Intertemporal Equilibria with Knightian Uncertainty

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Outline

1. Uncertainty versus Risk
2. Model
3. Efficiency and Equilibria in Bewley Economies
4. Non-Insurance of Knightian Uncertainty: A Case Study
Uncertainty versus Risk

- Roulette versus Horse Races

- objective probability versus no probabilities, just uncertain outcomes

- $(\Omega, \mathcal{F}, P)$ probability space versus $(S, \mathcal{I})$ measurable space,
  $X : (S, \mathcal{I}) \to \mathbb{R}$

Savage, Anscombe–Aumann
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Uncertainty II: Ellsberg

Asking for exact subjective probabilities too demanding

Example

Imagine a Cologne soccer fan. He has the choice between two bets. Situation 1:

- SF Giants win the World Series
- Der 1. FC Köln wird Pokalsieger 2012.

Situation 2:

- SF Giants do not win the World Series
- Der 1. FC Köln wird nicht Pokalsieger 2012

It is perfectly rational to go for the second bet in both cases; but this would contradict the additivity of probability.
Knight (1921): many economic decisions are of a one–shot nature and one cannot presume probabilities

- Probability fairly well known for
  - Car Insurance
  - Life Insurance (Mortality Risk)
  - “IBM”

- Probability less clear for
  - market entry
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Uncertainty IV: Two Formal Models

As “P” is not exactly known, work with a whole class of probability measures \( P \), (Huber, 1982, Robust Statistics)

Two Models

- **Pessimistic Multiple Priors (Gilboa–Schmeidler):**
  - complete preferences, pessimistic approach:
    \[ U(X) = \min_{P \in \mathcal{P}} E_P u(x) \]
  - Föllmer–Schied, Maccheroni, Marinacci, Rustichini generalize to variational preferences
    \[ U(X) = \min_{P} E_P u(X) + c(P) \]
    for a cost function \( c \)
  - special case: Hansen, Sargent, \( c(P) = E_P \log dP/dQ \) relative entropy with respect to a reference measure \( Q \)
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Uncertainty versus Risk Model Efficiency Breakdown

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Model: Bewley and Savage Economies

Definition

1. B–economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $P^i$ for agent $i$

2. Fix priors $Q^i \in P^i$.
   S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$= complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$

3. S for Savage, not a risk economy!
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- Priors admit densities with respect to a reference measure $P^0$
- Agents agree on null sets
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Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \ldots, T$

Agent $i$ weakly prefers $c^i$ over $d^i$ iff

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Efficiency in Savage Economies

- Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$
- A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

$$MRS^i_t = \frac{u^i_c(t, c^i_t)q^i_t}{u^i_c(c^i_0)} = \frac{u^j_c(t, c^j_t)q^j_t}{u^j_c(c^j_0)} = MRS^j_t$$
Efficiency in Savage Economies

Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$

A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

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More or less trivial:

**Lemma**

If $c$ is efficient in some $S$–economy with priors $Q$, then $c$ is efficient in the $B$–economy.

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Challenge: the converse!
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**MRS=Risk–Adjusted Prior + Subjective Interest Rate**

- Every MRS can be written as

\[ MRS_t^i = \frac{u^i_c(t, c_t^i)q_t^i}{u^i(c_0^i)} = M_t^i \exp \left( -\sum_{s=1}^{t} r_s^i \right) \]

for a martingale \( M^i \) with expectation 1 and a subjective interest rate \( r^i \)

- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- \( M^i \) density process of a new measure, the risk–adjusted prior or equivalent martingale measure
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An interior allocation $c$ is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. the agents’ share a common marginal rate of substitution,
2. the agents share a risk–adjusted prior and for a common risk–adjusted prior $Q$ all individual interest rates are equal, i.e.
   \[ r^i(Q, c^i)_t = r^j(Q, c^j)_t \]
   for all $i, j = 1, \ldots, l$ and $t = 0, \ldots, T$,
3. for some selection of priors $Q^i \in \mathcal{P}^i, i = 1, \ldots, l$, $c$ is efficient in the Savage economy with priors $Q = (Q^1, \ldots, Q^l)$. 
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Samet’s Theorem

Theorem (Samet, Games and Economic Behavior, 1998)

Let $K_1, \ldots, K_n$ be convex, closed, nonempty subsets of $\Delta^m$ (the simplex in $\mathbb{R}^m$).

$\bigcap K_i = \emptyset$ iff there are $f_1, \ldots, f_n \in \mathbb{R}^m$ such that $\sum f_i = 0$, and $f_i \cdot x_i > 0$ for each $x_i \in K_i, i = 1, \ldots, n$. 
Samet’s Theorem for $L^\infty$

**Theorem**

Let $(S, \mathcal{I}, P)$ be a probability space. Let $(K_i)_{i=1,...,n}$ be nonempty, convex, and $\sigma(L^1(S, \mathcal{I}, P), L^\infty(S, \mathcal{I}, P))$-compact subsets of $\Delta = \{ D \in L^1_+(S, \mathcal{I}, P) : E D = 1 \}$. Then $\bigcap K_i = \emptyset$ if and only if there exists $g_i \in L^\infty(S, \mathcal{I}, P)$ with $\sum g_i = 0$ such that $\int g_i x_i dP > 0$ for all $x_i \in K_i$, $i = 1, \ldots, n$. 
Corollary

Any interior equilibrium \((p^*, c^*)\) of the Bewley economy is an interior equilibrium for some Savage economy with priors \(Q^i \in \mathcal{P}^i, i = 1, \ldots, I\) and vice versa.

Remark

- Huge number of equilibria if uncertainty is nontrivial
- Indeterminacy (compare Rigotti–Shannon)
Equilibria in Bewley economies

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Equilibria with Inertia: Existence and Variational Preferences

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer $c^i$ over $\omega^i$ under all $P \in \mathcal{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level $\omega^i$

$$V^i(x) = \min_{Q \in \mathcal{P}^i} E^Q \left( (U^i(x) - U^i(\omega^i)) \right)$$

Theorem

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the B–economy). In particular, equilibria with inertia exist.

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Such variational preferences are Mackey–continuous.
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**Story in a Nutshell**

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- aggregate endowment is zero
- agent 1 has endowment \( \omega_t^1 = R_t + U_t \)
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- \( R_t = \sum_{s=1}^{t} \varepsilon_s, \quad \varepsilon_s \sim N(0,1), \text{ i.i.d.} \)
- \( U_t = \sum_{s=1}^{t} \nu_s, \) (\( \nu_t \)) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors

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q_t = \exp \left( \sum_{s=1}^{t} \left( \alpha_s \nu_s - \frac{1}{2} \alpha_s^2 \right) \right)
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for some \( U \)–predictable process \( (\alpha_s) \) with values in \([-\kappa, \kappa]\)
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$$q_t = \exp \left( \sum_{s=1}^{t} \left( \alpha_s \nu_s - \frac{1}{2} \alpha_s^2 \right) \right)$$

for some $U$–predictable process $(\alpha_s)$ with values in $[-\kappa, \kappa]$

why only $U$–predictable? Agents agree on independence of $R$ and $U$ under all priors!
Case Study: Details

- two agents with CARA utility, \( u(x) = -\exp(-x) \)
- aggregate endowment is zero
- agent 1 has endowment \( \omega_1 = R_t + U_t \)
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hence an equilibrium in the B–economy

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Intuition: every agent finds one very optimistic prior where she prefers uncertainty over insurance

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Theorem

*The above Bewley economy has an equilibrium with inertia in which agent 1 consumes*

\[ x_t^1 = U_t. \]

*The equilibrium price is*

\[ p_t^* = \exp \left( - \left( \rho + \frac{1}{2} \right) t \right). \]
Case Study: Equilibrium, Uniqueness

Remark

- Risk $R_t$ is fully insured
- Uncertainty $U_t$ not traded at all
- No uniqueness, however

- Uncertainty can be traded, but not “too much”
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Regulation of Financial Markets

Regulation can be interpreted as “imposing preferences”

- **stress testing**: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- **worst–case approach**: compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better than the worst–case outcome of the status–quo

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A Related Motivation: Control of Investment Banks

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Conclusion

1. General Equilibrium Analysis for Bewley’s Incomplete Preference Approach
2. Link to Variational Expectations
3. Samet’s Theorem for $L^\infty$
4. Link to Regulation of Financial Markets:
   - Regulation is a way to impose preferences on banks
   - imposing “objective” (incomplete + inertia) preferences might lead to market breakdown
   - argument in favor of “subjective” (complete, pessimistic) preferences
5. Case Study: Knightian uncertainty remains uninsured
Case Study: Computations

Note that

\[-E \exp (-U_t + \alpha U_t - \alpha^2 / 2t) = -\exp ((1/2 - \alpha) t)\]

Hence, agent 1 prefers $U$ to 0 for $\alpha > 1/2$ and prefers full insurance to $U$ for $\alpha < 1/2$. So, full insurance is not better than keeping $U$. 