CORRIGENDUM

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Our article on "Order statistics for Value at Risk estimation and option pricing" [Wilmott Magazine no. 26 (2006), pp. 46-50] contains two unfortunate errors, which, however, do not affect the qualitative results or conclusion of our paper.

First, the right hand side of the first equation on p. 49 is in fact a formula for the expectation $\mathbb{E}\left[X_{(k)}+X_{(k+1)}, \left\{X_{(k)}+X_{(k+1)}<z\right\}\right]$ (where, moreover, the integration limits $\int_{-\infty}^{z}$ should be replaced by $\int_{-\infty}^{\infty}$).

The correct formula for the distribution of $X_{(k)}+X_{(k+1)}$ is significantly simpler. Using the abbreviation $C := n(n-1)\binom{n-2}{k-1} = \frac{n!}{(k-1)!(n-k-1)!}$, equation (11) yields for any bounded measurable function $g:\mathbb{R}\to\mathbb{R}$:

$$\begin{aligned}
&\mathbb{E}\left[g\left(X_{(k)}+X_{(k+1)}\right)\right] = \int_{\mathbb{R}} g(z)\mathbb{P}\left\{X_{(k)}+X_{(k+1)}\in dz\right\}\\
=&C\int_{-\infty}^{\infty}\int_{y=x}^{y=\infty} g(x+y)f(x)F(x)^{k-1}f(y)\left(1-F(y)\right)^{n-k-1}dy\ dx\\
=&C\int_{-\infty}^{\infty}\int_{z=2x}^{z=\infty} g(z)f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}dz\ dx\\
=&C\int_{-\infty}^{\infty} g(z)\int_{-\infty}^{z/2} f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}dx\ dz.
\end{aligned}$$

This, however, is tantamount to

$$\begin{aligned}
&\mathbb{P}\left\{X_{(k)}+X_{(k+1)}\in dz\right\}\\
=&C\int_{-\infty}^{z/2} f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}dx\ dz
\end{aligned}$$

and thus for all $w\in\mathbb{R}$,

$$\begin{aligned}
&\mathbb{P}\left\{X_{(k)}+X_{(k+1)}<w\right\}\\
=&C\int_{-\infty}^{w}\int_{-\infty}^{z/2} f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}dx\ dz.
\end{aligned}$$

Consequently, the formulæ for the mean and variance of the implied VaR level of the estimator $\frac{X_{(k)}+X_{(k+1)}}{2}$ also need to be corrected:

$$\begin{aligned}
&\mathbb{E}\left[F\left(\frac{X_{(k)}+X_{(k+1)}}{2}\right)\right] = \int_{-\infty}^{\infty} F(z/2)\mathbb{P}\left\{X_{(k)}+X_{(k+1)}\in dz\right\}\\
=&C\int_{-\infty}^{\infty} F(z/2)\int_{-\infty}^{z/2} f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}\ dx\ dz,
\end{aligned}$$

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and

$$\begin{aligned}
&\mathbb{V}\left[\frac{X_{(k)}+X_{(k+1)}}{2}\right]\\
&=\int_{-\infty}^{\infty}\frac{z^2}{4}\mathbb{P}\left\{X_{(k)}+X_{(k+1)}\in dz\right\}-\left(\int_{-\infty}^{\infty}\frac{z}{2}\mathbb{P}\left\{X_{(k)}+X_{(k+1)}\in dz\right\}\right)^2\\
=&\frac{C}{4}\int_{-\infty}^{\infty}z^2\int_{-\infty}^{z/2}f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}\ dx\ dz\\
&-\frac{C^2}{4}\left(\int_{-\infty}^{\infty}z\int_{-\infty}^{z/2}f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}\ dx\ dz\right)^2.
\end{aligned}$$

The results on the right half of p. 49 are unaffected by this. For example, if the X_i are uniformly distributed on $[0,1]$, one has $F=\mathrm{id}$, hence $\mathbb{E}\left[F\left(\frac{X_{(k)}+X_{(k+1)}}{2}\right)\right]=\frac{\mathbb{E}\left[X_{(k)}\right]+\mathbb{E}\left[X_{(k+1)}\right]}{2}$ which equals $\frac{1}{2}\frac{2k+1}{n+1}$ by equation (6).

Secondly, Equation (10) is erroneous and has to be rewritten as follows:

$$\begin{aligned}
&\mathbb{E}\left[\frac{X_{(k)}+X_{(k+1)}}{2}\right]=\frac{\mathbb{E}\left[X_{(k)}\right]+\mathbb{E}\left[X_{(k+1)}\right]}{2}\\
=&\frac{n}{2}\int_{-\infty}^{\infty}xf(x)F(x)^{k-1}\left(1-F(x)\right)^{n-k-1}\left(\binom{n-1}{k-1}\left(1-F(x)\right)+\binom{n-1}{k}F(x)\right)dx\\
=&\frac{n}{2}\binom{n-1}{k-1}\int_{-\infty}^{\infty}xf(x)F(x)^{k-1}\left(1-F(x)\right)^{n-k-1}\left(1+\left(\frac{n-k}{k}-1\right)F(x)\right)dx
\end{aligned}$$

(using $\binom{n-1}{k}/\binom{n-1}{k-1}=\frac{n-k}{k}$).

Consequently, the second term on the right-hand side of the last equation in the left column on p. 49 has to be changed; the equation now reads:

$$\begin{aligned}
&\mathbb{V}\left[\frac{X_{(k)}+X_{(k+1)}}{2}\right]\\
=&\frac{n(n-1)}{4}\binom{n-2}{k-1}\int_{\mathbb{R}}\begin{pmatrix}z^3\int_{-\infty}^{z}f(x)F(x)^{k-1}f(z-x)\left(1-F(z-x)\right)^{n-k-1}dx\\+z^2f(z)F(z)^{k-1}\int_{z}^{0}(z+y)f(y)\left(1-F(y)\right)^{n-k-1}dy\end{pmatrix}dz\\
&-\left|\frac{n}{2}\binom{n-1}{k-1}\int_{-\infty}^{\infty}xf(x)F(x)^{k-1}\left(1-F(x)\right)^{n-k-1}\left(1+\left(\frac{n-k}{k}-1\right)F(x)\right)dx\right|^2
\end{aligned}$$

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