Optimal Stopping under Multiple Priors and Variational Expectations in Continuous Time

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Workshop on Risk Measures and Robust Optimization in Finance,
National University of Singapore, November 2009

Outline

1. Optimal Stopping for Risk Measures

2. Optimal Stopping under g–expectations: General Theory
    - Snell Envelope and Optimal Rules
    - Relation with RBSDE
    - Worst–Case Priors and Duality
    - PDE Approach: Modified HJB Equation

3. Examples
    - Monotone Problems
    - Exotic Options
    - American Straddle

Optimal Stopping under Risk Measures

Problem Formulation

- Let $(\rho_t)_{t\in[0,T]}$ be a dynamic risk measure
- let $(X_t)_{t\in[0,T]}$ be a loss process
- choose a stopping time $\tau \le T$ that minimizes the risk

$$\rho_0(X_\tau)$$

or, more generally, $\tau \in [t, T]$ that minimizes

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Equivalent Formulations

By the representation theorems for (conditional) coherent or convex risk measures,

- choose a stopping time $\tau \leq T$ that maximizes the minimal expected gain
$$\min_{P \in \mathscr{P}} E^P(X_\tau)$$
for a suitable class of probability measures $\mathscr{P}$,
- Artzner, Delbaen, Eber, Heath, R., Detlefsen/Scandolo, Artzner et.al., Cheridito/Kupper
- ambiguity aversion, Gilboa/Schmeidler, Epstein/Schneider
- or, more generally,
$$\min_{P \in \Delta} E^P(X_\tau) + c(P)$$
for a penalty function c defined on the set Δ of all probability measures
- Föllmer/Schied, Fritelli, Rosazza Giannin, Föllmer/Penner
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Diffusion Setup

Probability space $(\Omega, \mathscr{F}, P_0)$ on which $B = \{B_t, 0 \leq t \leq T\}$ is a d-dimensional standard Brownian motion. $(\mathscr{F}_t)_{t\geq 0}$ the filtration generated by B augmented by the P_0–null sets.

Assumption

The payoff process $(X_t)_{t\geq 0}$ is a rightcontinuous adapted process that is leftcontinuous over stopping times. The payoff process $(X_t)_{t\geq 0}$ is also bounded in L^2:

$$E \sup_{t\in[0,T]} X_t^2 < +\infty \,. \tag{1}$$

κ–ambiguity as a reference model

Philosophical Points

- in diffusion models, volatility usually observable
- because quadratic variation is observable
- if we assume that the agent understands the sets of measure 1 and 0
- consequence: in our model — if we stick to P_0–equivalent measures — there can only be ambiguity about the drift

κ–ambiguity, "drift ambiguity"

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κ–ambiguity as g–expectation

g–expectations as standard time–consistent, nonlinear expectations

- Chen/Epstein: The nonlinear conditional expectation

$$\mathscr{E}_t(X) = \min_{P \in \mathscr{P}^\kappa} E^P \left[X | \mathscr{F}_t \right]$$

solves the BSDE $\mathscr{E}_T(X) = X$ and

$$-d\mathscr{E}_t(X) = -\kappa |Z_t| \, dt - Z_t \, dB_t$$

for some volatility (square–integrable, adapted process) Z

- more generally, let $g(t, z)$ be concave in z, then the g–expectation $\mathscr{E}_t(X)$ is the unique solution of the BSDE with driver g
- Coquet, Hu, Mémin, Peng: all time–consistent nonlinear expectations that are dominated by some κ–ambiguity are of this type

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Assumptions on our g–expectations

- From now on, our nonlinear expectations is a g–expectation dominated by κ–ambiguity (g is κ–Lipschitz in z and concave)
- From duality theory for g–expectations

$$\mathscr{E}_t(X) = \inf_{(\theta_t)} E^{\theta}\left[X + \int_t^T f(\theta_u)du|\mathscr{F}_t\right],$$

where $f(x) = \sup_z g(z) - xz$ is the convex dual of g (Delbaen's talk!)

Remark

$g(z) = 1/2z^2$ (entropic risk) is excluded here. However, for entropic risk, our problem is equivalent to

$$\min_{\tau} \log E \exp(-X_{\tau}),$$

a standard problem.

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Optimal Stopping under g–expectations: Theory

Our Problem

Let (X_t) be rightcontinuous, leftcontinuous over stopping times, adapted, with $\sup_{t \leq T} |X_t| \in L^2$. Find a stopping time $\tau \leq T$ that maximizes

$$\mathscr{E}_0(X_\tau)\,.$$

Optimal Stopping under g–expectations: General Structure

Let

$$V_t = \operatorname*{ess\,sup}_{\tau \geq t} \mathscr{E}_t(X_\tau)\,.$$

be the value function of our problem.

Theorem

1. *(V_t) is the smallest right–continuous g–supermartingale dominating (X_t);*
2. *$\tau^* = \inf\{t \geq 0 : V_t = X_t\}$ is an optimal stopping time;*
3. *the value function stopped at τ^*, $(V_{t\wedge\tau^*})$ is a g–martingale.*

Proof.

Our proof uses the properties of g–expectations like regularity, time–consistency, Fatou, etc. to mimic directly the classical proof (as, e.g., in Peskir,Shiryaev) with one additional topping: rightcontinous versions of g–supermartingales □

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Rightcontinuous Versions

Lemma

Let $(X_t)_{t\in[0,T]}$ *be a g–supermartingale with*

$$E[\sup_{t\in[0,T]} X_t^2] < +\infty\,.$$

Assume that the mapping

$$t \in [0,\, T] \mapsto \mathscr{E}_0\left(X_t\right)$$

is continuous. Then there exists an event $\Omega^* \subset \Omega$ *with* $P\left(\Omega^*\right) = 1$ *on which the (right–continuous!) process*

$$Y_t = X_{t+} = \lim_{s\downarrow t, s\in\mathbb{Q}} X_s \qquad (t \in [0,\, T]) \tag{2}$$

is well–defined. We have $Y_t = X_t$ *a.s. for all* $t \in [0,\, T]$, *and* $\left(Y_t\right)_{t\in[0,T]}$ *is also a g–supermartingale.*

Relation with Reflected BSDE

RBSDE (El Karoui, Kapoudjian, Pardoux, Peng, Quenez)

(Y, Z, K) solves the reflected BSDE with driver g, obstacle X, and terminal condition ξ if

- $Y \geq X$
- $Y_T = \xi$
- $-dY_t = g(t, Z_t)dt - Z_t dW_t + dK_t$
- K is increasing and grows only when $Y_t = X_t$

The value function of our optimal stopping problem solves the RBSDE!

Corollary

RBSDE have unique solutions.

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Worst–Case Priors

Drift ambiguity

- V is a g–supermartingale
- from the Doob–Meyer–Peng decomposition

$$-dV_t = g(t, Z_t)dt - Z_t dW_t + dA_t$$

  for some increasing process A
- $= -\kappa|Z_t|dt - Z_t dW_t + dA_t$
- Girsanov: $= -Z_t dW_t^* + dA_t$ with kernel $\kappa \operatorname{sgn}(Z_t)$

Theorem (Duality for κ–ambiguity)

There exists a probability measure $P^* \in \mathscr{P}^\kappa$ such that $V_t = \operatorname{ess\,sup}_{\tau \geq t} \mathscr{E}_t\left(X_\tau\right) = \operatorname{ess\,sup}_{\tau \geq t} E^*\left[X_\tau | \mathscr{F}_t\right]$. *In particular:*

$$\max_{\tau} \min_{P \in \mathscr{P}^\kappa} = \min_{P \in \mathscr{P}^\kappa} \max_{\tau}$$

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$$-dV_t = g(t, Z_t)dt - Z_t dW_t + dA_t$$

  for some increasing process A
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- Girsanov: $= -Z_t dW_t^* + dA_t$ with kernel $\kappa \, \mathrm{sgn}(Z_t)$

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There exists a probability measure $P^* \in \mathscr{P}^\kappa$ such that $V_t = \operatorname{ess\,sup}_{\tau \geq t} \mathscr{E}_t(X_\tau) = \operatorname{ess\,sup}_{\tau \geq t} E^*[X_\tau | \mathscr{F}_t]$. In particular:

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General Duality

Let

$$f(t,\theta) = \sup_{z\in\mathbb{R}^d} g(t,z) - z\theta$$

be the convex dual of g. Choose an adapted process θ that solves

$$f(t,\theta_t) = g(t,Z_t) - Z_t\theta_t\,.$$

Theorem (General Duality)

We have

$$V_t = \operatorname*{ess\,sup}_{\tau\geq t} E^\theta\left[X_\tau + \int_t^\tau f(s,\theta_s)ds|\mathscr{F}_t\right]\,.$$

Markov Models

- the state variable S solves a *forward SDE*, e.g.

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t, \quad S_0 = 1\,.$$

- Let

$$\mathscr{L} = \mu(x)\frac{\partial}{\partial x} + \sigma^2(x)\frac{\partial^2}{\partial x^2}$$

  be the infinitesimal generator of S.

- By Itô's formula, $v(t, S_t)$ is a martingale if

$$v_t(t,x) + \mathscr{L}v(t,x) = 0 \tag{3}$$

- similarly, $v(t, S_t)$ is a *g–martingale* if

$$v_t(t,x) + \mathscr{L}v(t,x) + \mathbf{g}(\mathbf{t}, \mathbf{v_x}(\mathbf{t},\mathbf{x})\sigma(\mathbf{x})) = 0 \tag{4}$$

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PDE Approach: A Modified HJB Equation

Theorem (Verification Theorem)

Let v be a viscosity solution of the g–HJB equation

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Monotone problems

Model from now on

- κ–ambiguity (Epstein, Chen)
- the drift of Brownian motion is uncertain, i.e. we only know that our asset price has a drift μ_t with $|\mu_t| \leq \kappa$,
- $dS_t = \sigma S_t dW_t$ under our reference measure P_0 (drift 0)
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- the function $g(x) = \kappa|x|$ captures the corresponding multiple prior model
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Monotone problems ctd.

Guess

- If the payoff function if monotone increasing in the asset price, the worst case measure assigns drift $\mu_t = -\kappa$ to the asset
- compute the (classical) value function with drift $-\kappa$ and check our new HJB
- $$v_t + \mathscr{L} v - \kappa|\sigma x v_x| = 0 \tag{6}$$
- as $v_x > 0$ in monotone problems, back to the original HJB equation, bingo!

Remark

This covers and extends the results of Nishimura, Ozaki on irreversible investment and search problems under ambiguity. (JET 2004, 2007)

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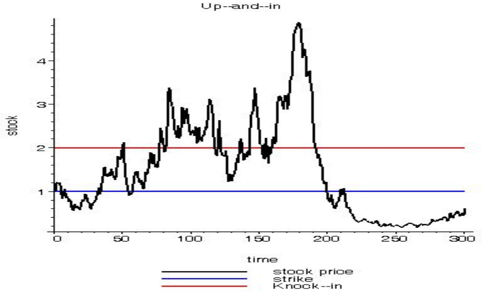
Barrier Options (with Jörg Vorbrink)

We consider up–and–in American Put Options. Let (S_t) be the price of a risky asset.

- there is a strike $K > 0$
- and a knock–out barrier $B > K$
- the option pays off $(K - S_\tau)^+$ if exercised at τ
- but only if it has been knocked in before
- knock-in occurs whenever $S_t > B$



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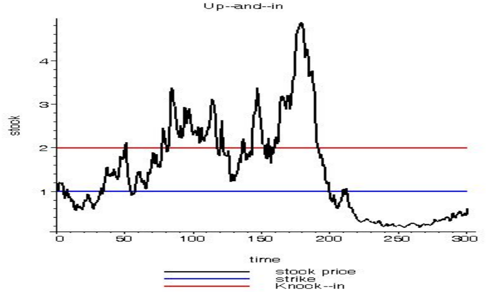

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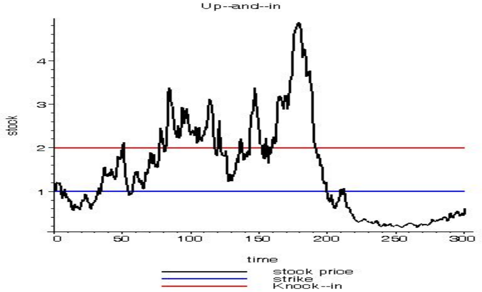

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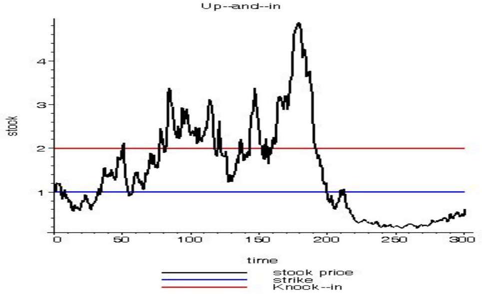

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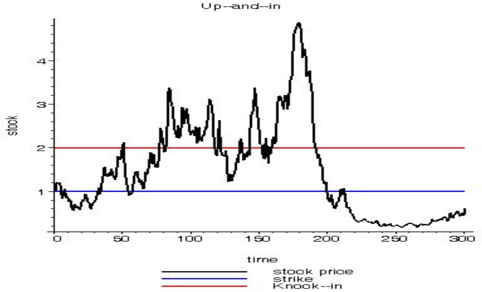

Optimal Stopping of Barrier Options

Worst–Case Prior

- before hitting the knock–in barrier, the worst–case prior has maximal negative drift
- afterwards, the worst case is maximal positive drift (American Put)
- investor presumes correlated returns in an iid setting!

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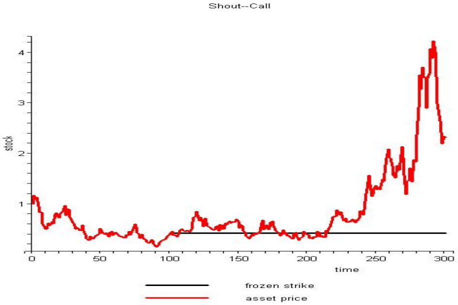
Shout Options (with Tatjana Chudjakow)

We consider Shout Options (of the Put type). Let (S_t) be the price of a risky asset.

- the buyer has the right to shout once in the interval $[0, T]$
- when she shouts at τ, the strike of a Put with maturity T is fixed at $K = S_\tau$
- the option pays off $(S_\tau - S_T)^+$ at maturity T



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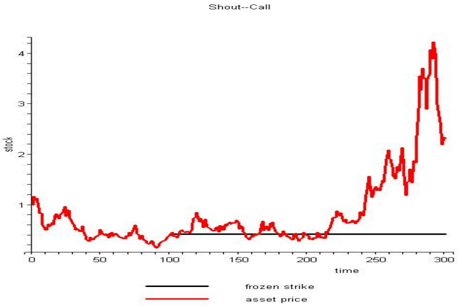

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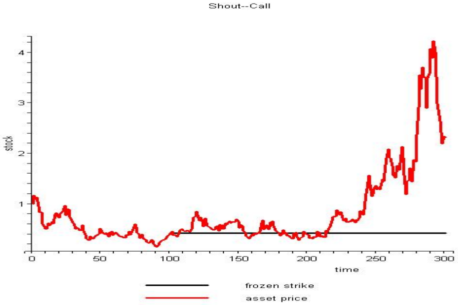

Optimal Stopping of Shout Options

Worst–Case Prior

- after fixing the strike at time τ, we have a European Put
- worst-case: maximal positive drift
- value at τ is thus

$$E^{\overline{P}}[(S_T - S_\tau)^+ | \mathscr{F}_\tau] = S_\tau E^{\overline{P}}[(S_T/S_\tau - 1)^+ | \mathscr{F}_\tau] = S_\tau E^{\overline{P}}[(S_T/S_\tau - 1)^+]$$

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American Straddle in the Bachelier Model for Drift Ambiguity

Suppose we want to stop $X_t = W_t$ under κ–ambiguity for an interest rate $r > 0$, i.e.

$$\max_\tau \mathscr{E}(|X_\tau| e^{-r\tau}) \, .$$

Claim: under the worst–case measure P^*, the process X has dynamics

$$dX_t = -\operatorname{sgn}(X_t)dt + dW_t^*$$

for the P^*–Brownian motion W^*.

American Straddle in the Bachelier Model for Drift Ambiguity

g–HJB equation: in the continuation set

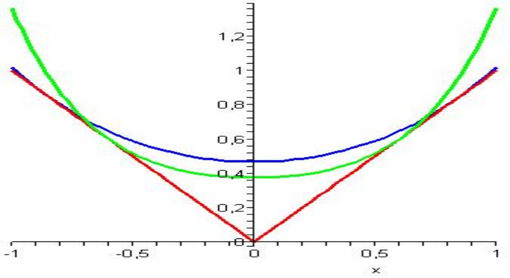
$$v_t + \frac{1}{2} v_{xx} - \kappa |v_x| = 0$$

Verification: solve the standard optimal stopping problem under P^*. There, the HJB equation reads

$$v_t + \frac{1}{2} v_{xx} - \kappa \operatorname{sgn}(x) v_x = 0$$

Show $\operatorname{sgn}(v_x) = \operatorname{sgn}(x)$, then this equation becomes the g–HJB equation and we are done.

American Straddle in the Bachelier Model for Drift Ambiguity



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