Intertemporal Equilibria with Knightian Uncertainty

Frank Riedel   Rose–Anne Dana

[1]Institute for Mathematical Economics
Bielefeld University

[2]CEREMADE
Université Paris–Dauphine

Workshop on General Equilibrium Theory, Krakow 2010

Outline

1. Motivation
2. Model
3. Efficiency and Equilibria in Bewley Economies
4. Non–Insurance of Knightian Uncertainty: A Case Study

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(X)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
  - agents move from status quo ω^i to x^i iff $E^P u(x^i) \geq E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
  - agents move from status quo ω^i to x^i iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
  - agents move from status quo ω^i to x^i iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
  - agents move from status quo ω^i to x^i iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$
- Question: what are the consequences for markets in dynamic settings?

Motivation

Two Kinds of Multiple Priors, or Worst–case versus stress–testing

- Gilboa–Schmeidler weaken independence axiom and introduce pessimism
  - agents use multiple priors $P \in \mathscr{P}^i$
  - complete preferences, worst–case approach: $U(X) = \min_{P \in \mathscr{P}^i} E^P u(x)$
- Bewley introduces multiple priors, but removes completeness
- and adds inertia (=stress-testing):
  - agents move from status quo ω^i to x^i iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$
- Question: what are the consequences for markets in dynamic settings?

A Related Motivation: Controling Finance

Regulation of Financial Markets

- stress testing: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
  - Convex Risk Measures ( ) ⇔ Variational Preferences

A Related Motivation: Controling Finance

Regulation of Financial Markets

- stress testing: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
  - Convex Risk Measures (Föllmer, Schied, Frittelli, Gianin) ⇔ Variational Preferences

A Related Motivation: Controling Finance

Regulation of Financial Markets

- stress testing: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
  - Convex Risk Measures (Föllmer, Schied, Fritelli, Giannin) ⇔ Variational Preferences

A Related Motivation: Controling Finance

Regulation of Financial Markets

- stress testing: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
  - Convex Risk Measures (Föllmer, Schied, Fritelli, Giannin) ⇔ Variational Preferences

A Related Motivation: Controling Finance

Regulation of Financial Markets

- stress testing: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
  - Convex Risk Measures (Föllmer, Schied, Fritelli, Giannin) ⇔ Variational Preferences

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Literature

- Bewley's papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part I in *Decisions in Economics and Finance*, 2002
  - Part II in Hildenbrand's Festschrift
- Rigotti, Shannon, *Econometrica*, 2005
- Rigotti, Shannon, Strzalecki, *Econometrica*, 2008

Model: Bewley and Savage Economies

Definition

1. Bewley economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $\mathscr{P}^i$ for agent i
2. Fix priors $Q^i \in \mathscr{P}^i$.
   Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$= complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$

Model: Bewley and Savage Economies

Definition

1. Bewley economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $\mathscr{P}^i$ for agent i
2. Fix priors $Q^i \in \mathscr{P}^i$.
   Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$= complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$

Model: Bewley and Savage Economies

Definition

1. Bewley economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $\mathscr{P}^i$ for agent i
2. Fix priors $Q^i \in \mathscr{P}^i$.
   Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$= complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q_t^i
- Consumption plans $c^i = (c_t^i(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c_t^i\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d_t^i\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega_t^i(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q_t^i
- Consumption plans $c^i = (c_t^i(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^i over d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c_t^i\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d_t^i\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega_t^i(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q^i_t
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \dots, T$
- agent i weakly prefers c^i over d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c^i_t\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d^i_t\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q^i_t
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c^i_t\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d^i_t\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q^i_t
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c^i_t\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d^i_t\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q_t^i
- Consumption plans $c^i = (c_t^i(\omega))$ for $t = 0, 1, \dots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c_t^i\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d_t^i\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega_t^i(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q^i_t
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \dots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c^i_t\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d^i_t\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q^i_t
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \dots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c^i_t\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d^i_t\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations

Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q_t^i
- Consumption plans $c^i = (c_t^i(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^iover d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c_t^i\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d_t^i\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega_t^i(\omega))$ are strictly positive
- focus on interior allocations

Efficiency in Savage Economies

Efficiency in Savage economies

- Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$
- A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

$$MRS_t^i = \frac{u_c^i(t, c_t^i)q_t^i}{u^i(c_0^i)} = \frac{u_c^j(t, c_t^j)q_t^j}{u^i(c_0^j)} = MRS_t^j$$

Efficiency in Savage Economies

Efficiency in Savage economies

- Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$
- A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

$$MRS_t^i = \frac{u_c^i(t, c_t^i)q_t^i}{u^i(c_0^i)} = \frac{u_c^j(t, c_t^j)q_t^j}{u^i(c_0^j)} = MRS_t^j$$

Efficiency in Savage Economies

Efficiency in Savage economies

- Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$
- A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the Savage economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

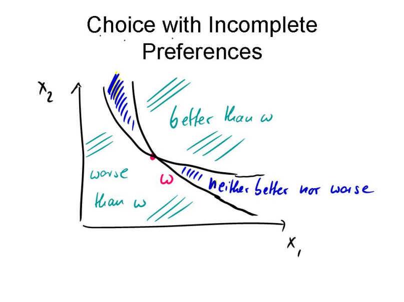
$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u^i(c_0^i)} = \frac{u_c^j(t, c_t^j) q_t^j}{u^i(c_0^j)} = MRS_t^j$$

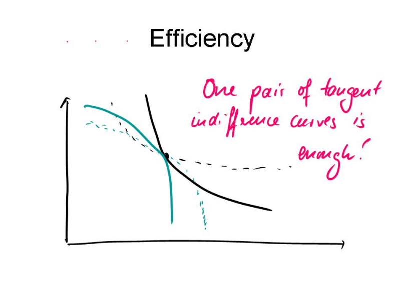
Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some Savage economy with priors Q, then c is efficient in the Bewley economy.





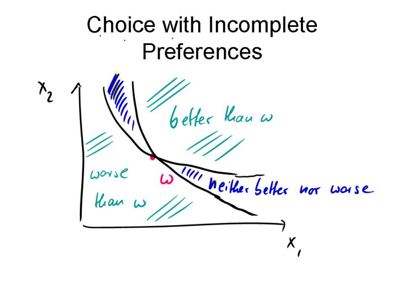
- Challenge: the converse!

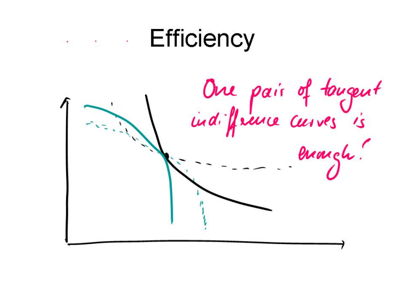
Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some Savage economy with priors Q, then c is efficient in the Bewley economy.





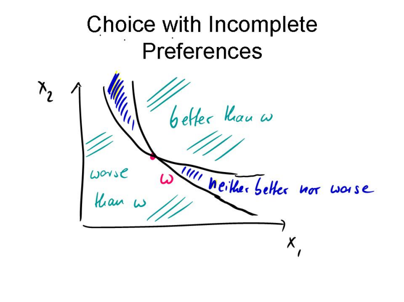
- Challenge: the converse!

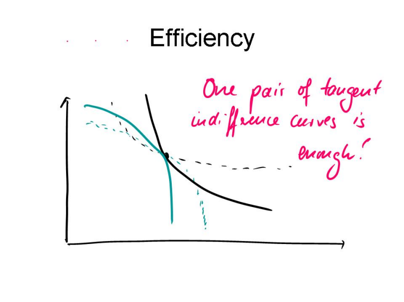
Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some Savage economy with priors Q, then c is efficient in the Bewley economy.





- Challenge: the converse!

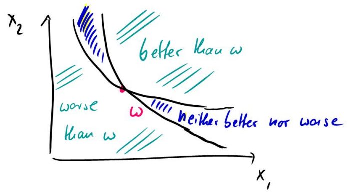
Efficiency in Bewley economies

More or less trivial:

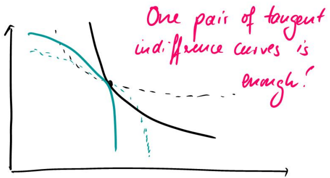
Lemma

if c is efficient in some Savage economy with priors Q, then c is efficient in the Bewley economy.

Choice with Incomplete Preferences



Efficiency



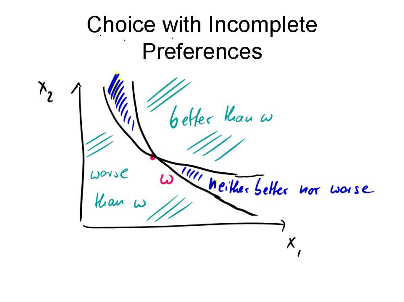
- Challenge: the converse!

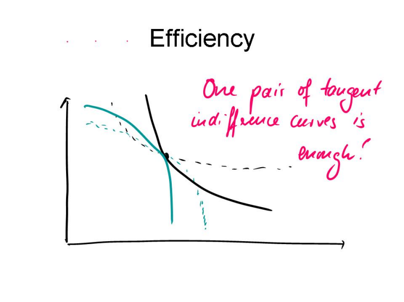
Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some Savage economy with priors Q, then c is efficient in the Bewley economy.





- Challenge: the converse!

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i)q_t^i}{u^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i\right)$$

  for a martingale M^i with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i\right)$$

  for a martingale M^i with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u^i(c_0^i)} = M_t^i \exp\left(- \sum_{s=1}^{t} r_s^i \right)$$

  for a martingale M^i with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i \right)$$

  for a martingale M^i with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i)q_t^i}{u^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i\right)$$

  for a martingale M^i with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,*
2. *the agents share a risk–adjusted prior and for a common risk–adjusted prior Q all individual interest rates are equal, i.e.*
$$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$
for all $i, j = 1, \ldots, I$ and $t = 0, \ldots, T$,
3. *for some selection of priors $Q^i \in \mathscr{P}^i$, $i = 1, \ldots, I$, c is efficient in the Savage economy with priors $Q = (Q^1, \ldots, Q^I)$.*

New version of Samet's separation theorem for L^∞.

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,*
2. *the agents share a risk–adjusted prior* and
   for a common risk–adjusted prior Q all individual interest rates are equal, i.e.
   $$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$
   for all $i, j = 1, \ldots, I$ *and* $t = 0, \ldots, T$,
3. *for some selection of priors* $Q^i \in \mathscr{P}^i$, $i = 1, \ldots, I$, *c is efficient in the Savage economy with priors* $Q = (Q^1, \ldots, Q^I)$.

Remark

New version of Samet's separation theorem for L^∞.

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,*
2. *the agents share a risk–adjusted prior* and *for a common risk–adjusted prior Q all individual interest rates are equal, i.e.*

$$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$

for all $i, j = 1, \ldots, I$ and $t = 0, \ldots, T$,

3. *for some selection of priors $Q^i \in \mathscr{P}^i$, $i = 1, \ldots, I$, c is efficient in the Savage economy with priors $Q = (Q^1, \ldots, Q^I)$.*

Remark

New version of Samet's separation theorem for L^∞.

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,*
2. *the agents share a risk–adjusted prior* and *for a common risk–adjusted prior Q all individual interest rates are equal, i.e.*
$$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$
for all $i, j = 1, \dots, I$ and $t = 0, \dots, T$,
3. *for some selection of priors $Q^i \in \mathscr{P}^i$, $i = 1, \dots, I$, c is efficient in the Savage economy with priors $Q = (Q^1, \dots, Q^I)$.*

Remark

New version of Samet's separation theorem for L^∞.

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,*
2. *the agents share a risk–adjusted prior* and
   for a common risk–adjusted prior Q all individual interest rates are equal, i.e.
   $$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$
   for all $i, j = 1, \dots, I$ and $t = 0, \dots, T$,
3. *for some selection of priors $Q^i \in \mathscr{P}^i$, $i = 1, \dots, I$, c is efficient in the Savage economy with priors $Q = (Q^1, \dots, Q^I)$.*

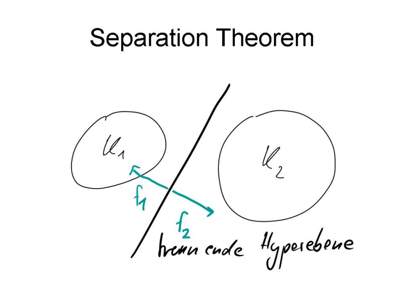
Remark

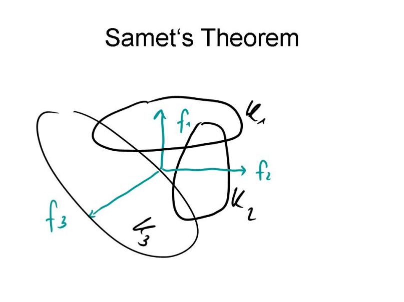
New version of Samet's separation theorem for L^∞.

Samet's Theorem

Theorem (Samet, *Games and Economic Behavior*, 1998)

Let $K_1, \ldots, K_n$ be convex, closed, nonempty subsets of Δ^m (the simplex in $\mathbb{R}^m$).
$\bigcap K_i = \emptyset$ iff there are $f_1, \ldots, f_n \in \mathbb{R}^m$ such that $\sum f_i = 0$, and $f_i \cdot x_i > 0$ for each $x_i \in K_i, i = 1, \ldots, n$.





Samet's Theorem for L^∞

Theorem

Let $(S, \mathscr{S}, P)$ be a probability space. Let $(K_i)_{i=1,\ldots,n}$ be nonempty, convex, and $\sigma(L^1, L^\infty)$–compact subsets of $L^1(S, \mathscr{S}, P)$.
Then $\bigcap K_i = \emptyset$ if and only if there exists $g_i \in L^\infty(S, \mathscr{S}, P)$ with $\sum g_i = 0$ such that $\int g_i x_i dP > 0$ for all $x_i \in K_i, i = 1, \ldots, n$.

Equilibria in Bewley economies

Corollary

Any interior equilibrium (p^*, c^*) of the Bewley economy is an interior equilibrium for some Savage economy with priors $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ and vice versa.

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*

Equilibria in Bewley economies

Corollary

Any interior equilibrium (p^*, c^*) of the Bewley economy is an interior equilibrium for some Savage economy with priors $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ and vice versa.

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*
- *in many equilibria, agents consume plans they cannot compare to their endowment: implausible!*

Equilibria in Bewley economies

Corollary

Any interior equilibrium (p^*, c^*) of the Bewley economy is an interior equilibrium for some Savage economy with priors $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ and vice versa.

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*
- *in many equilibria, agents consume plans they cannot compare to their endowment: implausible!*

Equilibria in Bewley economies

Corollary

Any interior equilibrium (p^*, c^*) *of the Bewley economy is an interior equilibrium for some Savage economy with priors* $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ *and vice versa.*

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*
- *in many equilibria, agents consume plans they cannot compare to their endowment: implausible!*

Equilibria in Bewley economies

Corollary

Any interior equilibrium (p^*, c^*) *of the Bewley economy is an interior equilibrium for some Savage economy with priors* $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ *and vice versa.*

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*
- *in many equilibria, agents consume plans they cannot compare to their endowment: implausible!*

Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q \left(\left(U^i(x) - U^i(\omega^i) \right) \right) \tag{1}$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the economy with Bewley preferences). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.

Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q\left(\left(U^i(x) - U^i(\omega^i)\right)\right) \tag{1}$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the economy with Bewley preferences). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.

Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q \left((U^i(x) - U^i(\omega^i)) \right) \tag{1}$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the economy with Bewley preferences). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.

Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q \left((U^i(x) - U^i(\omega^i)) \right) \tag{1}$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the economy with Bewley preferences). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.

Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q \left((U^i(x) - U^i(\omega^i)) \right) \quad (1)$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the economy with Bewley preferences). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy
  - both risk and uncertainty are fully insured
  - Intuition: for a pessimist, full insurance is better than uncertainty

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy
  - both risk and uncertainty are fully insured
  - intuition: for a pessimist, full insurance is better than uncertainty

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy
  - both risk and uncertainty are fully insured
  - intuition: for a pessimist, full insurance is better than uncertainty

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy
  - both risk and uncertainty are fully insured
  - intuition: for a pessimist, full insurance is better than uncertainty

Market Breakdown (No Trade Equilibria): A Case Study

Story in a Nutshell

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- equilibrium with inertia:
  - risk is completely insured
  - uncertainty is not traded at all (market breakdown, liquidity crisis)
  - Intuition: inertia and incomplete preferences avoid trade
  - under some (quite optimistic) prior, uncertainty is better than full insurance
- equilibrium in the corresponding Gilboa–Schmeidler economy
  - both risk and uncertainty are fully insured
  - intuition: for a pessimist, full insurance is better than uncertainty

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$

  for some predictable process (α_s) with values in $[-\kappa, \kappa]$ (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
-
$$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$
  for some predictable process (α_s) with values in $[-\kappa, \kappa]$
  (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
-
$$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$
for some predictable process (α_s) with values in $[-\kappa, \kappa]$ (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$

  for some predictable process (α_s) with values in $[-\kappa, \kappa]$ (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$

  for some predictable process (α_s) with values in $[-\kappa, \kappa]$ (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$

  for some predictable process (α_s) with values in $[-\kappa, \kappa]$ (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega^1_t = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$
  for some predictable process (α_s) with values in $[-\kappa, \kappa]$
  (κ–ambiguity, Epstein, Schneider)

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega^1_t = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$

  for some predictable process (α_s) with values in $[-\kappa, \kappa]$
  (κ–ambiguity, Epstein, Schneider)

Case Study: Theorem

Theorem

If ambiguity is large enough, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$x_t^1 = U_t \, .$$

The allocation is uniquely determined.

Remark

- *Risk R_t is fully insured*
- *Uncertainty U_t not traded at all*

Case Study: Theorem

Theorem

If ambiguity is large enough, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$x_t^1 = U_t \ .$$

The allocation is uniquely determined.

Remark

- *Risk R_t is fully insured*
- *Uncertainty U_t not traded at all*

Case Study: Theorem

Theorem

If ambiguity is large enough, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$x_t^1 = U_t \, .$$

The allocation is uniquely determined.

Remark

- *Risk R_t is fully insured*
- *Uncertainty U_t not traded at all*

Case Study: Theorem

Theorem

If ambiguity is large enough, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$x_t^1 = U_t\ .$$

The allocation is uniquely determined.

Remark

- *Risk R_t is fully insured*
- *Uncertainty U_t not traded at all*

Conclusion

1. General Equilibrium Analysis for Bewley's Incomplete Preference Approach
2. Link to Variational Expectations: New Existence Proof
3. Samet's Theorem for L^∞
4. Link to Regulation of Financial Markets:
   - might be wrong to use stress–testing
5. Case Study: Knightian uncertainty remains uninsured