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The Meaning of a First-Order Formula, Compositionality and Alphabetic Innocence
Short Abstract

We focus on the concept of the meaning of a first-order formula. Initially, we discriminate between global and local approaches towards explicating the notion of the meaning of a first-order formula. The former approach derives from abstract model theory. Whereas the latter identifies the meaning of a first-order formula in a structure with a relation defined over the universe at hand. Primarily, local approaches to the meaning of a first-order formula stem from the algebraization of satisfaction relations. We single out the algebraization of the standard satisfaction relation for ordinary first-order languages relatively to the class of cylindric set algebras of dimension α. We survey the relevant results from this field and discuss two complaints raised in the literature. The first complaint is the so called representationalism. The second charge is the lack of alphabetic innocence. We find the official formulation of representationalism flawed. However, we point that it can be reinstated to a degree in our settings. Next, we direct attention to principle of alphabetic innocence. Admittedly, it is not a well-known semantic principle. Nonetheless, it is endorsed by couple of authors. We provide motivations underpinning alphabetic innocence and define it. Moreover, we argue that it can be interpreted as requiring invariance of meaning assignment under the group of permutation of variables. We state the following precise question " Is it possible to define an alphabetically innocent semantics for ordinary first-order languages ? ". We settle this issue positively by resorting to the well-known concept of definability in a structure.

The construction of our semantics proceeds in few stages. Firstly, we stress that there is no unique way to represent a formula in a finite variable context. We distinguish between orderings of variables parasitic on canonical ordering of variables and ordering suggested by the structure of a formula. We employ the latter ordering and define the operation of bijective simultaneous replacement of free and bound occurrences of variables in a formula. Subsequently, we state the substitution lemma for this operation. Finally, we define Hodges′ style semantics for ordinary first-order languages and prove by an appeal to our substitution lemma that this semantics is alphabetically innocent. Our semantics is non-inductively defined and consequently non-compositional.

Finally, we discuss some consequences of our construction. Firstly, we refute the claim that under an alphabetically innocent semantics it is impossible to distinguish within a language a relation and its converse. Secondly, we show that our semantics provides fine-grained criteria of the identity of meaning. There exist logically equivalent formulas (in the standard Tarskian sense) which are non-synonymous.