

No-three-in-line for forty-seven and forty-nine

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Abstract

Examples are presented where 94 points of a 47 by 47 square, and 98 points of a 49 by 49 square of lattice points are selected with no three of them in a straight line.

It is an old problem to ask for the maximum number $D(n)$ of lattice points which can be selected from n by n points of the unit lattice such that no three are in a straight line [4, 8]. For large n it is conjectured $D(n) < cn$ with $c = \pi/\sqrt{3} \approx 1.81\dots$ [3]. Recent extensions of the problem include generalizations to higher dimensions [7] and to the infinite grid [6].

Examples proving $D(n) = 2n$ are known for $n \leq 46$, and for $n = 48, 50, 52$ [1, 2]. Early work by this author [5] indicated that using symmetry to reduce the search space was a sensible strategy, which was also key in the later works [1, 2], where known solutions for large n had 90-degree rotational symmetry for n even, or 90-degree rotational symmetry except for the diagonal for n odd (augmented by two symmetric points on one diagonal).

As result of a computer search using a constraint satisfaction approach [9], new solutions with $D(n) = 2n$ are presented for $n = 43, 44, \dots, 50$. With the newly found solutions presented below, the minimum n with $D(n) < 2n$ is at least 51 [10].

1 New Solutions for $n = 43, 44, \dots, 50$

No-three-in-line solutions with $2n$ points on the grid $[0, \dots, n-1]^2$ for $n = 43$ to 50 are as follows. Python code to verify their validity is given in the Appendix.

43. [(42, 26), (16, 42), (26, 0), (0, 16), (0, 24), (18, 0), (42, 18), (24, 42), (26, 41), (1, 26), (41, 16), (16, 1), (11, 1), (31, 41), (1, 31), (41, 11), (12, 2), (30, 40), (40, 12), (2, 30), (33, 40), (2, 33), (40, 9), (9, 2), (39, 23), (3, 19), (19, 39), (23, 3), (36, 39), (39, 6), (3, 36), (6, 3), (12, 38), (30, 4), (38, 30), (4, 12), (4, 15), (15, 38), (27, 4), (38, 27), (25, 5), (5, 17), (17, 37), (37, 25), (37, 6), (36, 37), (6, 5), (5, 36), (22, 7), (20, 35), (35, 22), (7, 20), (7, 32), (35, 10), (10, 7), (32, 35), (8, 13), (34, 29), (13, 34), (29, 8), (34, 15), (8, 27), (27, 34), (15, 8), (10, 9), (32, 33), (33, 10), (9, 32), (25, 11), (31, 25), (17, 31), (11, 17), (18, 13), (29, 18), (24, 29), (13, 24), (28, 28), (14, 14), (28, 22), (20, 28), (22, 14), (14, 20), (23, 21), (19, 21), (21, 19), (21, 23)]
44. [(0, 13), (13, 43), (30, 0), (43, 30), (0, 23), (43, 20), (20, 0), (23, 43), (1, 23), (20, 1), (23, 42), (42, 20), (27, 42), (42, 16), (1, 27), (16, 1), (14, 2), (41, 14), (2, 29), (29, 41), (30, 41), (13, 2), (41, 13), (2, 30), (11, 40), (32, 3), (40, 32), (3, 11), (25, 40), (3, 25), (40, 18), (18, 3), (37, 4), (4, 6), (39, 37), (6, 39), (39, 31), (12, 39), (4, 12), (31, 4), (25, 5), (18, 38), (38, 25), (5, 18), (5, 21), (21, 38), (22, 5), (38, 22), (7, 37), (6, 7), (36, 6), (37, 36), (7, 32), (11, 7), (36, 11), (32, 36), (17, 8), (8, 26), (26, 35), (35, 17), (8, 31), (31, 35), (12, 8), (35, 12), (34, 27), (9, 16), (27, 9), (16, 34), (34, 15), (28, 34), (9, 28), (15, 9), (10, 21), (21, 33), (22, 10), (33, 22), (19, 10), (33, 19), (24, 33), (10, 24), (29, 15), (28, 29), (15, 14), (14, 28), (24, 26), (17, 24), (26, 19), (19, 17)]

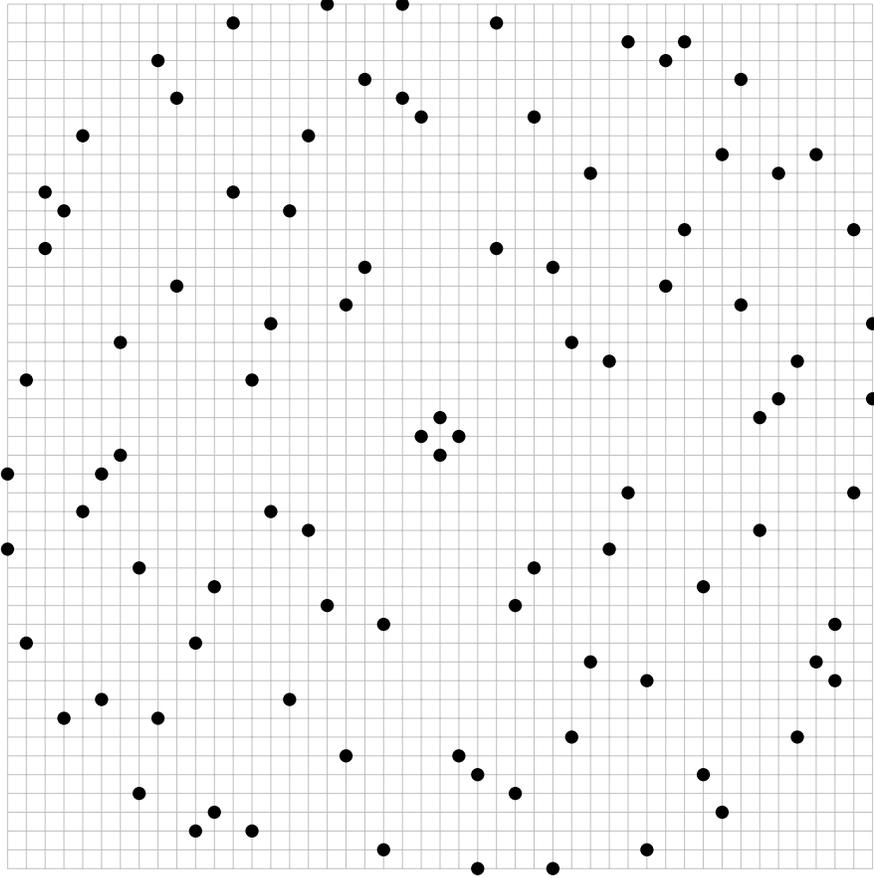


Figure 1: $2n$ lattice points, no three in line, for $n = 47$.

45. [(0, 7), (7, 44), (37, 0), (44, 37), (44, 23), (23, 0), (21, 44), (0, 21), (1, 17), (43, 27), (17, 43), (27, 1), (32, 43), (12, 1), (43, 12), (1, 32), (2, 10), (42, 34), (34, 2), (10, 42), (16, 42), (2, 16), (28, 2), (42, 28), (33, 41), (11, 3), (3, 33), (41, 11), (41, 6), (38, 41), (6, 3), (3, 38), (40, 25), (25, 4), (4, 19), (19, 40), (4, 26), (18, 4), (26, 40), (40, 18), (36, 5), (39, 36), (8, 39), (5, 8), (28, 39), (5, 28), (16, 5), (39, 16), (38, 23), (23, 6), (21, 38), (6, 21), (37, 32), (7, 12), (32, 7), (12, 37), (8, 8), (36, 36), (29, 9), (15, 35), (35, 29), (9, 15), (35, 13), (13, 9), (31, 35), (9, 31), (30, 34), (14, 10), (10, 30), (34, 14), (17, 33), (27, 11), (33, 27), (11, 17), (13, 30), (14, 13), (30, 31), (31, 14), (24, 15), (20, 29), (29, 24), (15, 20), (25, 26), (19, 18), (26, 19), (18, 25), (20, 22), (22, 20), (24, 22), (22, 24)]
46. [(45, 31), (31, 0), (0, 14), (14, 45), (0, 23), (23, 45), (22, 0), (45, 22), (38, 1), (7, 44), (1, 7), (44, 38), (1, 13), (13, 44), (44, 32), (32, 1), (2, 16), (43, 29), (16, 43), (29, 2), (2, 26), (43, 19), (19, 2), (26, 43), (42, 19), (19, 3), (3, 26), (26, 42), (3, 29), (16, 3), (42, 16), (29, 42), (41, 24), (4, 21), (24, 4), (21, 41), (33, 41), (12, 4), (4, 33), (41, 12), (40, 38), (38, 5), (7, 40), (5, 7), (5, 15), (15, 40), (30, 5), (40, 30), (39, 12), (12, 6), (6, 33), (33, 39), (39, 11), (34, 39), (11, 6), (6, 34), (8, 18), (27, 8), (18, 37), (37, 27), (13, 8), (32, 37), (8, 32), (37, 13), (9, 21), (36, 24), (24, 9), (21, 36), (35, 36), (9, 35), (10, 9), (36, 10), (10, 28), (35, 17), (28, 35), (17, 10), (30, 11), (15, 34), (11, 15), (34, 30), (14, 18), (31, 27), (27, 14), (18, 31), (25, 28), (20, 17), (17, 25), (28, 20), (25, 22), (20, 23), (22, 20), (23, 25)]
47. [(46, 29), (17, 46), (29, 0), (0, 17), (25, 0), (46, 25), (0, 21), (21, 46), (1, 12), (12, 45), (45, 34), (34, 1), (20, 1), (26, 45), (1, 26), (45, 20), (13, 2), (44, 13), (2, 33), (33, 44), (10, 2), (44, 10), (36, 44), (2, 36), (3, 8), (8, 43), (43, 38), (38, 3), (3, 35), (35, 43), (11, 3), (43, 11),

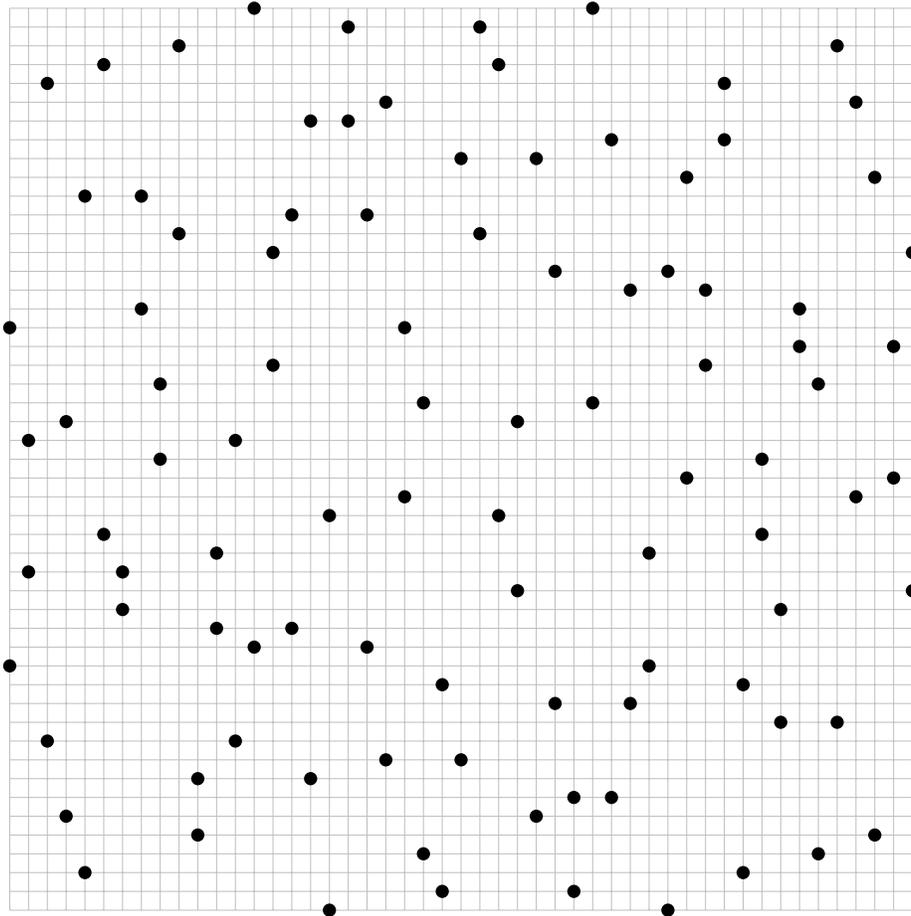


Figure 2: $2n$ lattice points, no three in line, for $n = 49$.

- (42, 27), (19, 42), (27, 4), (4, 19), (7, 4), (4, 39), (39, 42), (42, 7), (41, 37), (5, 9), (37, 5), (9, 41), (25, 5), (5, 21), (41, 25), (21, 41), (24, 6), (40, 24), (6, 22), (22, 40), (28, 40), (18, 6), (40, 18), (6, 28), (30, 7), (7, 16), (39, 30), (16, 39), (8, 8), (38, 38), (37, 15), (31, 37), (9, 31), (15, 9), (36, 34), (34, 10), (10, 12), (12, 36), (31, 11), (35, 31), (15, 35), (11, 15), (26, 33), (13, 26), (33, 20), (20, 13), (27, 14), (14, 19), (19, 32), (32, 27), (14, 29), (29, 32), (32, 17), (17, 14), (28, 16), (30, 28), (18, 30), (16, 18), (23, 22), (23, 24), (24, 23), (22, 23)]
48. [(0, 12), (35, 0), (47, 35), (12, 47), (0, 23), (24, 0), (47, 24), (23, 47), (1, 11), (46, 36), (36, 1), (11, 46), (28, 1), (1, 19), (46, 28), (19, 46), (45, 43), (2, 4), (4, 45), (43, 2), (45, 17), (17, 2), (30, 45), (2, 30), (30, 3), (44, 30), (17, 44), (3, 17), (3, 35), (35, 44), (44, 12), (12, 3), (43, 14), (4, 33), (33, 43), (14, 4), (20, 5), (27, 42), (5, 27), (42, 20), (37, 42), (5, 37), (10, 5), (42, 10), (41, 40), (6, 7), (40, 6), (7, 41), (25, 41), (6, 25), (41, 22), (22, 6), (7, 26), (40, 21), (26, 40), (21, 7), (8, 14), (33, 8), (14, 39), (39, 33), (31, 8), (16, 39), (39, 31), (8, 16), (27, 9), (38, 27), (9, 20), (20, 38), (13, 9), (34, 38), (9, 34), (38, 13), (11, 37), (10, 11), (36, 10), (37, 36), (29, 34), (34, 18), (13, 29), (18, 13), (24, 32), (15, 24), (32, 23), (23, 15), (26, 32), (32, 21), (21, 15), (15, 26), (19, 16), (31, 19), (28, 31), (16, 28), (22, 18), (25, 29), (29, 22), (18, 25)]
49. [(0, 13), (35, 0), (48, 35), (13, 48), (17, 0), (0, 31), (31, 48), (48, 17), (1, 18), (47, 30), (18, 47), (30, 1), (25, 47), (23, 1), (1, 25), (47, 23), (2, 9), (9, 46), (39, 2), (46, 39), (44, 46), (2, 44), (46, 4), (4, 2), (45, 43), (43, 3), (5, 45), (3, 5), (26, 45), (3, 26), (22, 3), (45, 22), (4, 38), (10, 4), (38, 44), (44, 10), (5, 20), (28, 5), (43, 28), (20, 43), (42, 32), (16, 42), (32, 6),

(6, 16), (30, 6), (6, 18), (42, 30), (18, 42), (7, 32), (16, 7), (41, 16), (32, 41), (10, 7), (7, 38), (38, 41), (41, 10), (8, 24), (24, 8), (40, 24), (24, 40), (40, 20), (28, 40), (20, 8), (8, 28), (36, 39), (39, 12), (9, 36), (12, 9), (33, 11), (37, 33), (11, 15), (15, 37), (11, 19), (19, 37), (29, 11), (37, 29), (36, 23), (23, 12), (25, 36), (12, 25), (13, 14), (35, 34), (14, 35), (34, 13), (14, 29), (19, 14), (29, 34), (34, 19), (33, 33), (15, 15), (21, 31), (31, 27), (27, 17), (17, 21), (26, 21), (27, 26), (21, 22), (22, 27)]

50. [(0, 18), (18, 49), (49, 31), (31, 0), (17, 0), (32, 49), (0, 32), (49, 17), (22, 1), (27, 48), (48, 22), (1, 27), (1, 39), (48, 10), (10, 1), (39, 48), (19, 47), (30, 2), (47, 30), (2, 19), (47, 23), (2, 26), (23, 2), (26, 47), (21, 3), (28, 46), (46, 21), (3, 28), (38, 46), (11, 3), (3, 38), (46, 11), (14, 45), (35, 4), (45, 35), (4, 14), (4, 43), (43, 45), (6, 4), (45, 6), (19, 5), (44, 19), (30, 44), (5, 30), (5, 33), (16, 5), (44, 16), (33, 44), (6, 40), (40, 43), (43, 9), (9, 6), (7, 12), (37, 7), (12, 42), (42, 37), (7, 15), (42, 34), (34, 7), (15, 42), (41, 35), (14, 41), (8, 14), (35, 8), (28, 8), (41, 28), (8, 21), (21, 41), (9, 10), (40, 39), (10, 40), (39, 9), (38, 24), (11, 25), (24, 11), (25, 38), (18, 37), (31, 12), (12, 18), (37, 31), (16, 36), (33, 13), (13, 16), (36, 33), (36, 17), (13, 32), (17, 13), (32, 36), (15, 27), (34, 22), (22, 15), (27, 34), (29, 20), (20, 29), (20, 20), (29, 29), (24, 23), (26, 24), (23, 25), (25, 26)]

Python Code for Configuration Testing

```
def testing(pts):
    import math
    import itertools
    def linekey(point1, point2):
        a=point2[1]-point1[1]; b=point1[0]-point2[0]
        c=point1[1]*point2[0]-point1[0]*point2[1]
        f=math.gcd(a,b,c)
        f=1 if f==0 else -f if a<0 else f if a>0 else -f if b<0\
            else f if b>0 else -f if c<0 else f
        return (a//f,b//f,c//f)
    lines=set()
    for point1,point2 in itertools.combinations(pts,2):
        l=linekey(point1,point2)
        if l in lines:
            break
        else:
            lines.add(l)
    return len(lines)==len(pts)*(len(pts)-1)//2
```

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