# Other–Regarding Preferences in Competitive Markets

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October 19, 2007

# Empirical Evidence

## Experimental Observations

- Ultimatum Game: People give more than (subgame perfect) Nash predicts
- Envy, fairness concerns, altruism widely documented

## Two Quotations

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# Simple Models

## Models that can explain experimental data

- Fehr–Schmidt introduce fairness and envy: $U_i = m_i - \frac{\alpha_i}{I-1} \sum_k \max\{(m_k - m_i), 0\} - \frac{\beta_i}{I-1} \sum_k \max\{(m_i - m_k), 0\}$
- Charness–Rabin: $m_i + \frac{\beta_i}{I-1} \left[\delta_i \min\{m_1, \ldots, m_I\} + (1 - \delta_i) \sum_{j=1}^{I} m_j\right]$
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# The Claim and the Question

**Fehr–Schmidt:**

Fairness preferences matter in games, but they do not matter in competitive markets

- FS have a simple market model with one indivisible good
- Shaked: in such a simple model, any monotone utility gives the same outcome

**Our Program**

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- under what general circumstances is the claim true?

  Set up a general equilibrium model with preferences defined on allocations and study equilibria, core, and welfare.

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# A (too simple) Answer and a more Specific Question

- These are models with externalities
- externalities do matter in markets
- stop!
- specify the question:
- can one identify (hopefully large) classes of preferences for which the externalities do not matter in markets?

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# General Equilibrium Model

- $l = 1, \ldots, L$ commodities,
- $i = 1, \ldots, I$ agents with consumption set $X_i = \mathbb{R}^L_+$ and endowments $e_i \in \mathbb{R}^L_{++}$
- (production possible)
- preferences $\succeq^i$ are defined **over allocations, not consumption plans**
- $\succeq^i$ complete, transitive, continuous relation on $\prod_{i=1}^I X_i = \mathbb{R}^{IL}_+$
- utility $U_i$ exists
- we assume that $\succeq^i$ is strictly monotone and strictly convex in own consumption $x_i$
- not convex or monotone in general! discuss!

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# Separability

- Let prices $p \gg 0$, wealth $w > 0$ and consumption plans $(x_j)_{j \neq i} = x^{-i}$ be given
- demand of agent $i$:

$$d_i\left(p, w, x^{-i}\right) = \arg\max_{x_i | px_i \leq w} U_i\left(x_i, x^{-i}\right)$$

- demand function exists

**Definition**

We say that agent $i$ behaves as if selfish if her demand function $d_i\left(p, w, x^{-i}\right)$ does not depend on others' consumption plans $x^{-i}$.

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# Main Theorem

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*Agent $i$ behaves as if selfish if and only if her preferences can be represented by a separable utility function*

$$V_i\left(m_i(x_i), x^{-i}\right)$$

*where $m_i : X_i \to \mathbb{R}$ is the* **internal utility function**, *continuous, strictly monotone, strictly quasiconcave, and $V_i : D \subseteq \mathbb{R} \times \mathbb{R}_+^{(I-1)L} \to \mathbb{R}$ is an* **aggregator**, *increasing in own utility $m_i$.*

## Technical Assumption

Preferences are smooth enough such that demand is continuously differentiable. Needed for the "only if".

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# Proof

- if $U_i = V_i(m_i, x^{-i})$ and $V_i$ is increasing, then the demand with $U_i$ is the same as the demand with $m_i$
- the only if part requires more work
- Take demand $d_i(p, w)$, independent of $x^{-i}$
- Apply an integrability theorem (Hurwicz, Uzawa) to obtain the internal utility function $m_i$
- Here one needs:
- $D :=$ image $m_i$, set

$$V_i(\mu, x^{-i}) = U_i(x_i, x^{-i})$$

for some $x_i$ with $m(x_i) = \mu$

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# Easy Consequences of Separability

Due to separability, we can define an **internal economy** with preferences $m_i$

- Walrasian equilibria=internal Walrasian equilibria
- Equilibrium allocations are internally efficient, and we have a modified Second Welfare Theorem
- But we need not have efficiency!

## Example

Edgeworth's example. Take two agents, two commodities with the same internal utility and $U_i = m_1 + m_2$. Take as endowment an internally efficient allocation close to the edge of the box. Unique Walrasian equilibrium, but not efficient, as the rich agent would like to give endowment to the poor. Markets cannot make gifts!

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# Well–Being Externalities and Welfare

## Bergsonian Preferences

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  with $V_i$ increasing in $m_i$, but not necessarily in $m_j, j \neq i$.
- Cardinal model
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## Example

- Subsitutable Altruism $m_1 + \beta m_2$
- Complementary altruism $\min\{m_1, \beta m_2\}$
- Fehr–Schmidt: mixture of substitutable and complementary altruism

# Well–Being Externalities and Welfare

## Bergsonian Preferences

- $$U_i = V_i\left(m_1(x_1), \ldots, m_I(x_I)\right)$$
  with $V_i$ increasing in $m_i$, but not necessarily in $m_j, j \neq i$.
- Cardinal model
- Material well–being of others exerts an externality on your own well–being

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# Second Welfare Theorem

## Earlier Literature

- Winter 69: Altruism allows for Second Welfare Theorem
- extended to envy by Rader, Borglin
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Hateful Society: $U_i = m_i - 2m_j$ for two agents $i \neq j$. No consumption is efficient.

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In order to exclude the previous example, we introduce

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For $z \in \mathbb{R}^L_+ \setminus \{0\}$ and any allocation $x$, there exists a redistribution $(z_i)$ with $\sum z_i = z$ such that

$$U_i(x + z) > U_i(x)$$

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*Under social monotonicity, the set of Pareto ptima is included in the set of internal Pareto optima.*

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# Core Equivalence

## Definition of the Core

- $x$ given allocation, $C \subseteq \{1, \ldots, I\}$ coalition
- what does it mean that $C$ prefers a $C$–allocation $y$ over $x$?
- $y$ is feasible for $C$: $\sum_{i \in C}(y_i - e_i) = 0$
- for every $i \in C$

$$U_i\left(\left(y_j\right)_{j \in C}, \left(x_j\right)_{j \notin C}\right) > U_i(x)$$

**Theorem**

*Strengthen Social Monotonicity to Coalitions, then the core is a subset of the internal core.*

**Theorem (Debreu–Scarf–type Theorem)**

*Under replication, the core shrinks. In the limit, it is a subset (not necessarily equal) of the set of Walrasian equilibria.*

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# Empty Core

## Example

Take an economy with three agents.

$$U_1 = m_1 + 2m_2$$
$$U_2 = m_2 + 2m_3$$
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- Group Social Monotonicity holds true
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# Large Numbers and the First Welfare Theorem

- We know already that equilibria need not be efficient
- Is there any hope for the special preferences by Fehr–Schmidt and others if the economy grows large?

**Redistributional Loser Property (RLP)**

Let $x \neq y$ be two feasible allocations. Let $p \gg 0$ be a price vector. Define $w_i = px_i$ and $w_i' = py_i$. Let $j$ be the agent who loses most from going to $y$, i.e. $w_j - w_j' \geq w_k - w_k'$ for all $k = 1, \ldots, I$. Then

$$U_j(x) > U_j(y)$$

Who loses most in income, loses also in utility.

**Theorem**

*Under RLP, Walrasian equilibria are efficient.*

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*For*

- *altruism:* $U_i = m_i + \beta/(I-1)\sum_{j \neq i} m_j$
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- *Charness–Rabin:* $U_i = m_i + \frac{\beta_i}{I-1}\left[\delta_i \min\{m_1, \ldots, m_I\} + (1-\delta_i)\sum_{j=1}^{I} m_j\right]$

*(RLP) is satisfied for large $I$.*

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# Conclusion

Merci de votre attention !