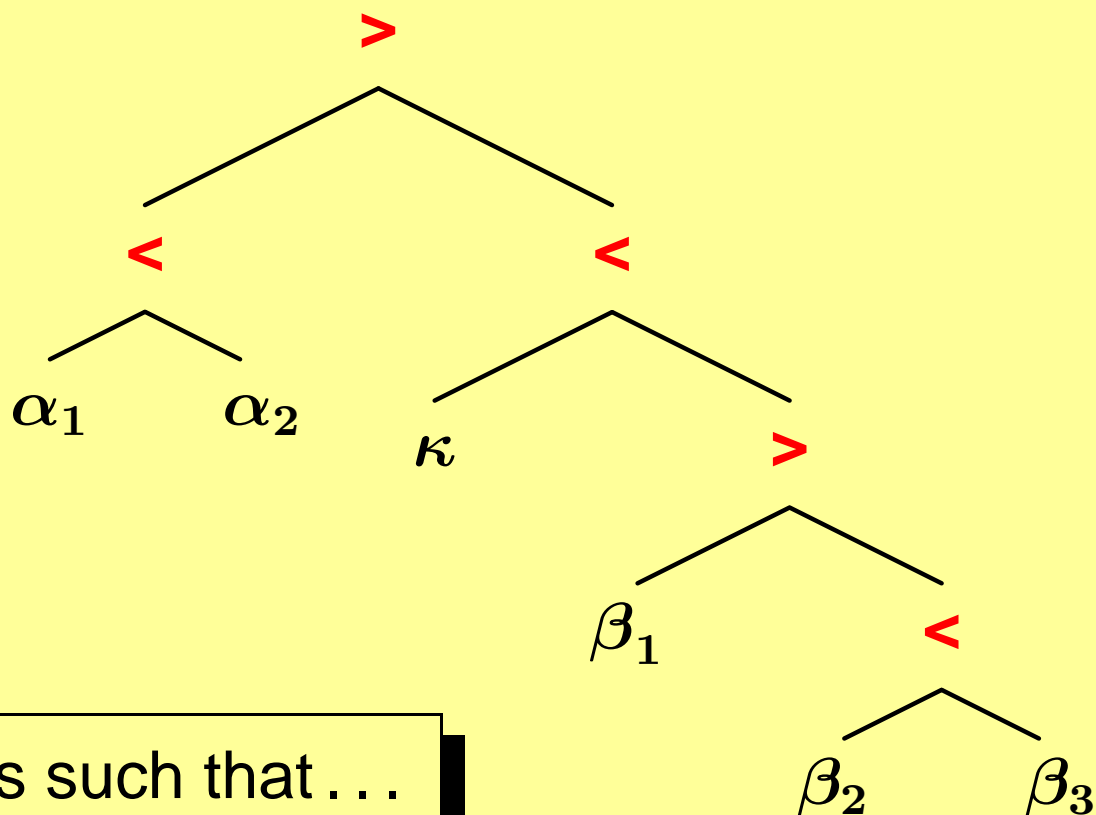


finite, binary labeled trees such that...

- non-leaf-labels are from  $\{<, >\}$  [“projection”]

# Minimalist expressions

- < “left daughter projects”
- > “right daughter projects”

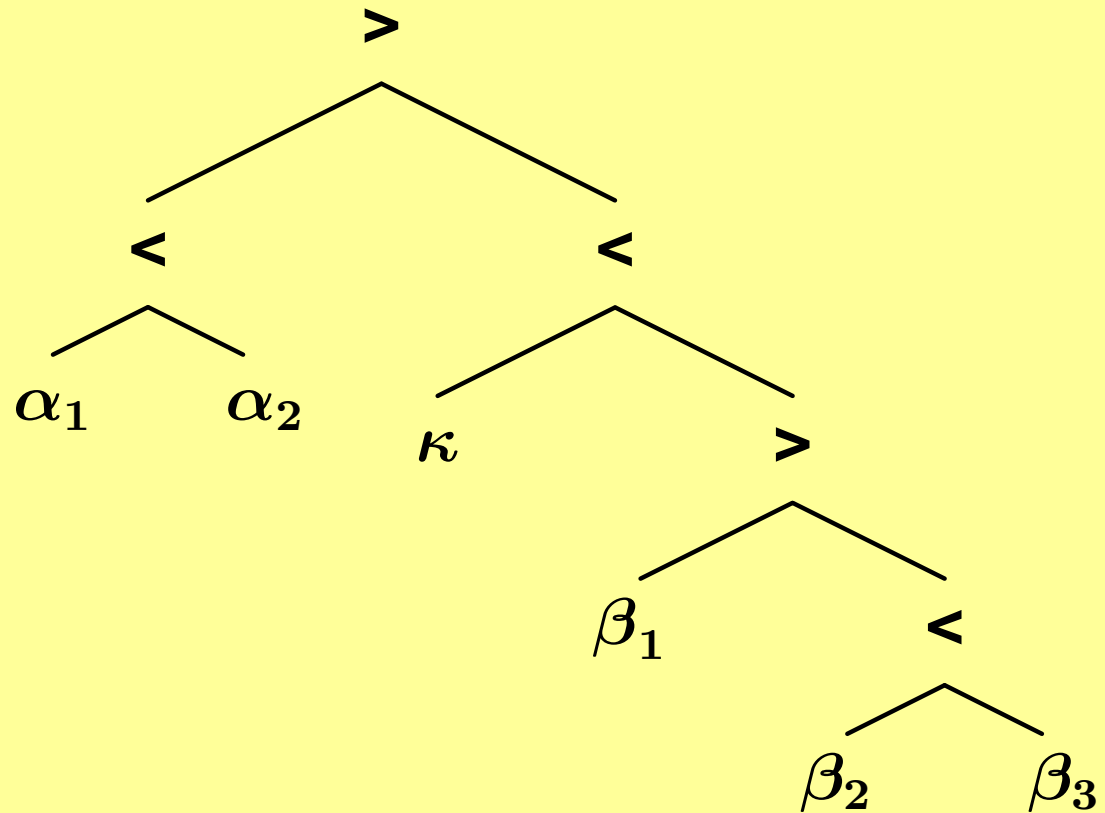


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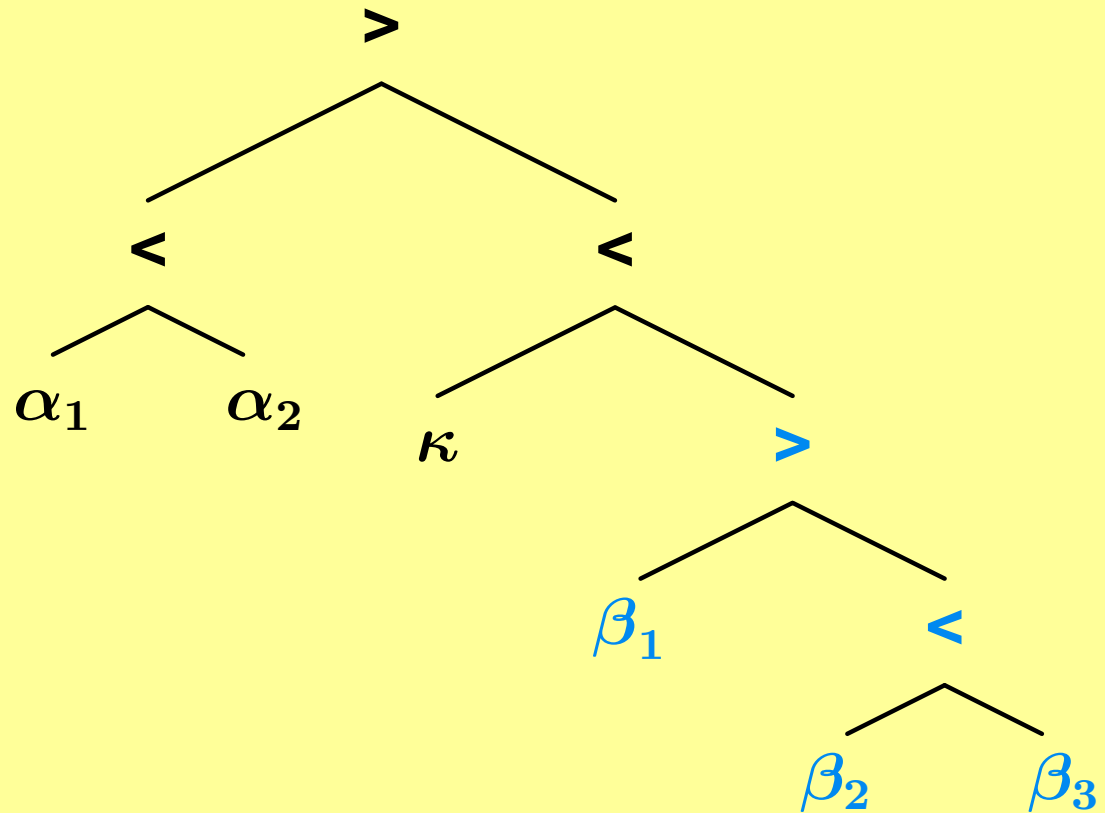


maximal projections :

each subtree whose root does not project

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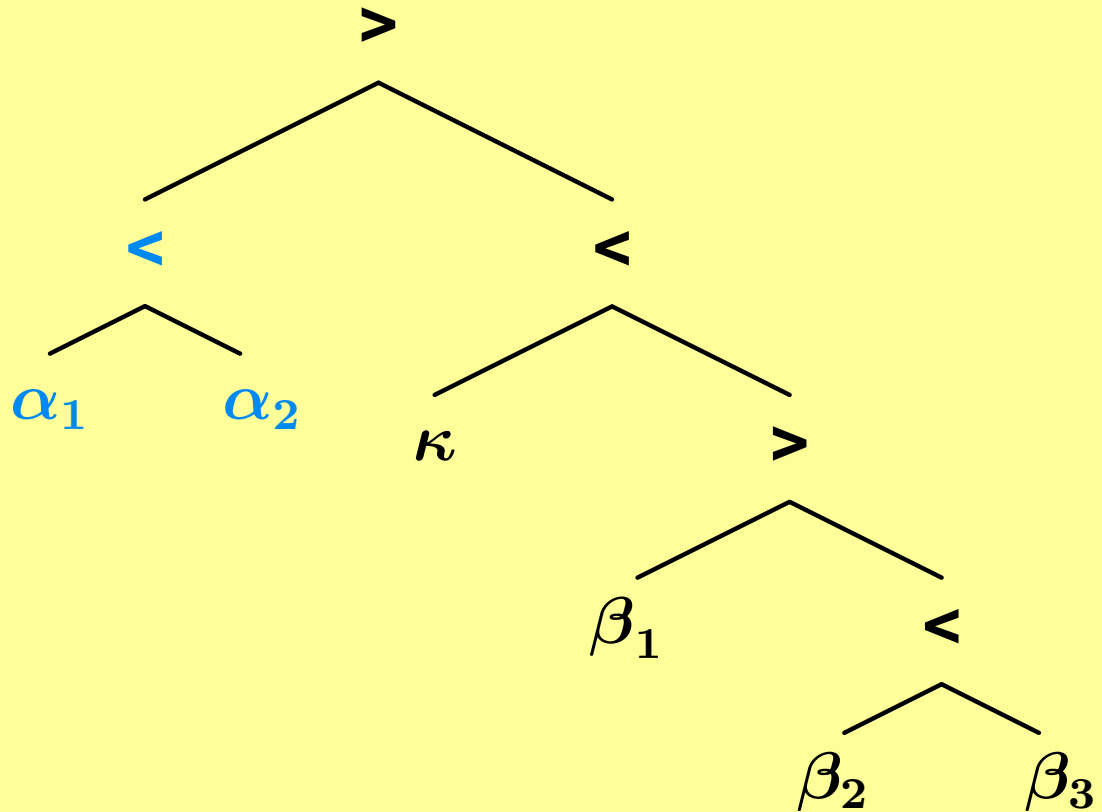


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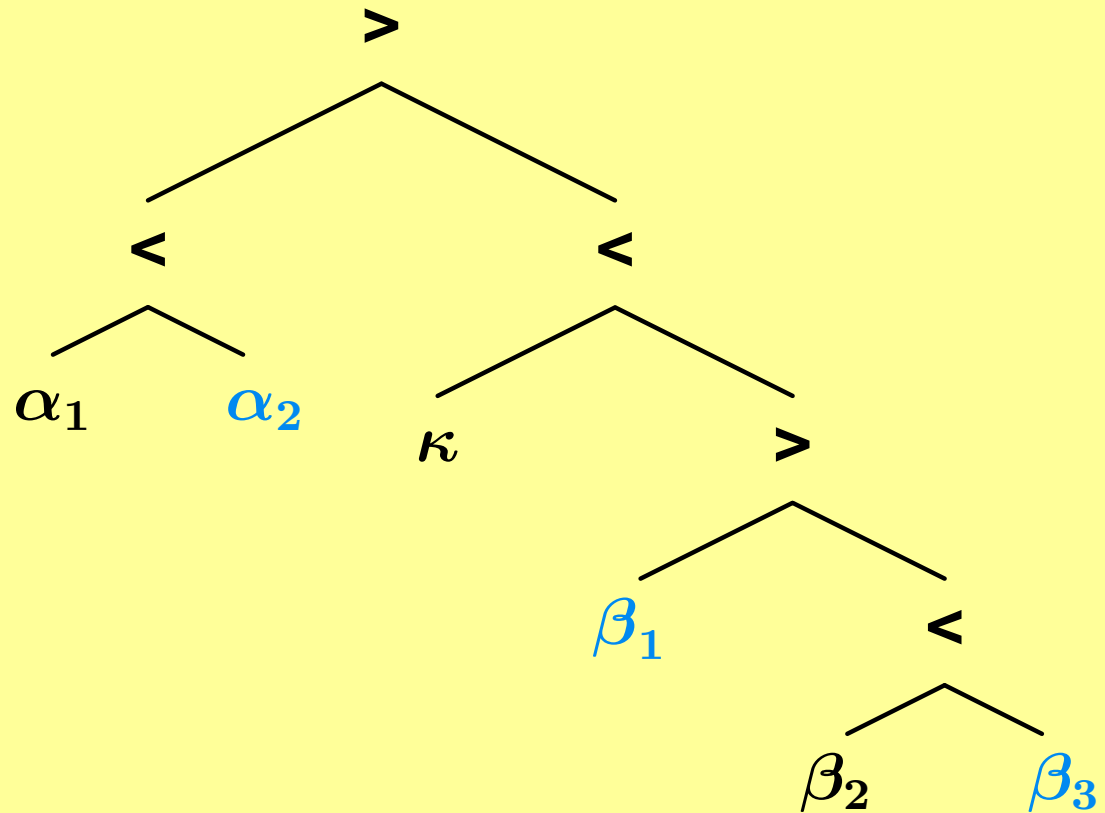


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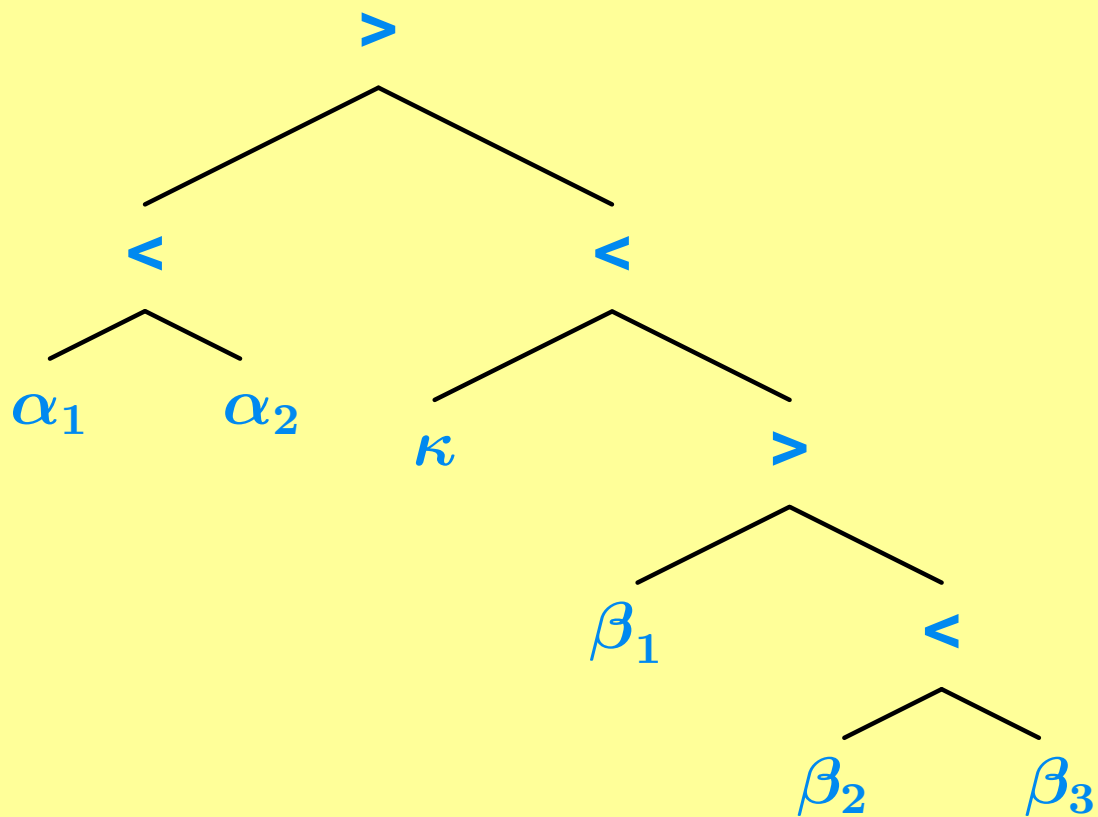


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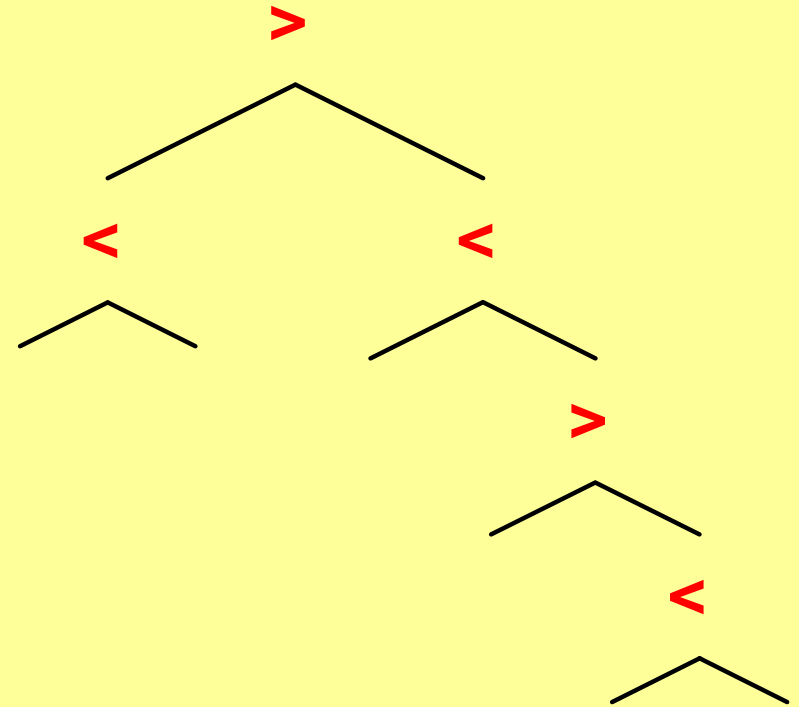
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# Minimalist expressions



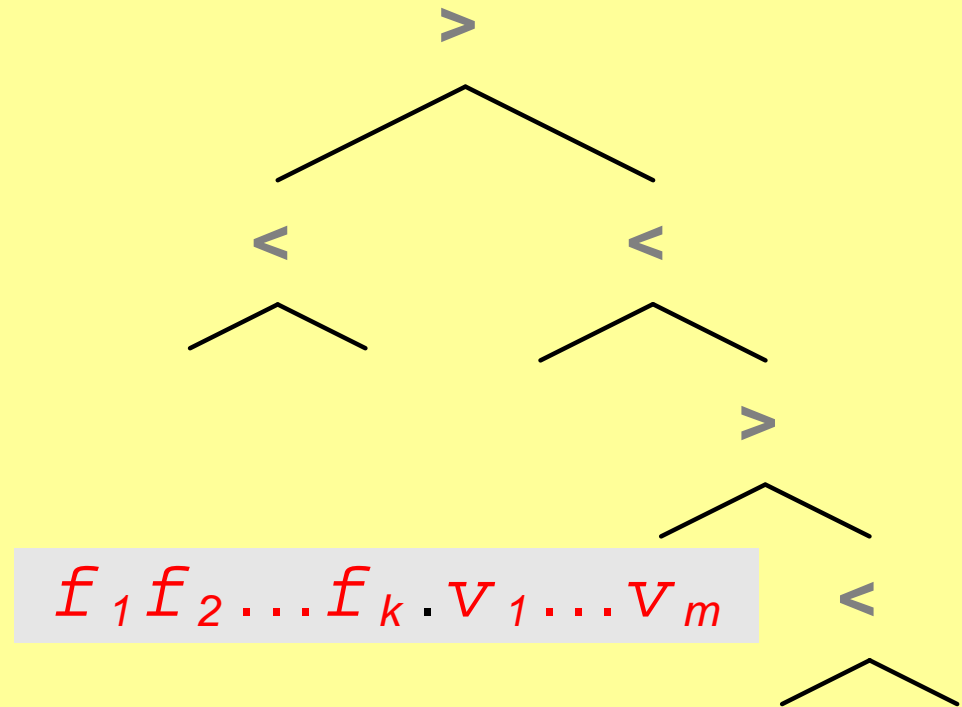
finite, binary labeled trees such that...

- non-leaf-labels are from  $\{<, >\}$  ["projection"]

# Minimalist expressions

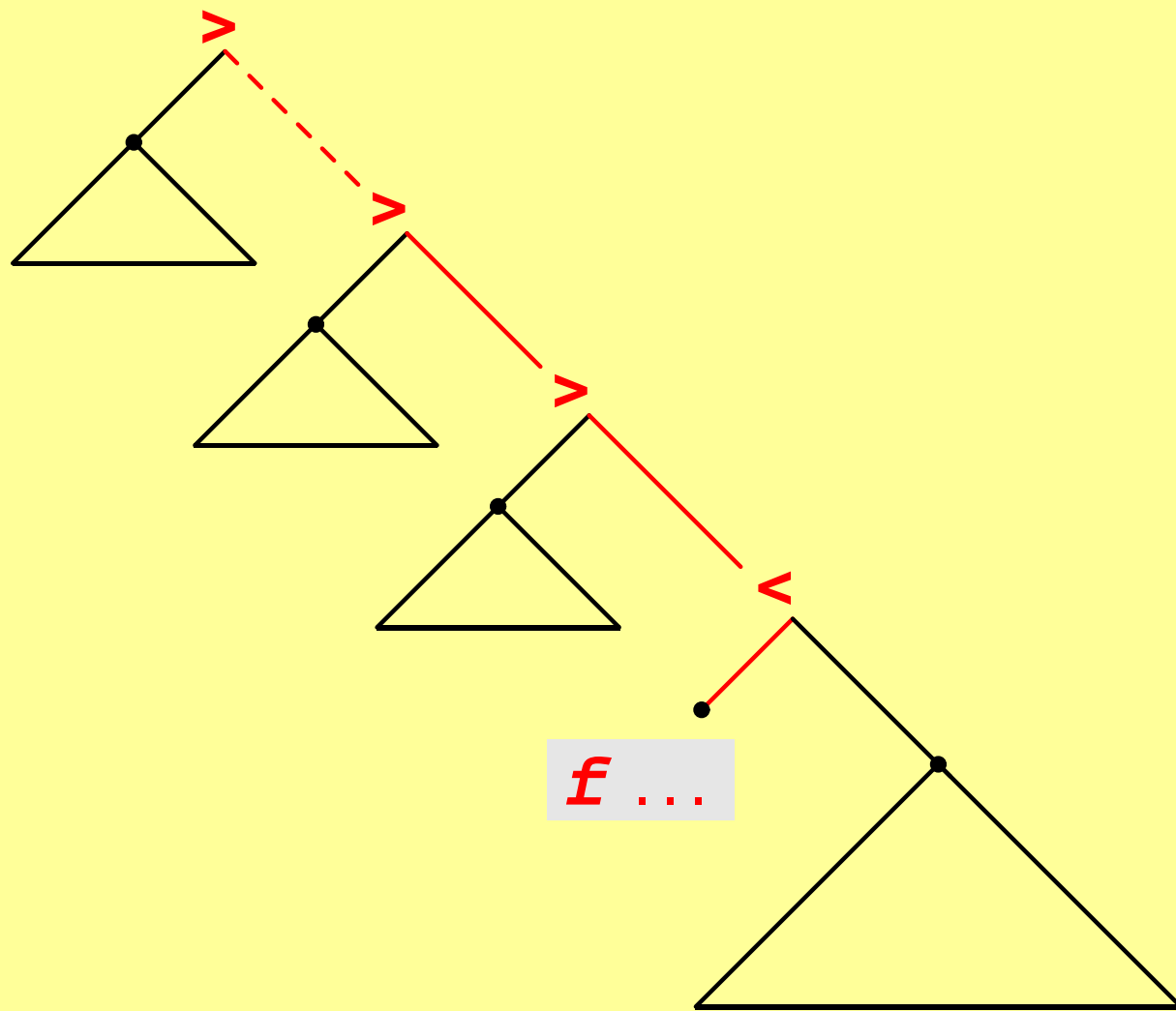
**Vocabulary** (terminals)

**SynFeatures** (syntactic features)



finite, binary labeled trees such that ...

- leaf-labels are from  $\text{SynFeatures}^* . \text{Vocabulary}^*$



tree displays feature  $\mathcal{F}$  :  $\iff$  head-label is of the form  $\mathcal{F} \dots$

- There are different types of syntactic features.

*(basic) categories:*             $x, y, z, \dots$             [ Base        ]

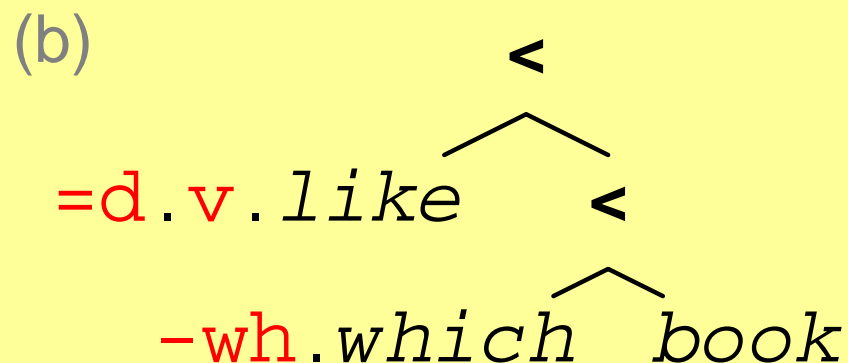
*(merge-) selectors:*             $=x, =y, =z, \dots$             [ Selectors   ]

*(move-) licensees:*             $-x, -y, -z, \dots$             [ Licensees   ]

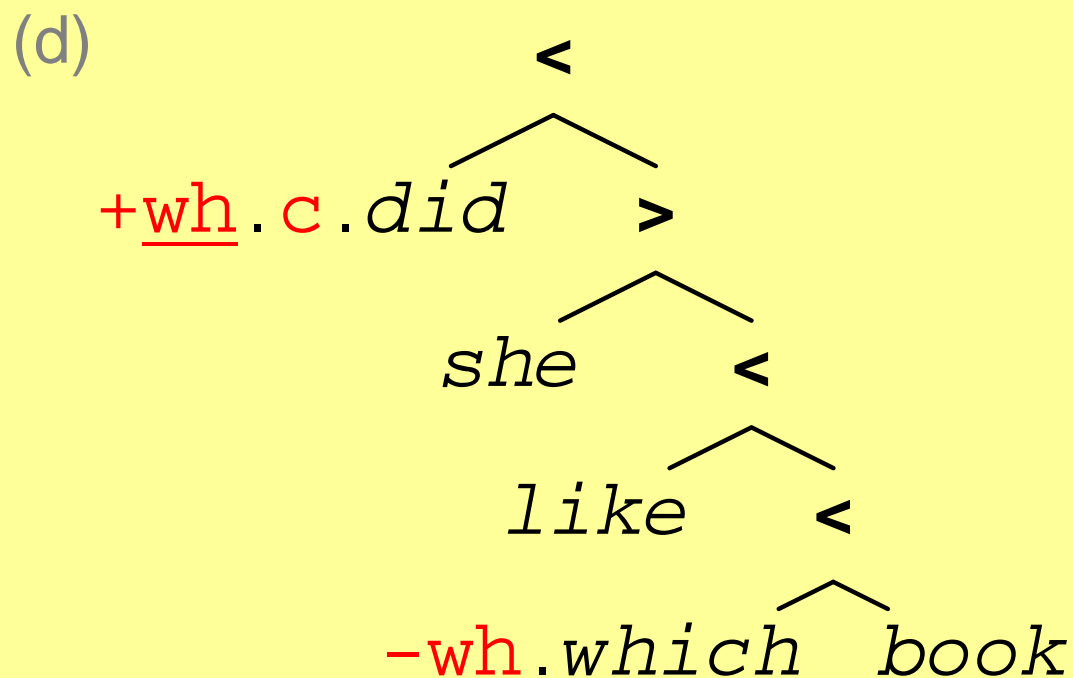
*(move-) licensors:*             $+\underline{x}, +\underline{y}, +\underline{z}, \dots$             [ Licensors   ]

...

(a) =d. =d.v. like



(c) d.she



# Building minimalist expressions

- Starting from a finite set of simple expressions (a lexicon),  
minimalist expressions can be built up recursively
  - by applying structure building functions  
checking off instances of syntactic features “from left to right,”  
  
where, after having applied a structure building function, the  
triggering feature instances are canceled.
- Different types of syntactic features trigger different structure  
building functions.

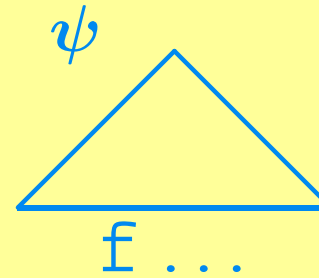
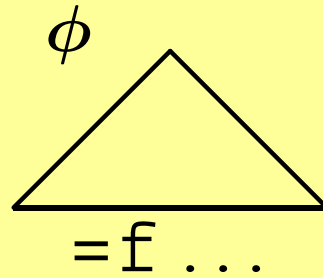
# Structure building functions

$\text{merge} : \text{Trees} \times \text{Trees} \xrightarrow{\text{part}} \text{Trees}$

- $\langle \phi, \psi \rangle \in \text{Domain}(\text{merge}) : \iff$ 
  - $\psi$  displays feature  $f \in \text{Base}$
  - $\phi$  displays feature  $=f \in \text{Selectors}$

# Structure building functions

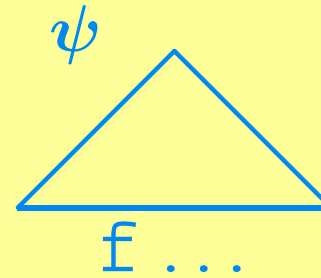
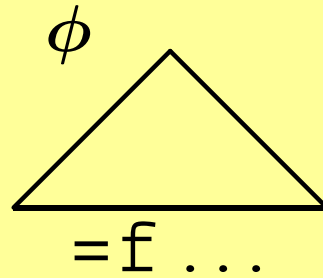
`merge : Trees × Trees  $\xrightarrow{\text{part}}$  Trees`





# Structure building functions

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees

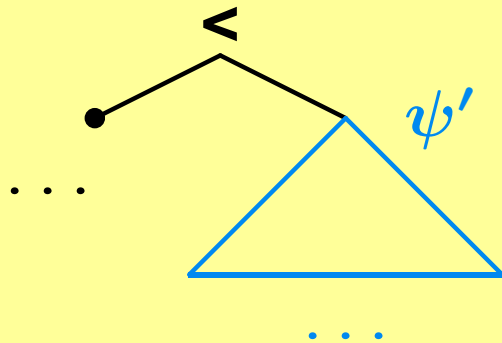
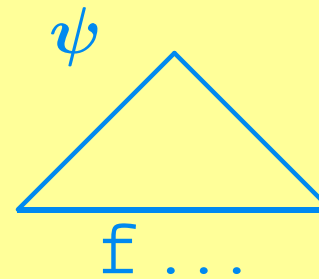
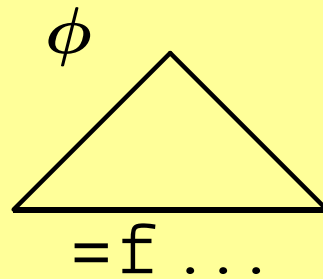


selecting  $\phi$  simple

selecting  $\phi$  complex

# Structure building functions

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees

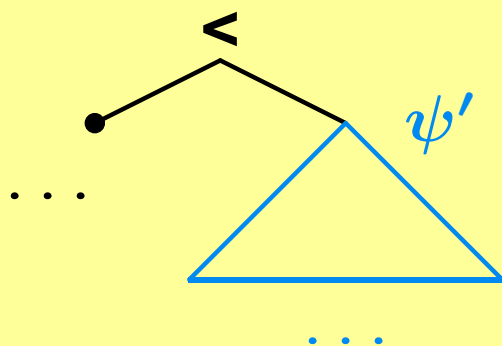
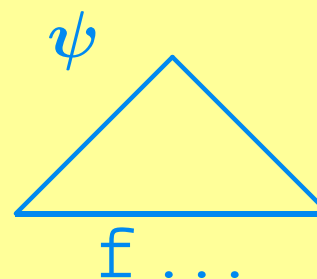
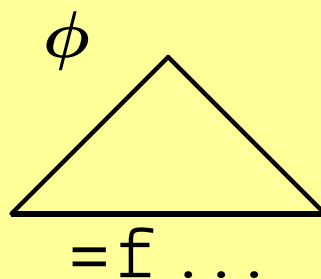


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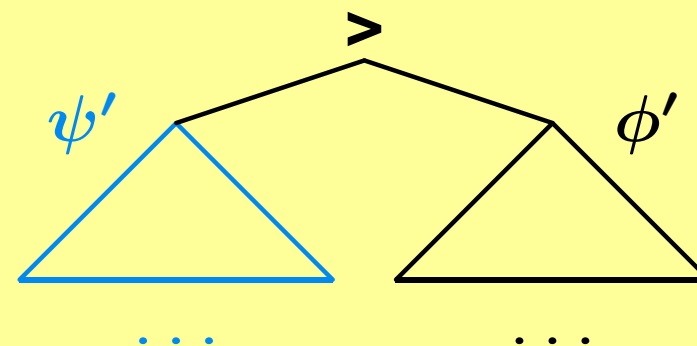
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merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees



selecting  $\phi$  simple



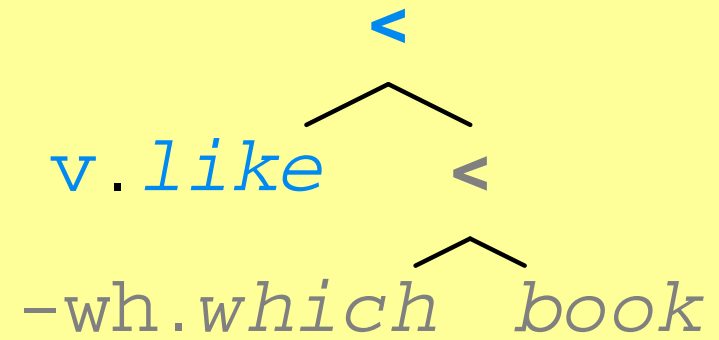
selecting  $\phi$  complex

merge

(selecting tree is simple)

=v . =d . i . ∅

+

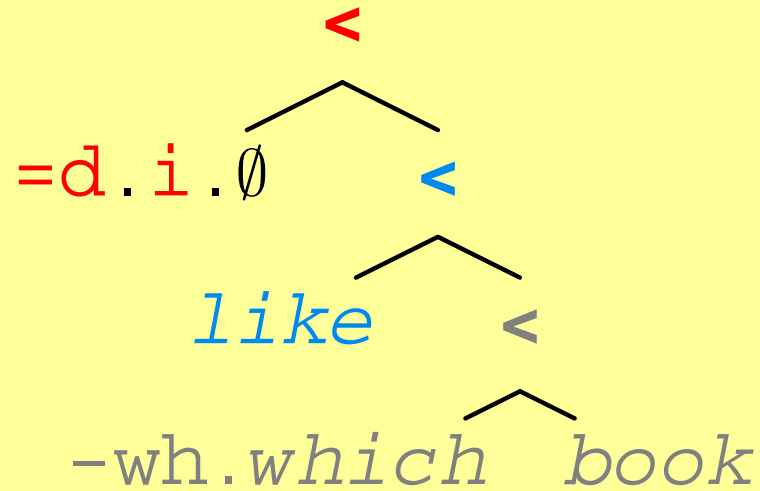
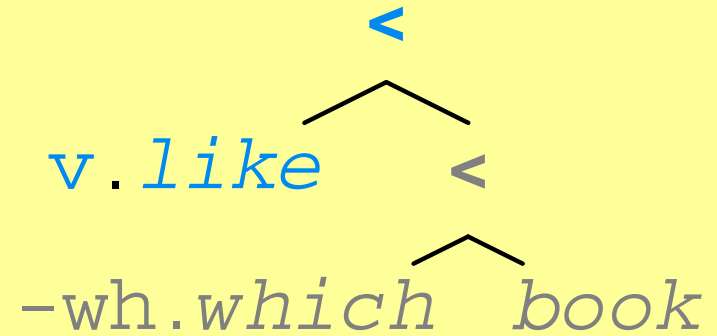


merge

(selecting tree is simple)

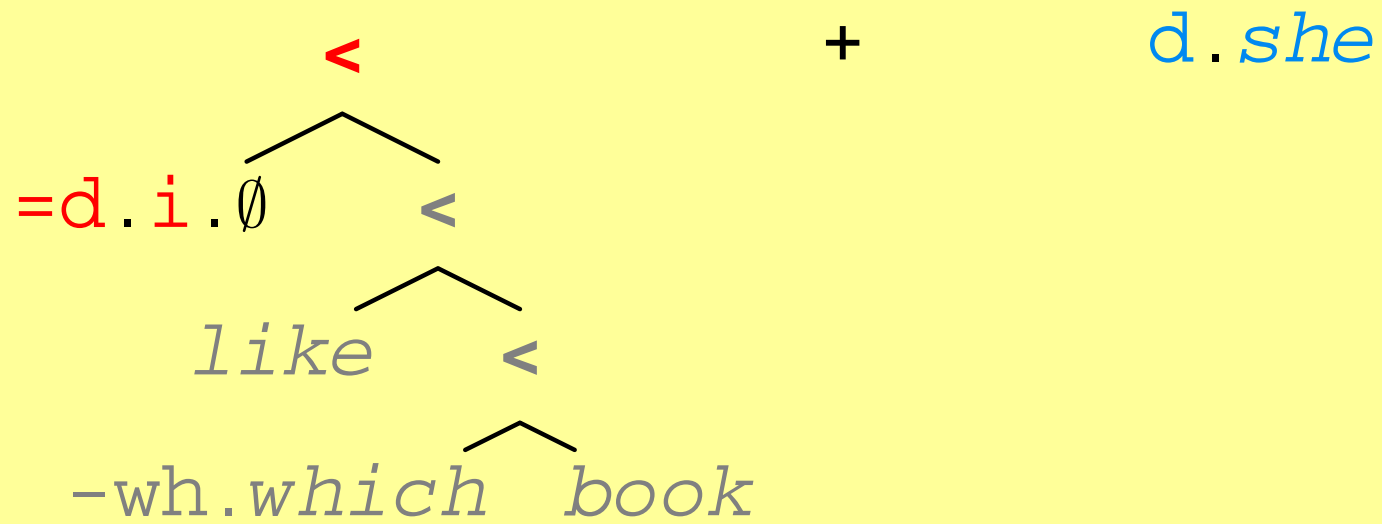
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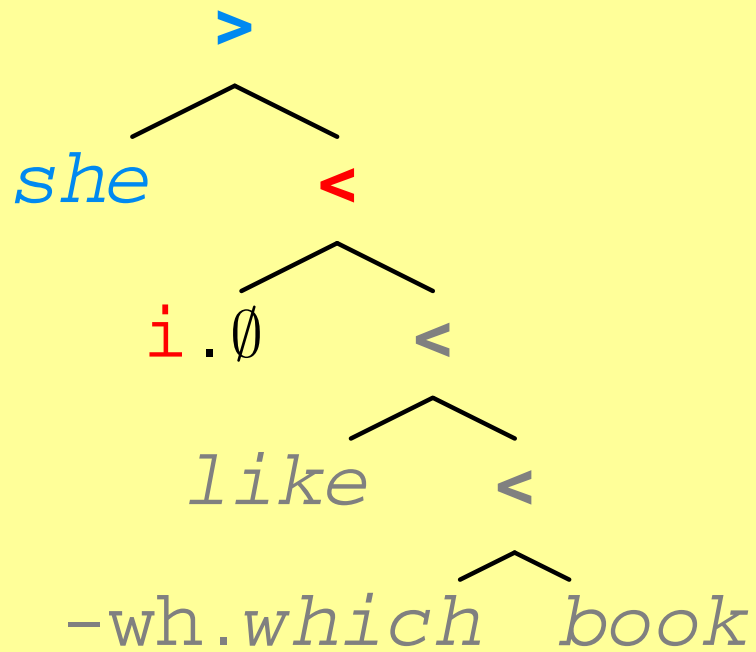
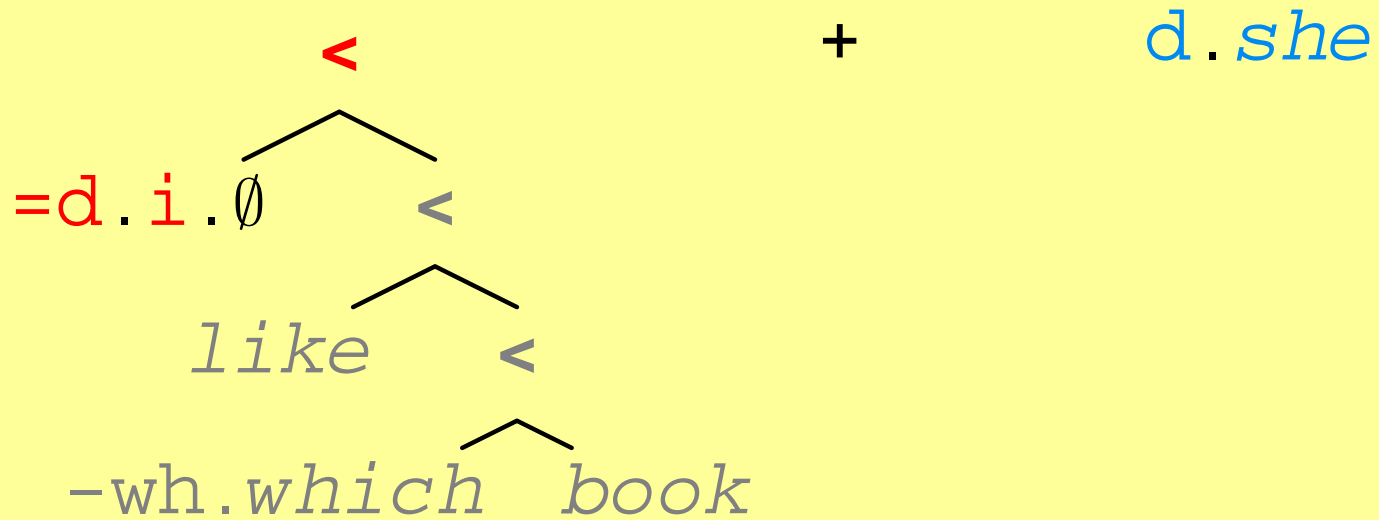
merge

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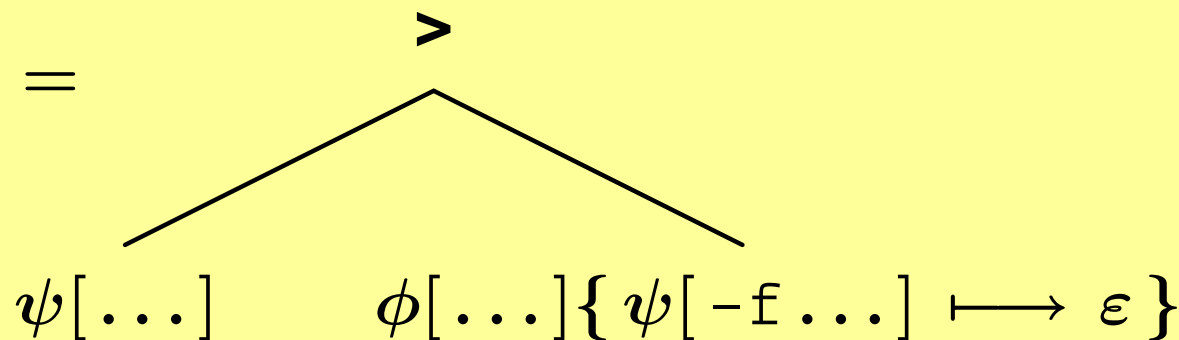


$$\text{move} : \text{Trees} \xrightarrow{\text{part}} 2^{\text{Trees}}$$

■  $\phi \in \text{Domain}(\text{move}) : \iff$

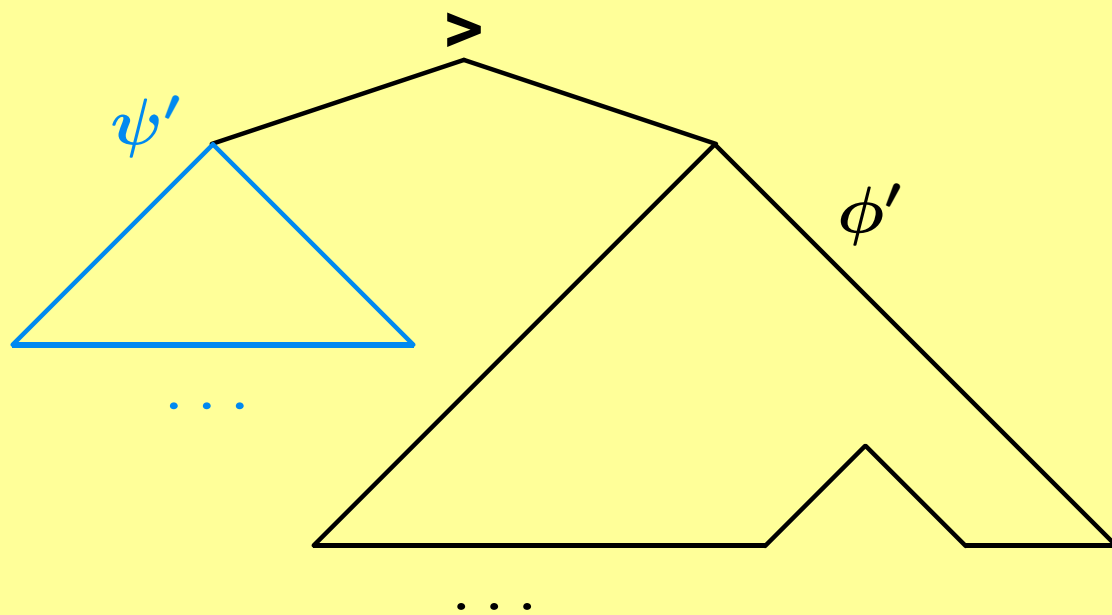
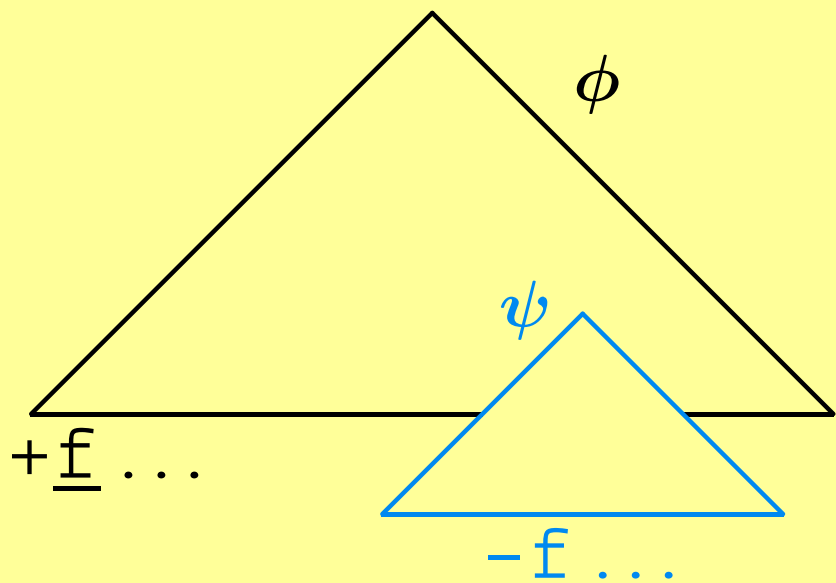
- $\phi$  displays feature  $+\underline{f} \in \text{Licensors}$
- there is a maximal projection  $\psi$  within  $\phi$  that displays feature  $-\underline{f} \in \text{Licensees}$

■  $\text{move}(\phi[+\underline{f}\dots]) =$

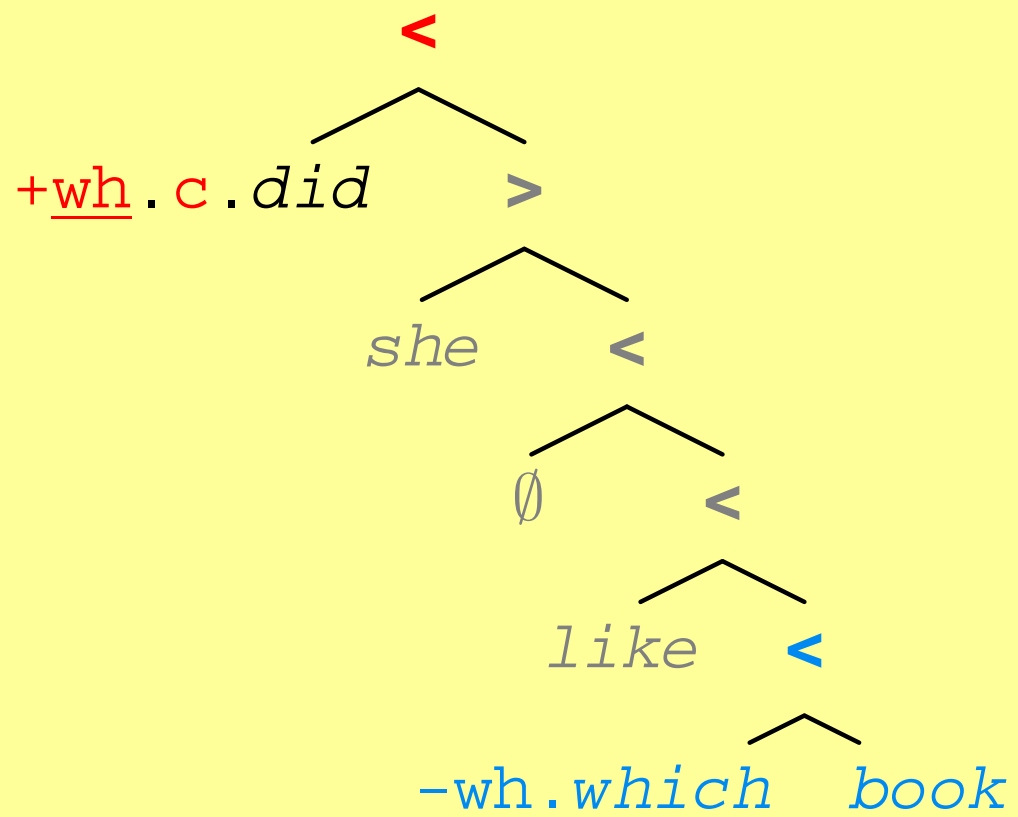




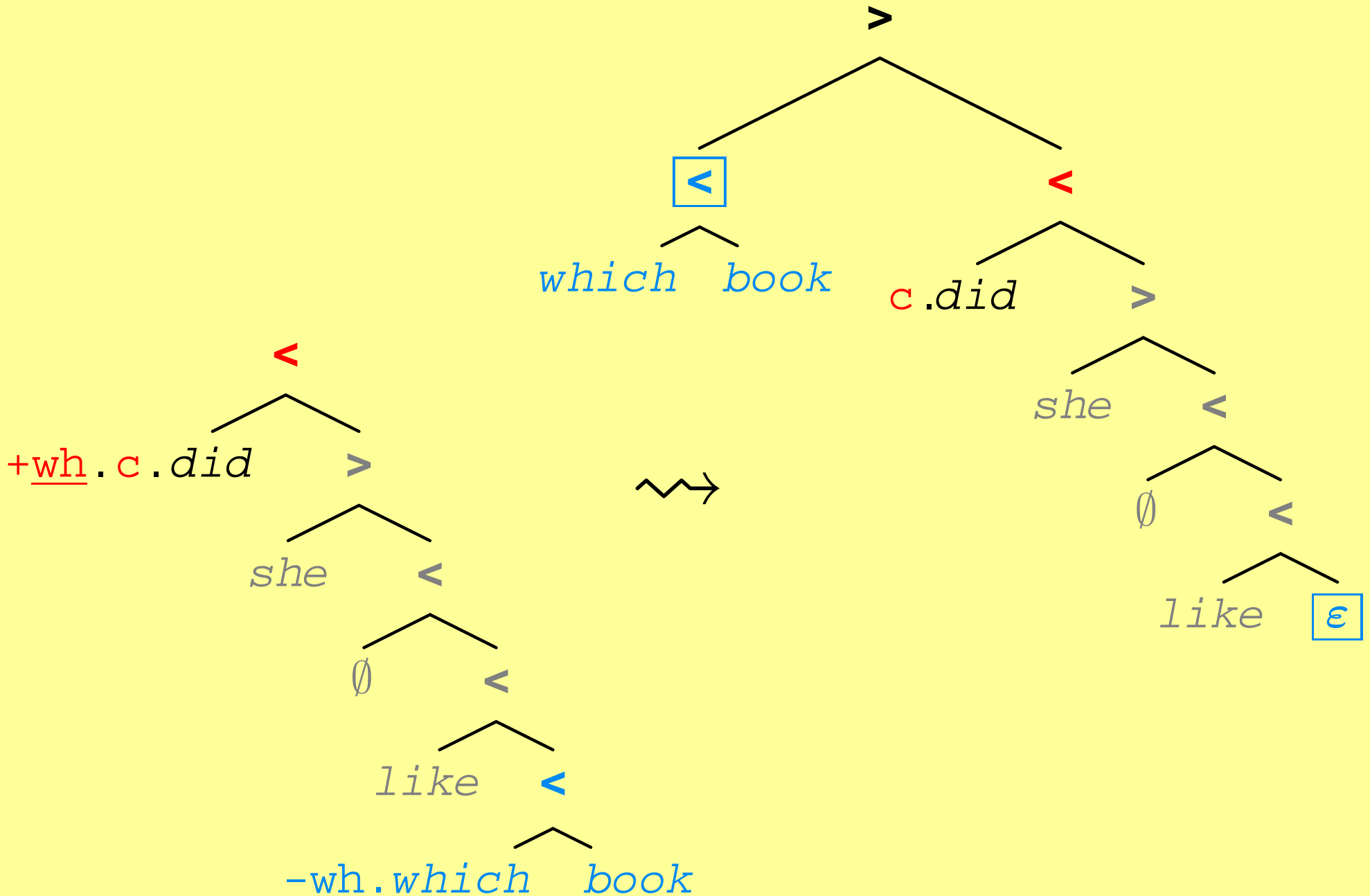
move : Trees  $\xrightarrow{\text{part}}$  2Trees



# move



# move



- There are different types of syntactic features.

*(basic) categories:*       $x, y, z, \dots$       [ Base      ]

*(merge-) selectors:*       $\dots$       [ Selectors      ]

*(move-) licensees:*       $-x, -y, -z, \dots$       [ Licensees      ]

*(move-) licensors:*       $+\underline{x}, +\underline{y}, +\underline{z}, \dots$       [ Licensors, strong ]

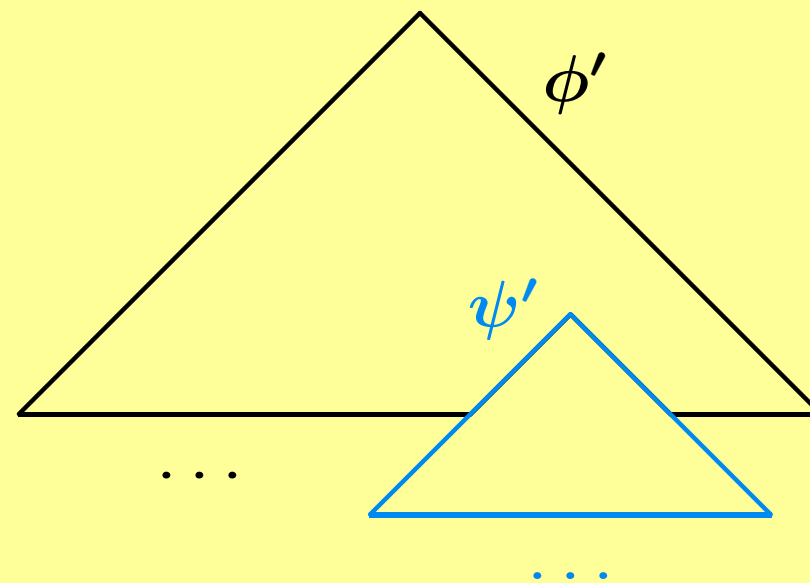
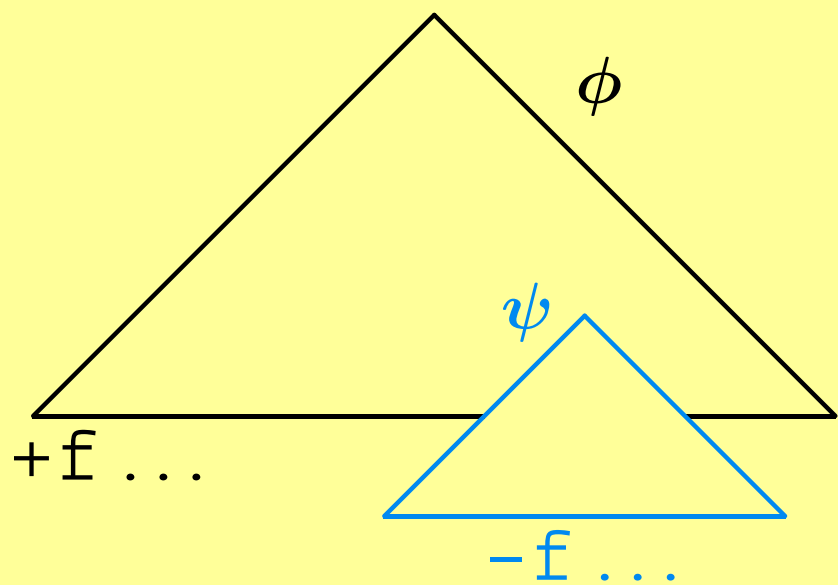
$+x, +y, +z, \dots$       [ Licensors, weak ]

$\dots$

$$\text{agree} : \text{Trees} \xrightarrow{\text{part}} 2^{\text{Trees}}$$

- $\phi \in \text{Domain}(\text{agree}) : \iff$ 
  - $\phi$  displays feature  $+f \in \text{Licensors}$
  - there is a maximal projection  $\psi$  within  $\phi$  that displays feature  $-f \in \text{Licensees}$
- $\text{agree}(\phi[+f \dots]) = \phi[\dots]\{\psi[-f \dots] \mapsto \psi[\dots]\}$

agree : Trees  $\xrightarrow{\text{part}}$   $2^{\text{Trees}}$



- There are different types of syntactic features.

*(basic) categories:*       $x, y, z, \dots$       [ Base      ]

*(merge-) selectors:*       $=x, =y, =z, \dots$       [ Selectors, weak ]

$=>x, =>y, =>z, \dots$       [ Selectors, strong ]

$x<=, y<=, z<=, \dots$

*(move-) licensees:*       $-x, -y, -z, \dots$       [ Licensees      ]

*(move-) licensors:*       $+x, +y, +z, \dots$       [ Licensors      ]

...

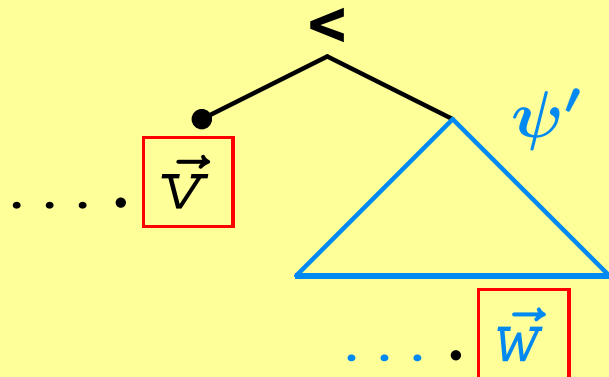
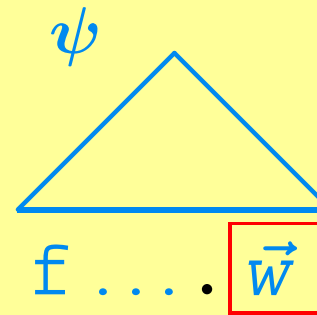
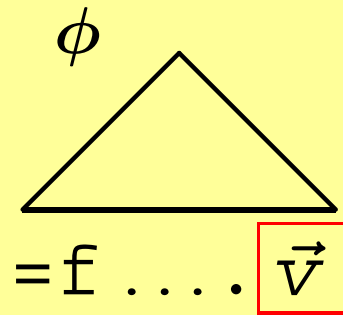
# Structure building functions

$\text{merge} : \text{Trees} \times \text{Trees} \xrightarrow{\text{part}} \text{Trees}$

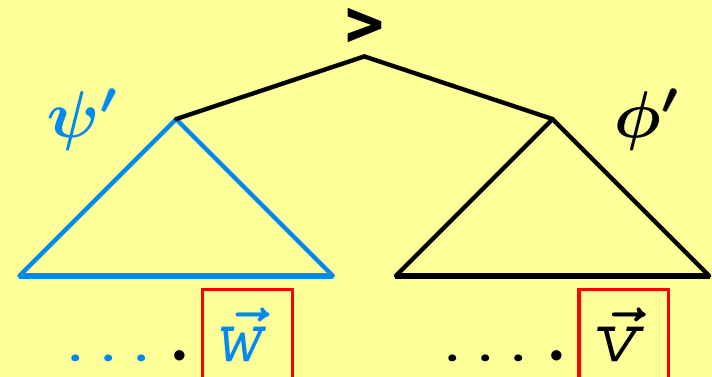
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  - $\psi$  displays feature  $f \in \text{Base}$
  - $\phi$  displays feature  $=f$ ,  $=>f$ , or  $f<= \in \text{Selectors}$



merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees

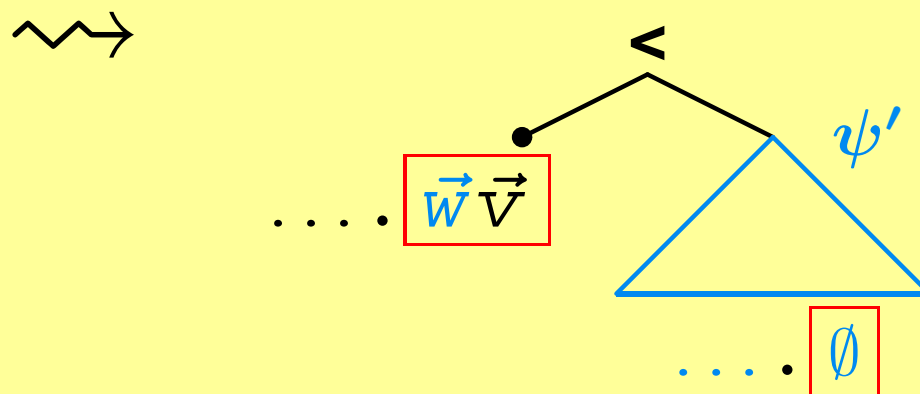
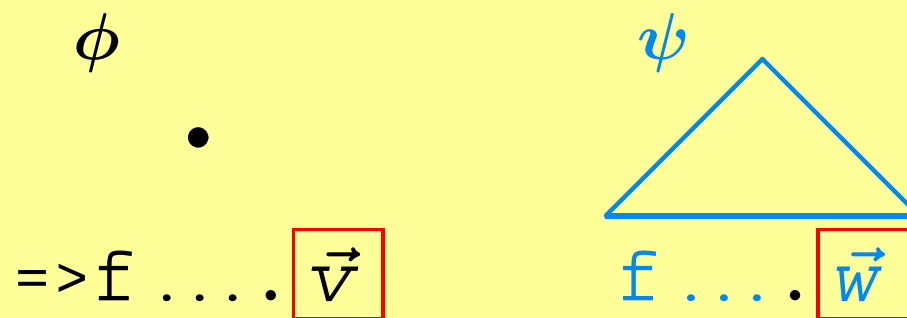


selecting  $\phi$  simple



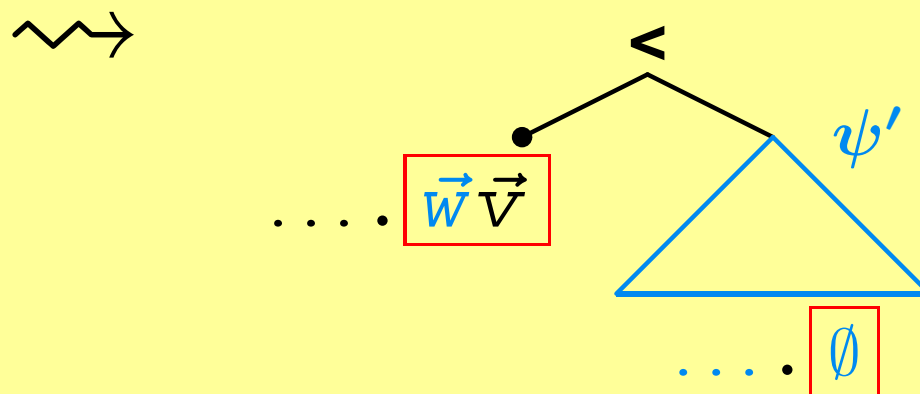
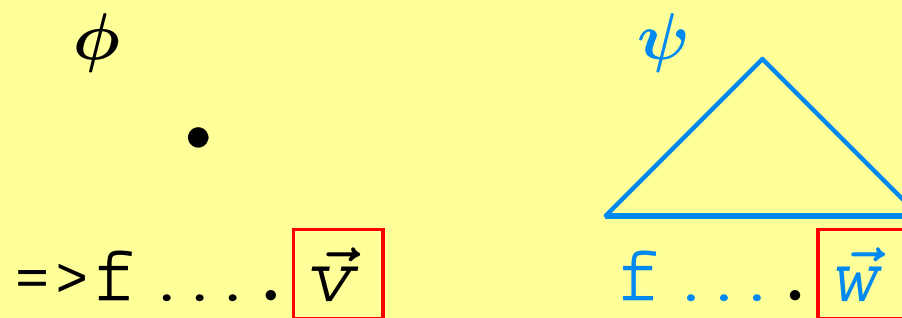
selecting  $\phi$  complex

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees



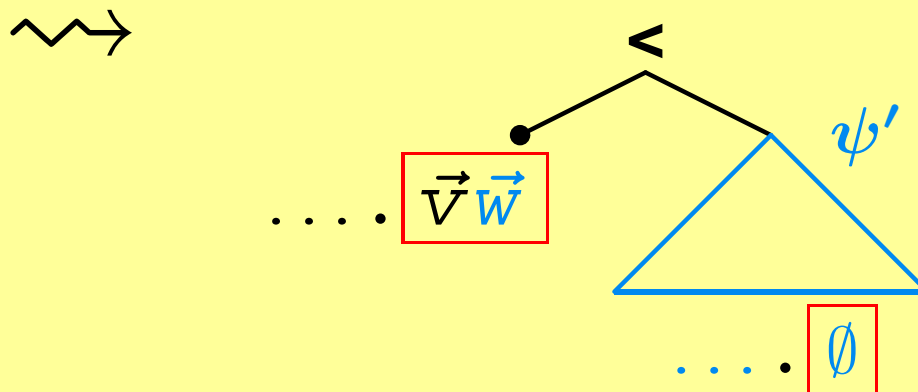
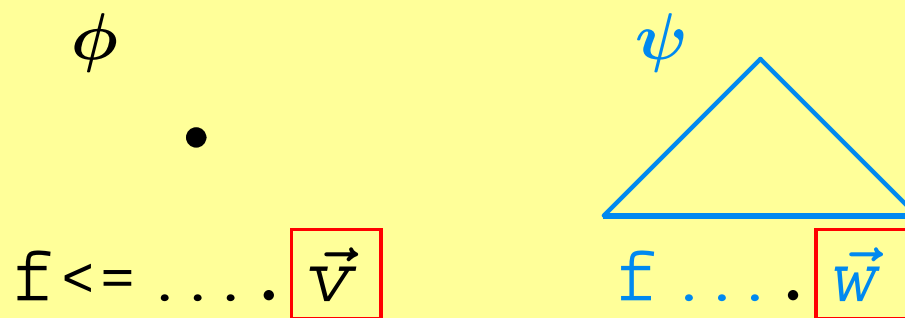
selecting  $\phi$  simple , head-incorporation left

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees (HMC)



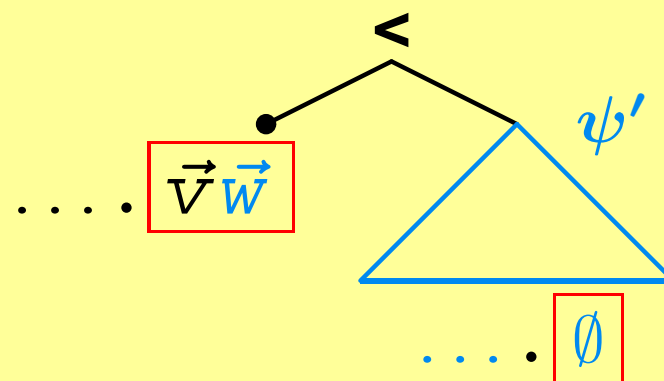
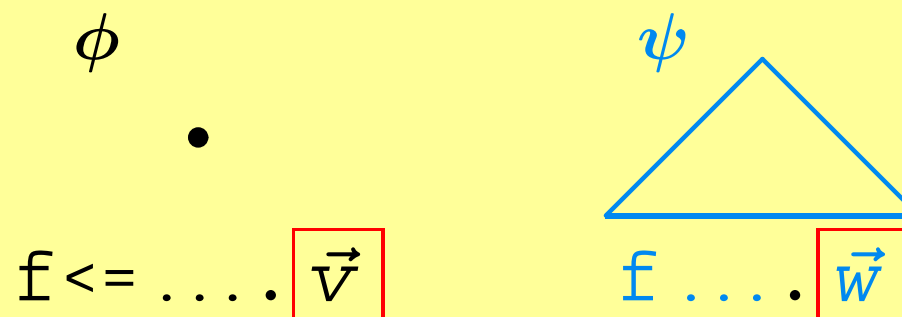
selecting  $\phi$  simple , head-incorporation left

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees



selecting  $\phi$  simple , head-incorporation right

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees (HMC)



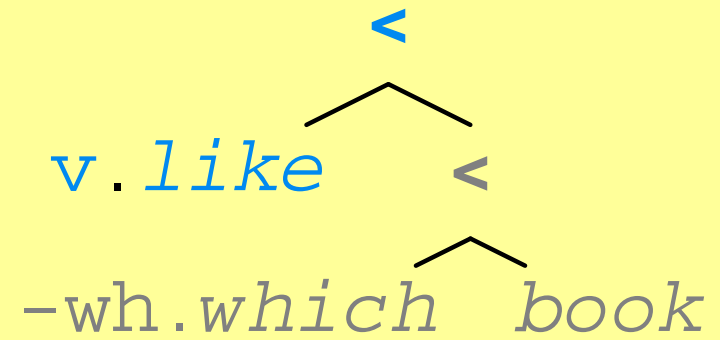
selecting  $\phi$  simple , head-incorporation right

merge

(head-incorporation left)

=>v . =d . i . -s

+

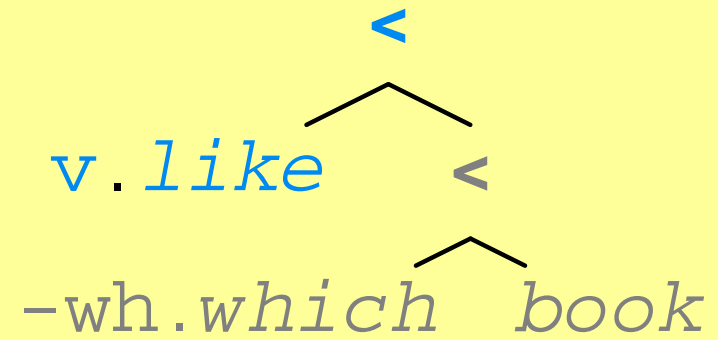


merge

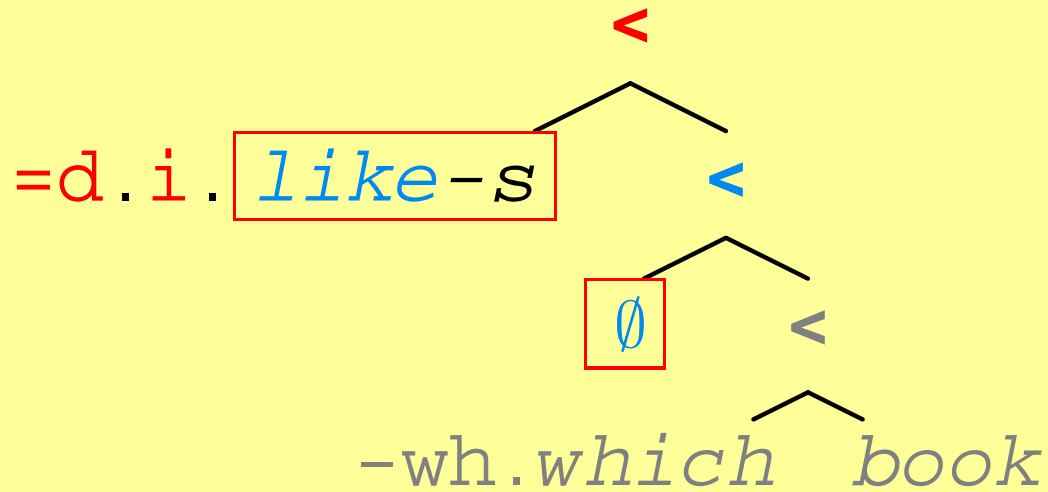
(head-incorporation left)

=>v . =d . i . -s

+



≈→

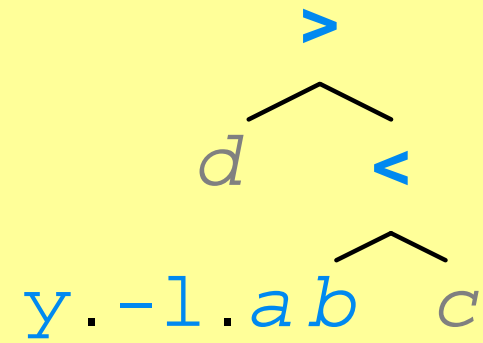


merge

(head-incorporation right)

$y \leq .x.a$

+



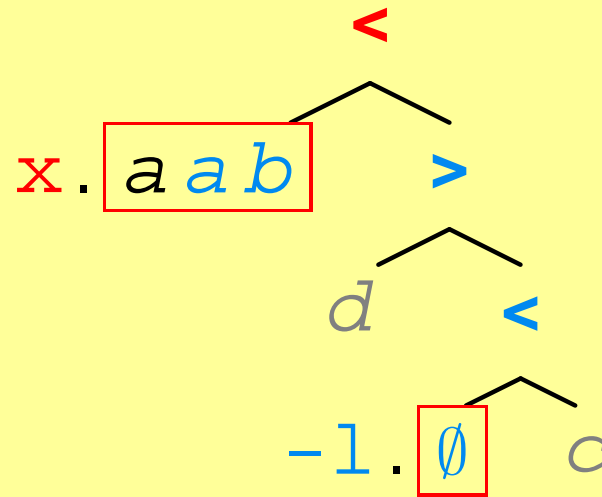
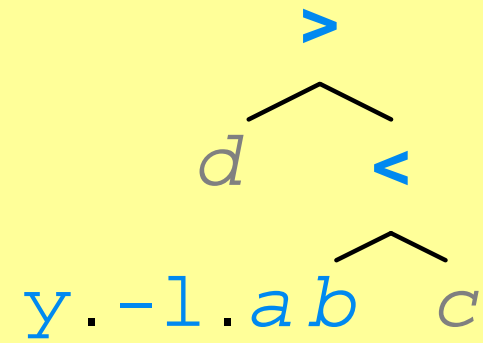


merge

(head-incorporation right)

$y \leftarrow .x.a$

+



# Minimalist grammars

$$G = \langle \text{Features}, \text{Lexicon}, \Omega, c \rangle$$

- Features = SynFeatures  $\cup$  Vocabulary [features]

$$\text{SynFeatures} = \text{Base} \cup \text{Selectors} \cup \text{Licensees} \cup \text{Licensors}$$

$$x = x, \Rightarrow x, x \leq \quad -x \quad +\underline{x}, +x$$

- Lexicon a finite set of simple expressions [lexicon]

- $\Omega = \{ \text{merge}, \text{move}, \text{agree} \}$  [structure building functions]

- $c \in \text{Base}$  [distinguished category]

The **closure** of  $G$  [ $\text{Closure}(G)$ ] :  $\iff$

closure of the lexicon under **finite applications of the functions in  $\Omega$** .

The **tree language** of  $G$  [ $T(G)$ ] :  $\iff$

trees in the closure with essentially **no unchecked syntactic features**  
— only head-label contains exactly one unchecked instance of  $c$ .

The **string language** of  $G$  [ $L(G)$ ] :  $\iff$

**(terminal) yields** of the trees belonging to the tree language.

# A simple MG-lexicon

*n . book*

*=d . v . like*

*=v . =d . i . ∅*

*d . she*

*=n . d . -wh . which*

*=i . +wh . c . did*

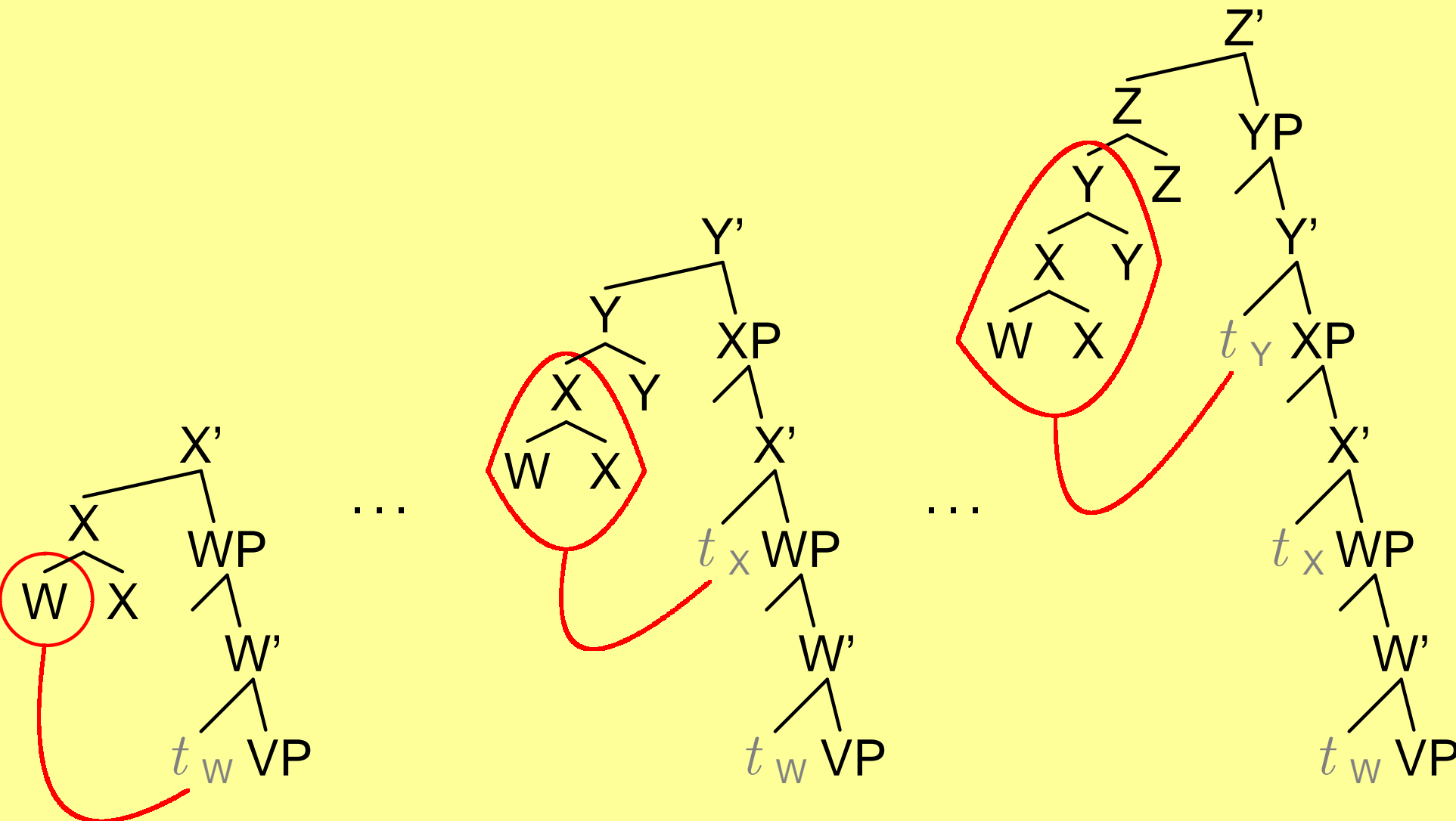
Vocabulary = { *book* , *did* , *like* , *she* , *which* }

- The implementation of head movement in MGs is in accordance with the HMC
  - demanding a moving head not to pass over the closest c-commanding head.

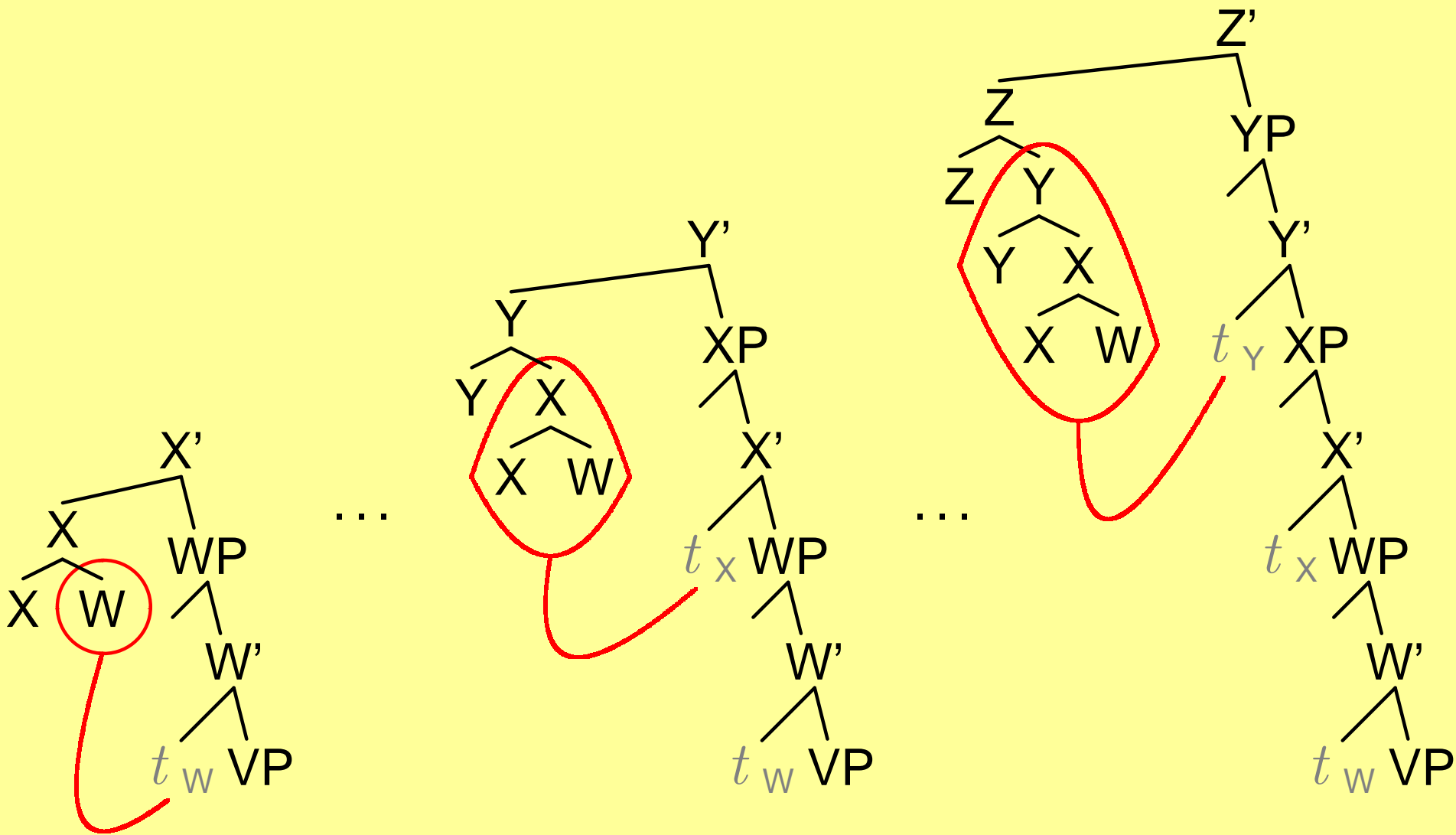
To put it differently,

whenever we are concerned with a case of successive head movement, i.e. recursive adjunction of a (complex) head to a higher head, it obeys strict cyclicity.

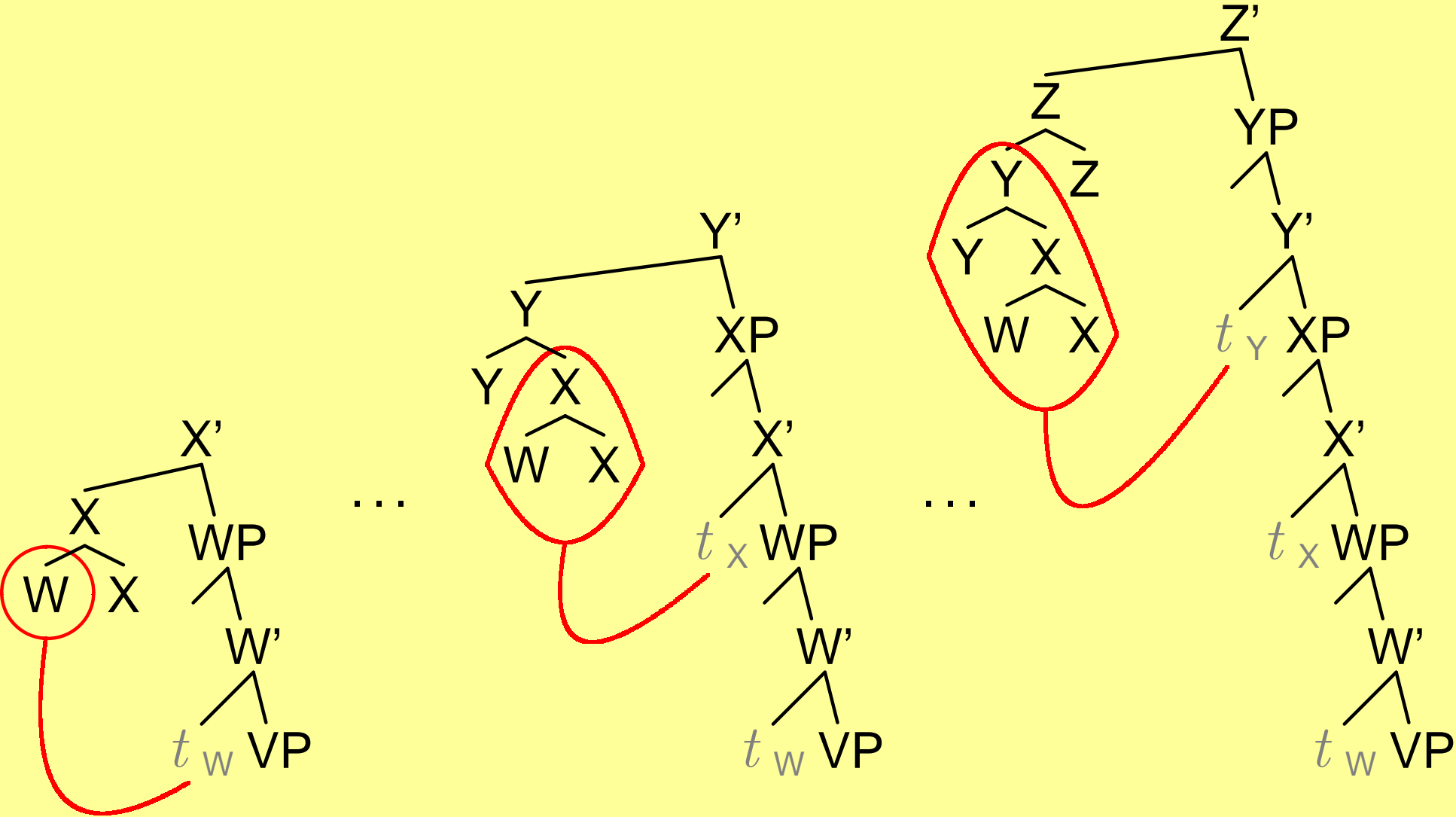
# Successive cyclic left head adjunction



# Successive cyclic right head adjunction



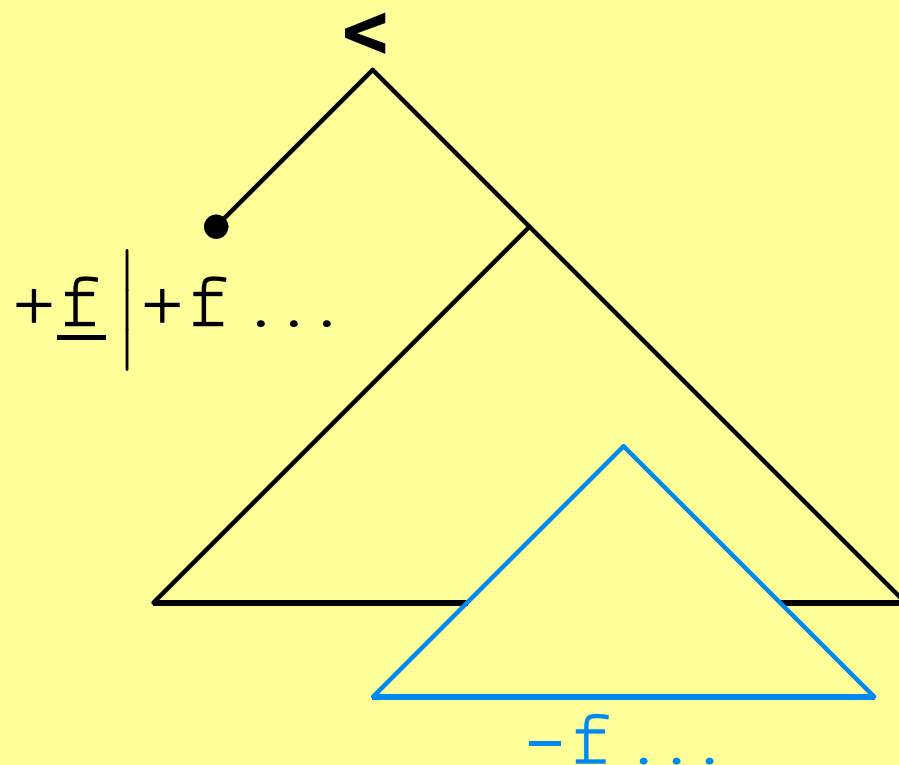
# Successive cyclic (mixed) head adjunction



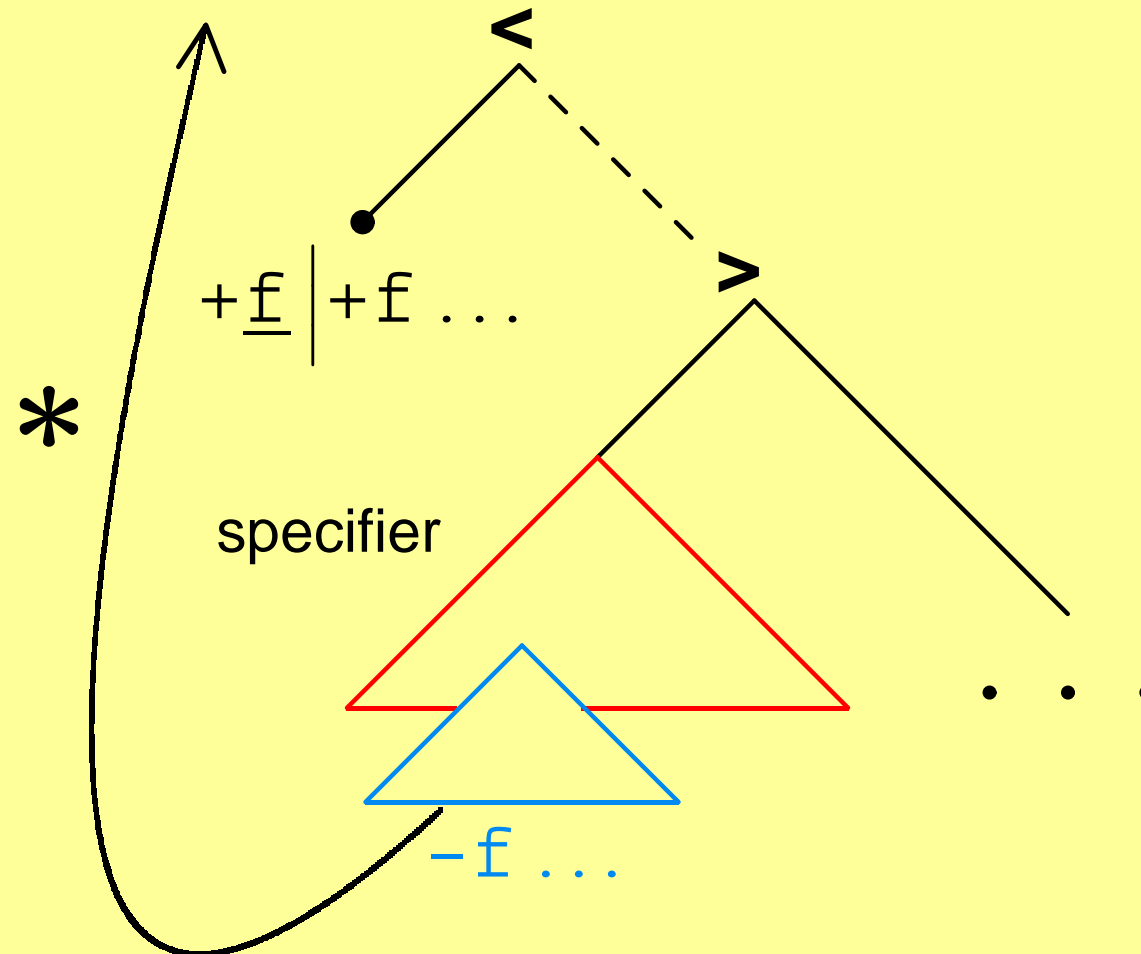


- The number of competing licensee features triggering a movement is (finitely) bounded by  $n$ .

In the strictest version  $n = 1$ , i.e., there is at most one maximal projection displaying a matching licensee feature:

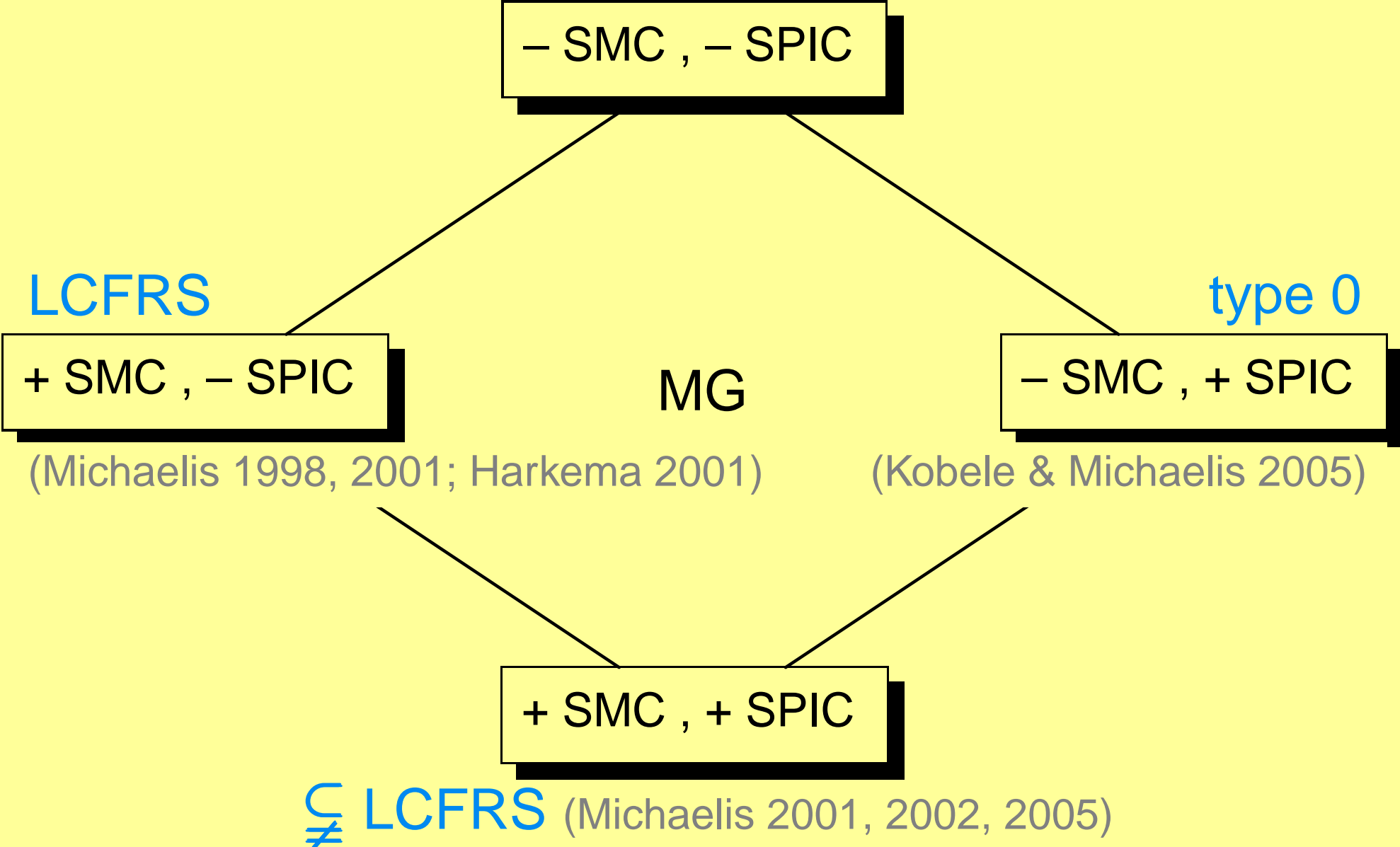


- Proper “extraction” from specifiers is blocked.



# SMC and SPIC — restricting the move-operator domain

## MELL-proof-search (Salvati 2008)

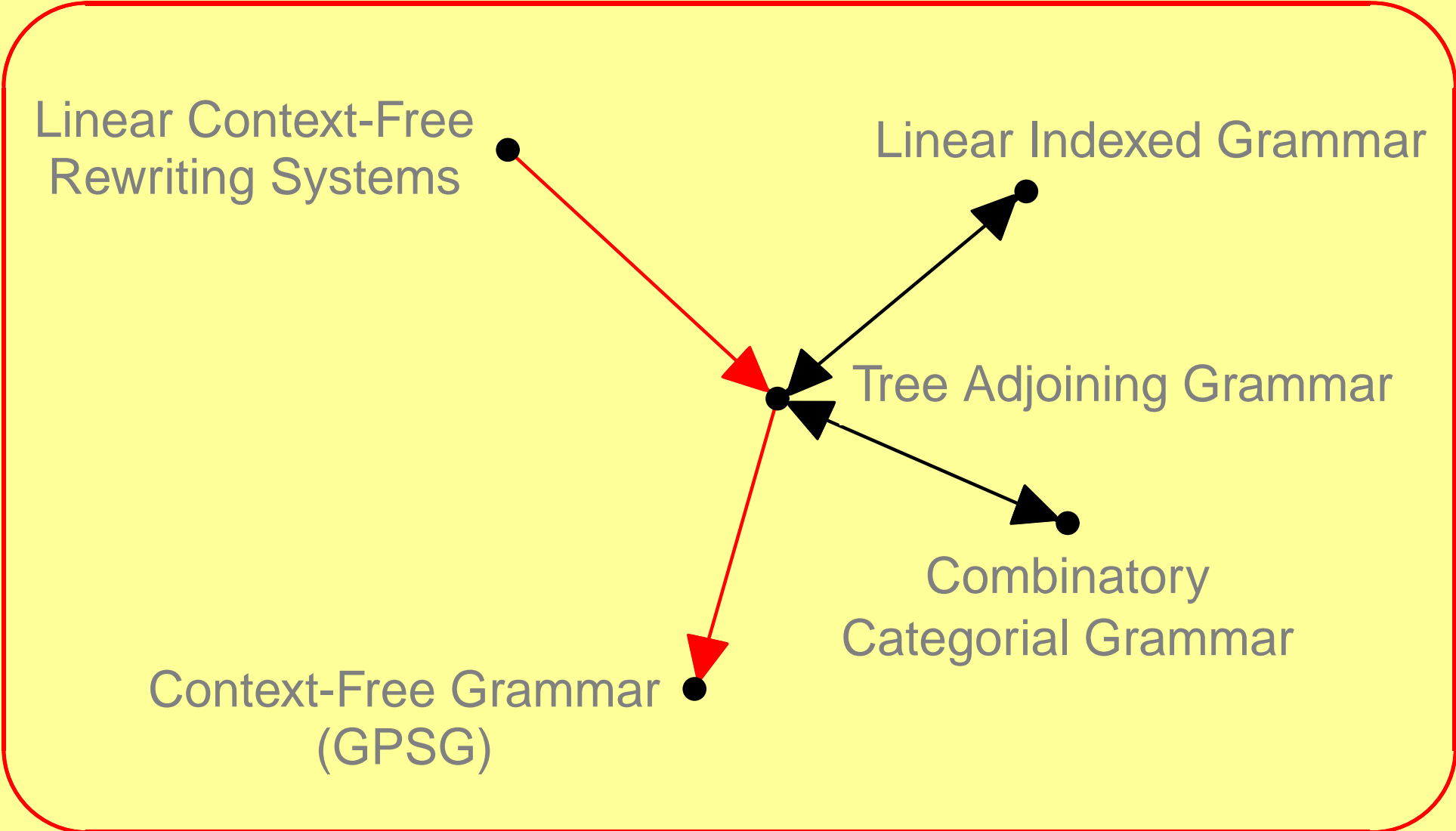


# MCSG-landscape

- Indexed Grammar

- Lexical Functional Grammar

**MCSG**

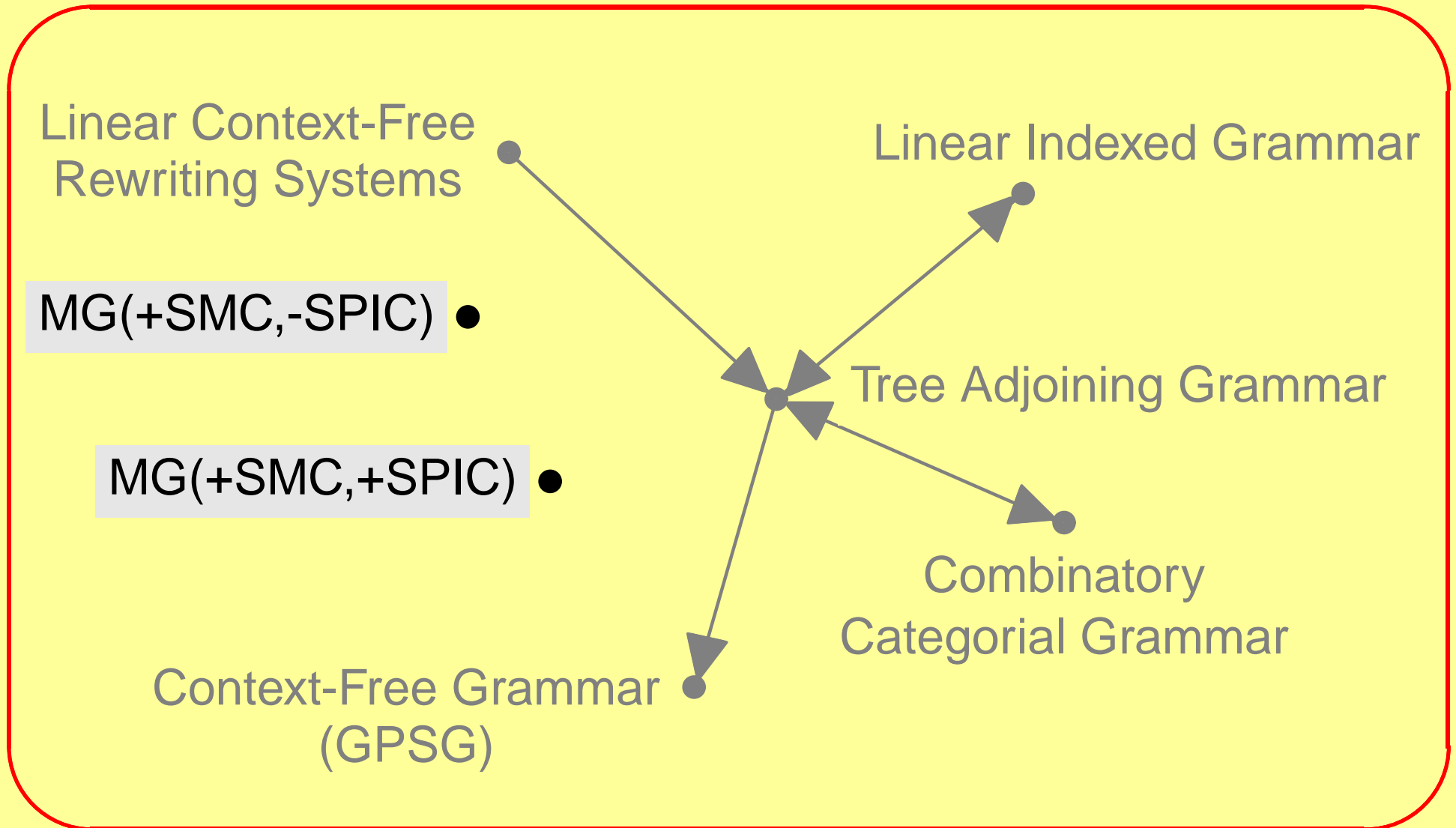


MG(-SMC,+/-SPIC) ●

● Indexed Grammar

● Lexical Functional Grammar

**MCSG**

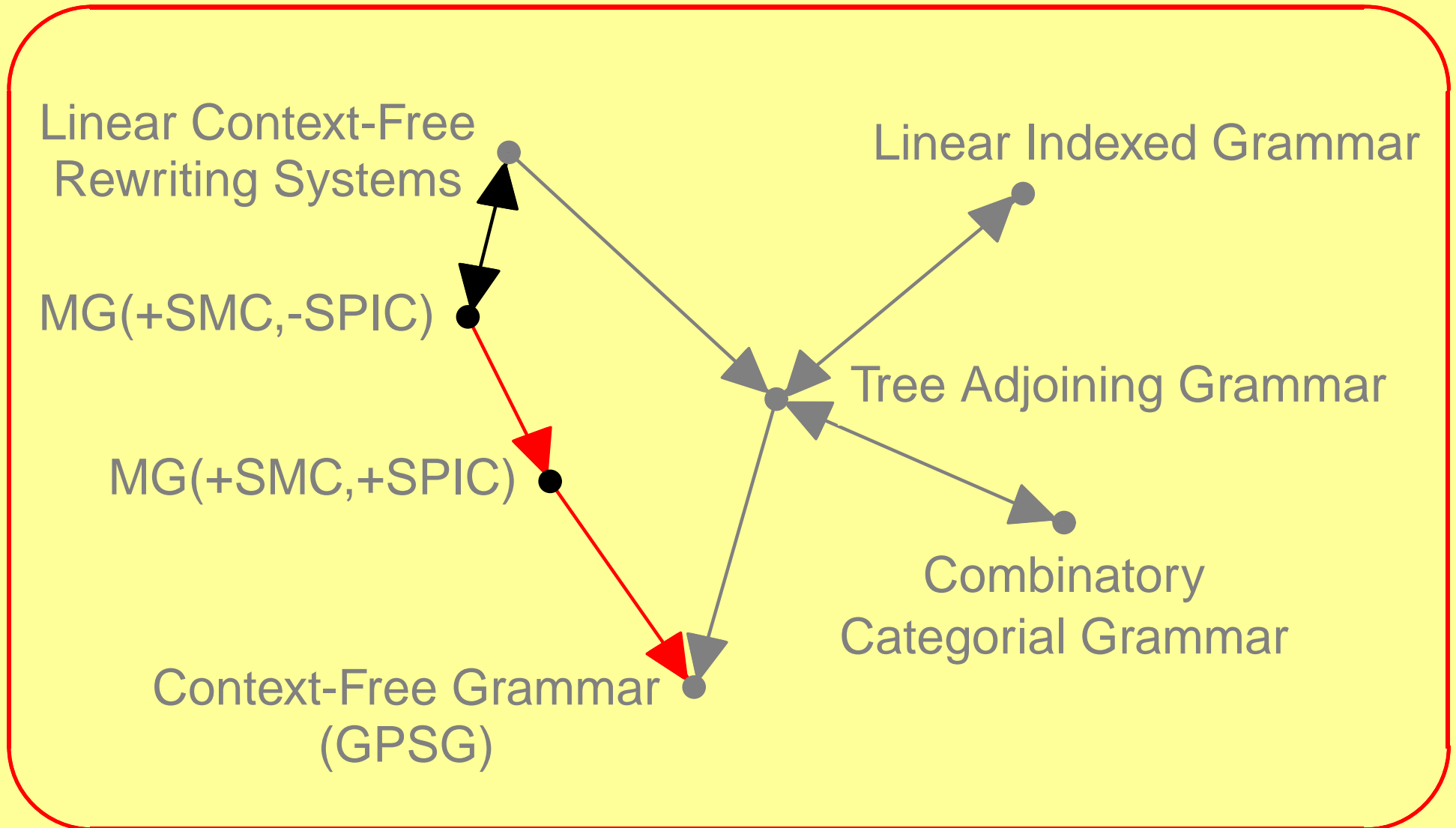


MG(-SMC,+/-SPIC) ●

● Indexed Grammar

● Lexical Functional Grammar

**MCSG**



- There are different types of syntactic features.

*(basic) categories:*      $x, y, z, \dots$      [ Base     ]

*(merge-) selectors:*      $=x(r), =y(r), =z(r), \dots$      [ Selectors, right ]

...

$=x(l), =y(l), =z(l), \dots$      [ Selectors, left ]

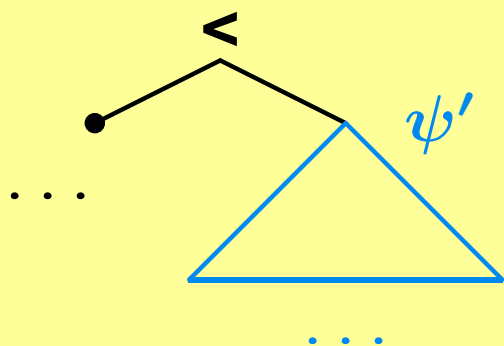
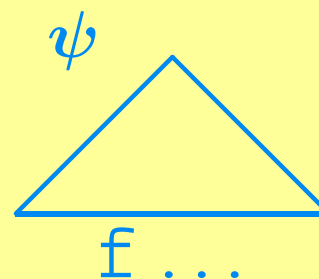
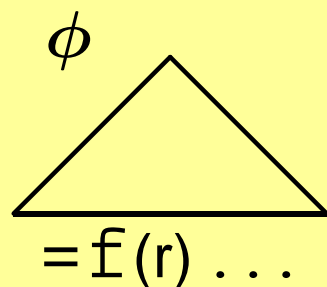
...

*(move-) licensees:*      $-x, -y, -z, \dots$      [ Licensees     ]

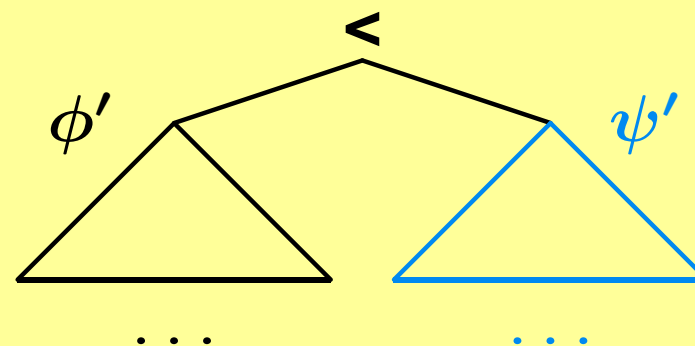
*(move-) licensors:*      $+x, +y, +z, \dots$      [ Licensors     ]

...

merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees



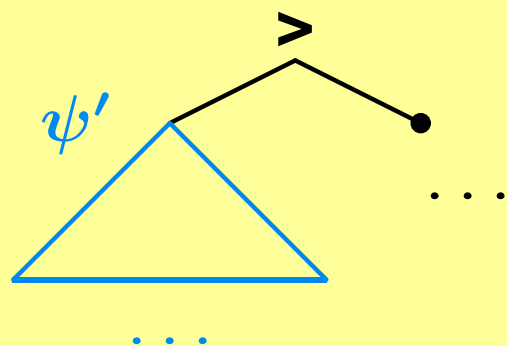
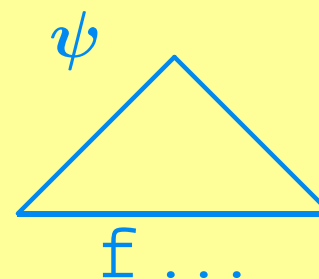
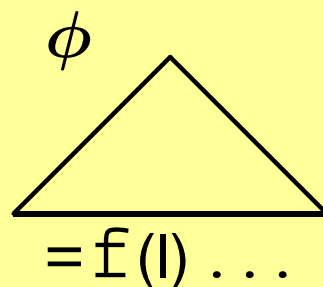
selecting  $\phi$  simple



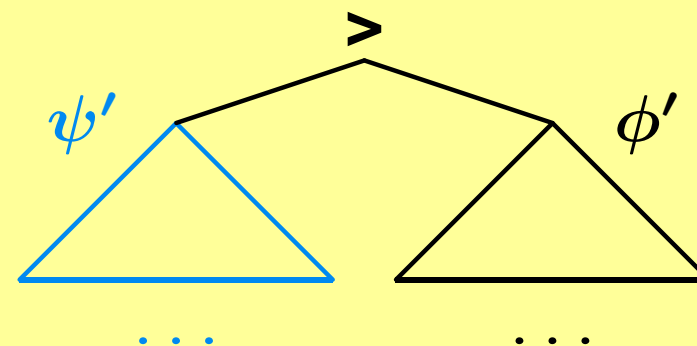
selecting  $\phi$  complex



merge : Trees  $\times$  Trees  $\xrightarrow{\text{part}}$  Trees



selecting  $\phi$  simple



selecting  $\phi$  complex

- There are different types of syntactic features.

*(basic) categories:*     $x, y, z, \dots$                     [ Base                    ]

*(merge-) selectors:*     $\dots$                                             [ Selectors                    ]

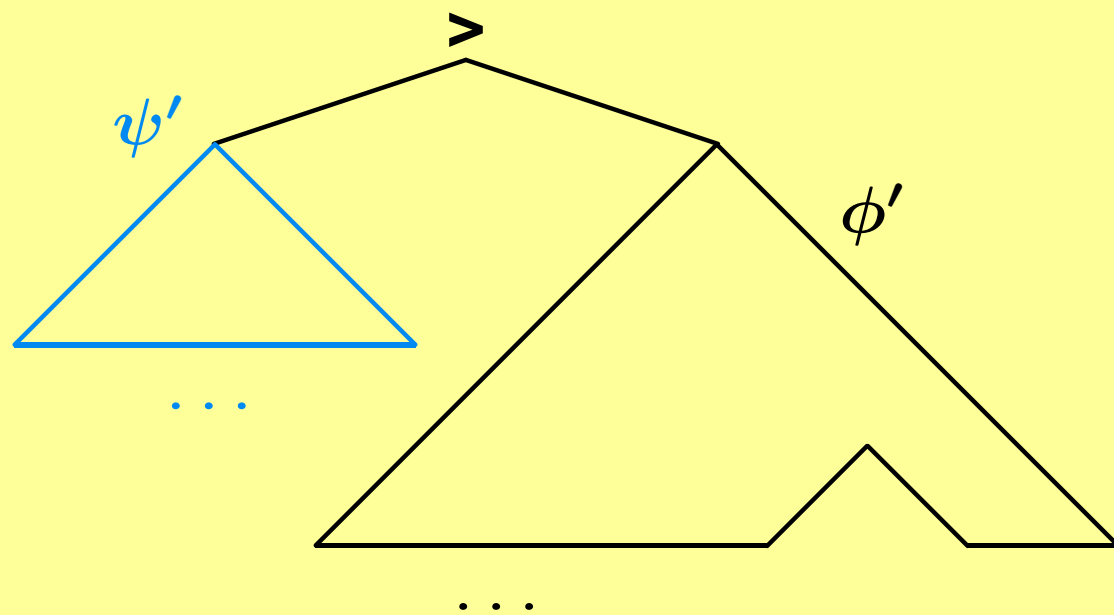
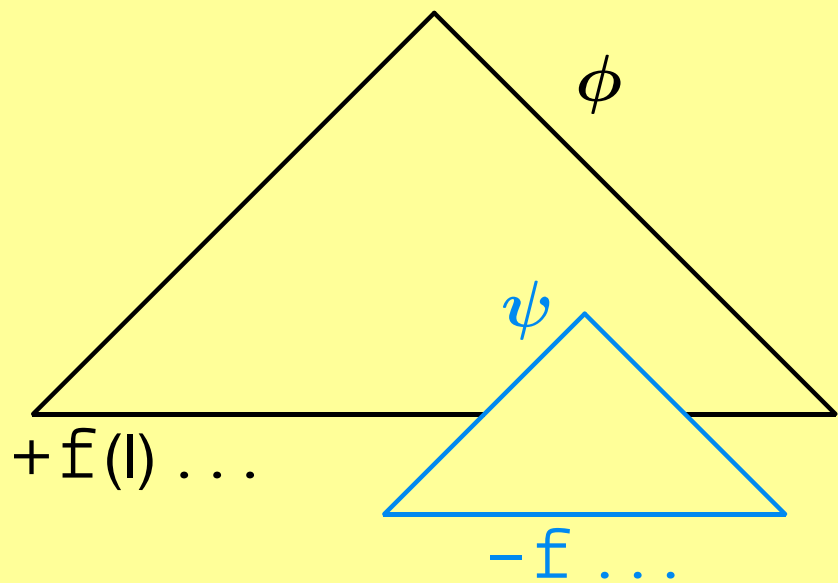
*(move-) licensees:*     $-x, -y, -z, \dots$                     [ Licensees                    ]

*(move-) licensors:*     $+x(l), +y(l), +z(l), \dots$                     [ Licensors, left                    ]

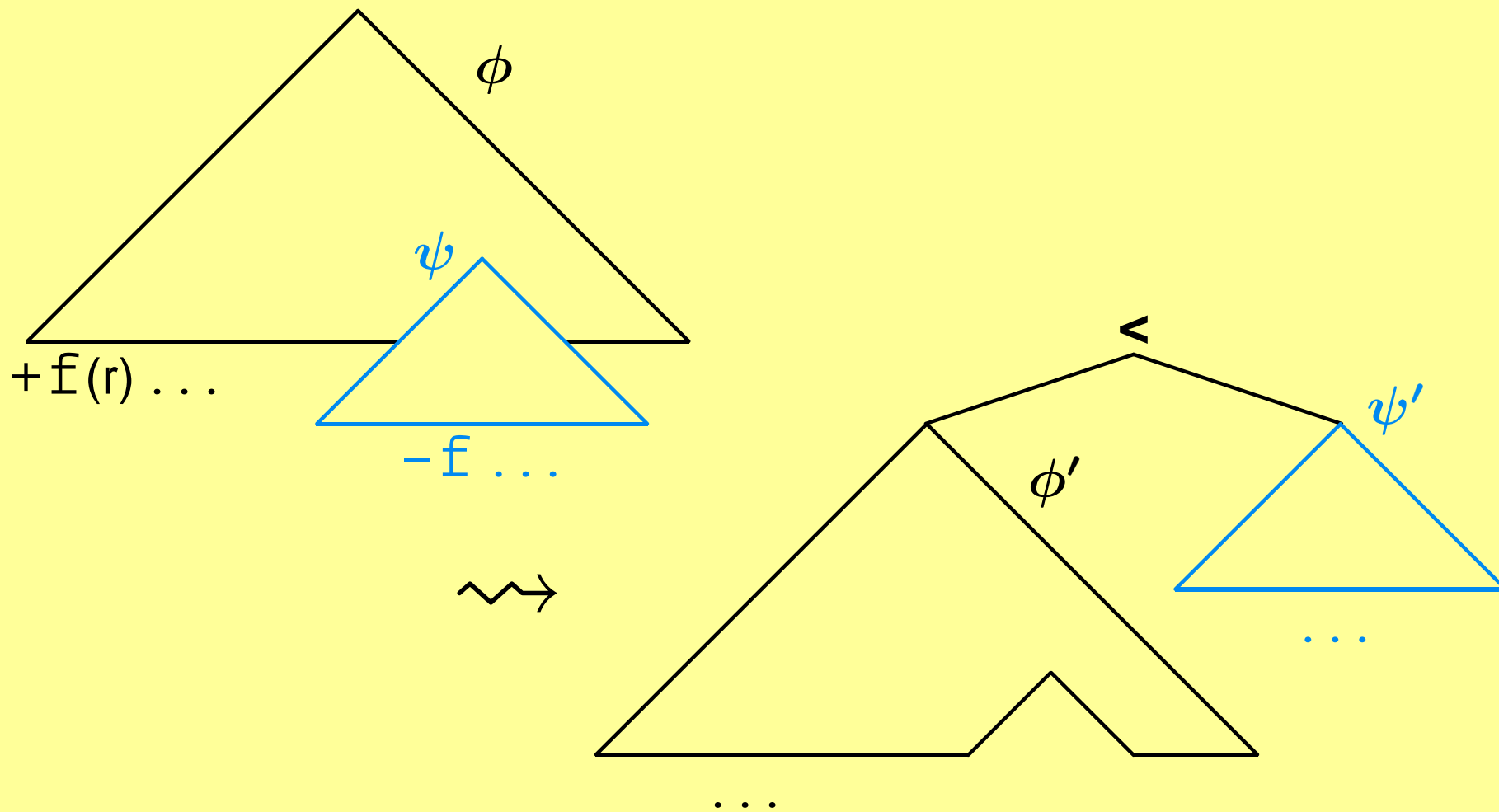
$+x(r), +y(r), +z(r), \dots$                     [ Licensors, right                    ]

$\dots$

move : Trees  $\xrightarrow{\text{part}}$  2Trees



move : Trees  $\xrightarrow{\text{part}}$  2Trees



- There are different types of syntactic features.

*(basic) categories:*     $x, y, z, \dots$                     [ Base                    ]

*(merge-) selectors:*     $\dots$                                                             [ Selectors                    ]

*(move-) licensees:*     $-x, -y, -z, \dots$                                                             [ Licensees                    ]

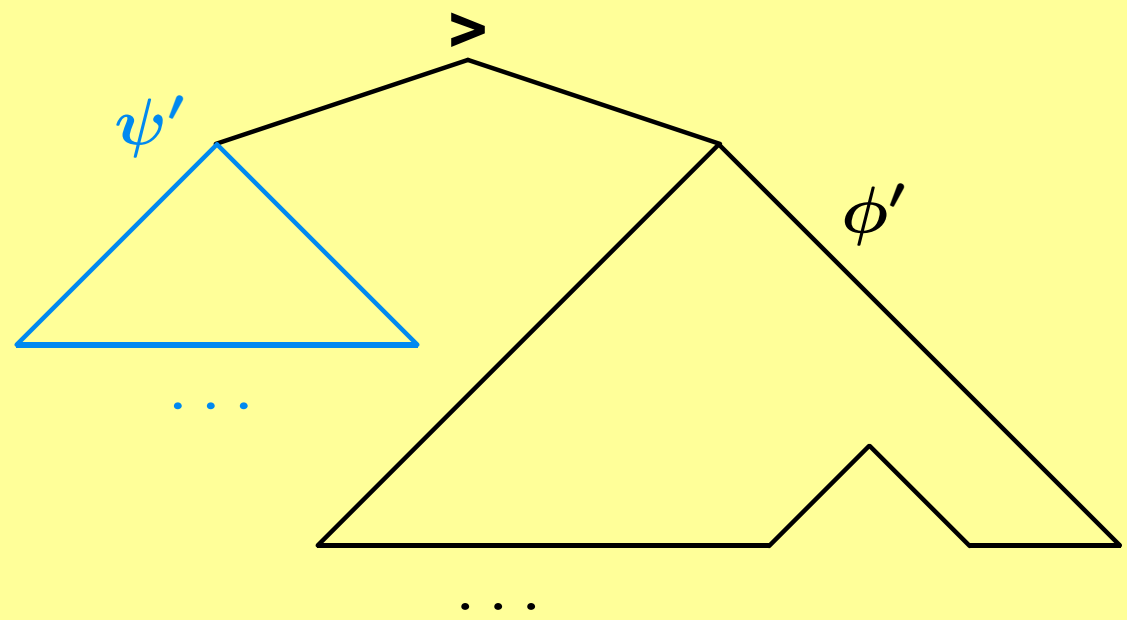
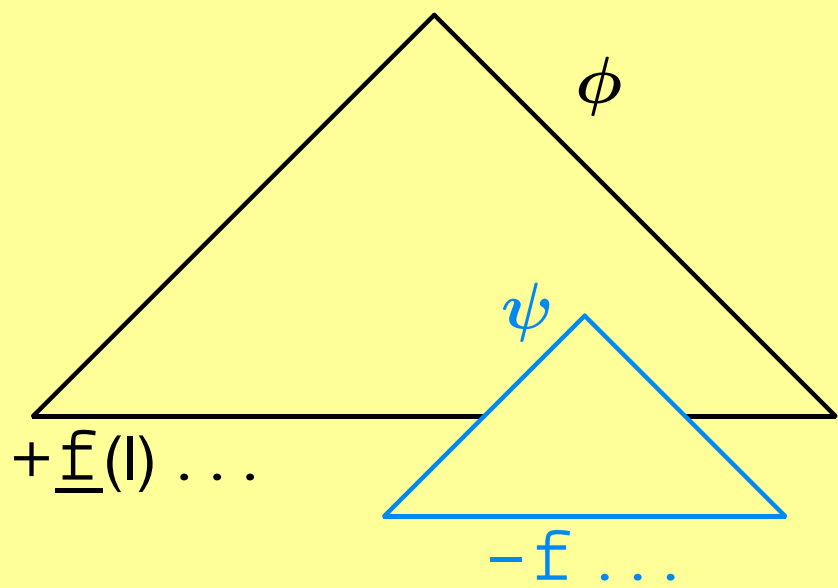
*(move-) licensors:*     $+x, +y, +z, \dots$                                                             [ Licensors, weak ]

$+\underline{x}(l), +\underline{y}(l), +\underline{z}(l), \dots$                                                             [ Licensors, strong ]

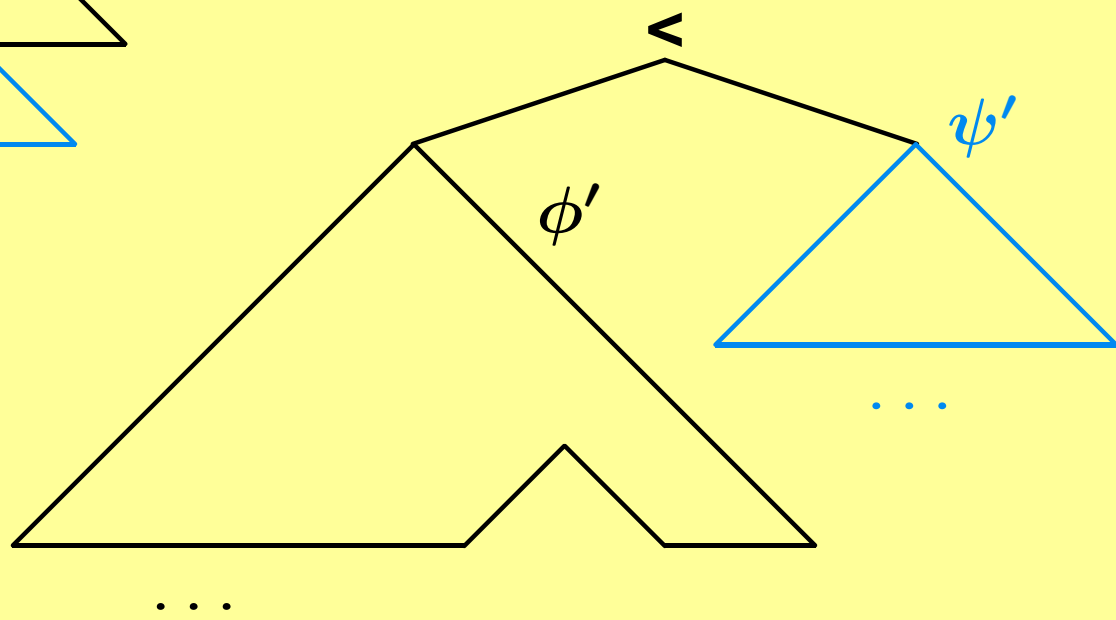
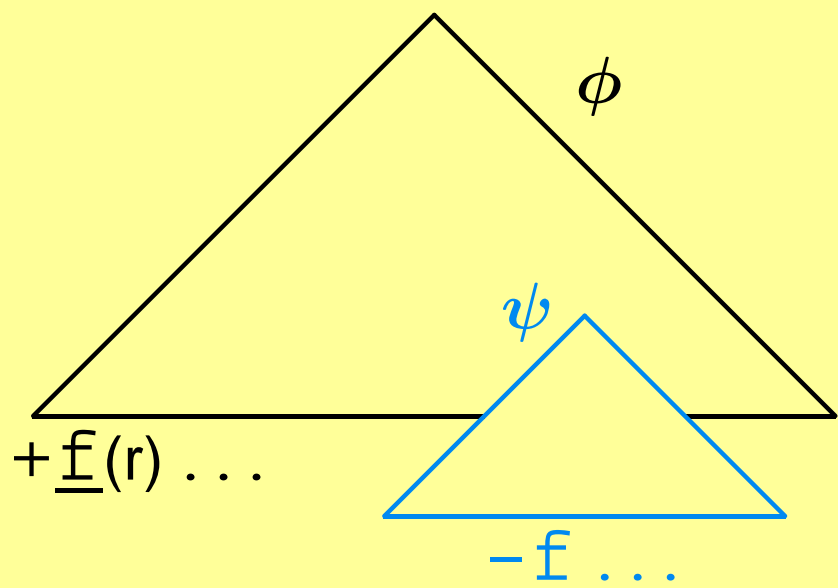
$+\underline{x}(r), +\underline{y}(r), +\underline{z}(r), \dots$

...

move : Trees  $\xrightarrow{\text{part}}$  2Trees



move : Trees  $\xrightarrow{\text{part}}$  2Trees



## Further outlook

- MGs can be extended with the operations **adjoin** and **scramble** involving two new types of syntactic features and a unilateral checking of their instantiations (Frey & Gärtner 2002, Gärtner & Michaelis 2003).
- If, in particular, categorial features are not deleted after checking, but marked as checked — and thus are still accessible — **acyclic (“late”) adjunction** can be defined as a subtype of adjoin.
- As to the interaction of the SMC and a corresponding **adjunct island constraint (AIC)**, the addition of the AIC has no effect, independently of the presence of the SMC.