Voronoi Languages

Equilibria in Cheap–Talk Games with Complex Types and Simple Signals

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[1]Institute for Mathematical Economics, Bielefeld and SFB 673 "Alignment in Communication"

joint work with Gerhard Jäger and Lars Koch

Motivation

Language

- sensation (Sinneseindruck) is complex: color, shape, size, location, temperature . . .
- only few words available

Job Market Signaling

Finance

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- cheap talk signaling game
- types ('Sinneseindrücke') are **complex**= from a continuum, $s \in \mathbb{R}^d$
- signals are **simple** = finitely many
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- hearer (receiver) interprets signal as a point in the type space
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Evolution of Languages: Formal Model

Communication in large population between two randomly matched players

- two roles for each player: speaker, hearer
- speaker gets a sensation (Sinneseindruck) $s \in S \subset \mathbb{R}^d$, convex, compact, nonempty interior
- sensations come with frequency $F(ds)$, atomless
- speaker chooses a word from a finite language $w \in W = \{w_1, \dots, w_n\}$
- hearer hears word w_j (so far, no errors here)
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- benchmark example: $l(x) = x^2$

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Efficient Languages

Cooperative approach

- players use a meta–language to find the best language
- minimize $E\ l\left(\|s - i_{w(s)}\|\right) = \int_S l\left(\|s - i_{w(s)}\|\right) F(ds)$ over measurable functions $w : S \to W$ and $i : W \to S$

Theorem

Efficient languages exist.

Remark

Proof requires analysis of optimal signaling systems w given some interpretation i; then essentially compactness and continuity ...

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Efficient Languages: Optimal Signaling

Best Choice of Words

- Suppose the hearer interprets word w_j as point i_j
- suppose sensation s is given
- which word is the best?
- choose the word w_j such that the distance from interpretation i_j to sensation s is minimal
- $w^* = \arg\min \{||s - i_j|| : j = 1, \ldots, n\}$
- this leads to a **Voronoi tesselation**

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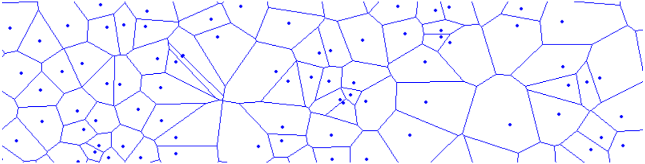
Voronoi Tesselations

Definition

Given distinct points $i_1, \ldots, i_n \in [0,1]^d$, the Voronoi tesselation assigns to (almost all) points $s \in [0,1]^d$ the unique closest point i_j to s. The convex set

$$C_j = \left\{ s \in [0,1]^d : \|s - i_j\| = \min_{k=1,\ldots,n} \|s - i_k\| \right\}$$

is called the Voronoi cell for i_j



Strict Nash Equilibria: Speaker

Best Choice of Words

as for optimal languages: choose the Voronoi tesselation corresponding to the hearer's interpretations i_j!

Strict Nash Equilibria: Hearer

- In strict Nash equilibrium, speaker uses a Voronoi tesselation
- what is the best interpretation of a Voronoi tesselation?
- we can identify words w_j with cells C_j
- given a cell C_j, find the point $i \in C_j$ that has the lowest average distance to all other points
- minimize $E\left[l(s-i)|C_j\right]$ over $i \in C_j$
- solution: best (conditional) estimate of s given C_j
- quadratic loss:

$$i^* = E\left[s|C_j\right]$$

- best interpretation is the average sensation of the cell

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A Voronoi language consists of a Voronoi tesselation for the speaker and a best estimator interpretation for the hearer.

Theorem

Strict Nash equilibria are Voronoi languages with full vocabulary and vice versa.

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Speaker can say $w \in \{left, right\}$
uniform distribution

- Speaker chooses a threshold $\theta \in (0, 1)$
- says "left" if $s < \theta$, else "right", or vice versa
- Hearer interprets "left" as $i_1 = \theta/2$, "right" as $i_2 = (1 + \theta)/2$
- in equilibrium, i_1, i_2 must generate the Voronoi tesselation with boundary θ
- $(x_1 + x_2)/2 = \theta \Leftrightarrow \theta = 1/2$
- unique strict Nash equilibrium
- maximizes social welfare
- evolutionarily stable

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- Hearer interprets "left" as $i_1 = \theta/2$, "right" as $i_2 = (1 + \theta)/2$
- in equilibrium, i_1, i_2 must generate the Voronoi tesselation with boundary θ
- $(x_1 + x_2)/2 = \theta \Leftrightarrow \theta = 1/2$
- unique strict Nash equilibrium
- maximizes social welfare
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Case Study: $d = 1$, quadratic loss, Two Words

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Case Study: $d = 2$, square, quadratic loss, Two Words

Speaker can say $w \in \{left, right\}$
uniform distribution

- Voronoi tesselations correspond to trapezoids
- there are only three (!) Voronoi languages (up to symmetry)
- only two with full vocabulary
  - left and right rectangle
  - left and right triangle
  - no language
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Case Study: $d = 2$, square, quadratic loss, 4^k Words

Speaker can say $w \in \{1, \dots, 4^k\}$
Some Voronoi language partitions the square into 4^k squares of equal size.
For $k > 1$, "bee hives" (hexagonal partitions) are optimal and evolutionarily stable.

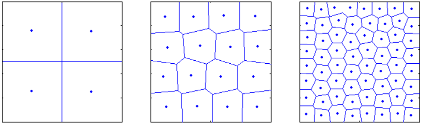

Figure: 4, 16, and 64 words, picture after 1000 iterations.

Case Study: $d = 2$, square, quadratic loss, 3 Words

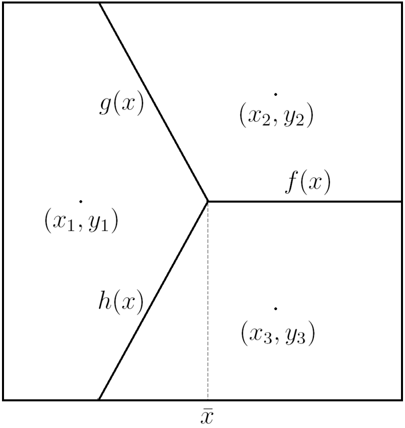

Figure: Mercedes star. **Tessels do not need to be of equal weight or shape.**

Evolutionary Dynamics in Banach Spaces

Foundations for Evolutionary Dynamics

Bomze, 1991, Oechssler, Riedel, 2001, 2002 , Cressman, Hofbauer, R., 2006, Heifetz, Shannon, Spiegel, 2007, Hofbauer, Oechssler, R., 2007

Lemma

Fundamental Law of Natural Selection holds true; payoff is a Lyapunov function (for payoff–monotone and BNN dynamics).

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Locally optimal languages are dynamically stable.

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Two words in a rectangle with unequal sides

- Two obvious Voronoi languages with full vocabulary
- Both are local minima of the loss function, hence stable
- only one is efficient

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Non–Uniform Sensations

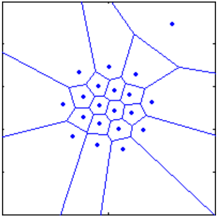

Figure: Tesselation of normally distributed types

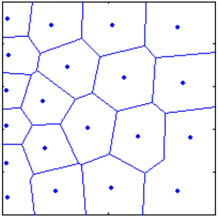

Figure: Tesselation of asymmetrically distributed types